Can you hear the shape of a drum ? and Deformational Spectral Rigidity

V. Kaloshin

February 7, 2019

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- M. Kac 'Can you hear the shape of a drum?'
- Laplace spectrum, Inverse problems
- Length spectrum and Laplace spectrum
- Deformational Spectral Rigidity and Main Results
- Ideas of proofs

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Can you hear the shape of a drum?

M. Kac'66: Can you hear the shape of a drum?

Can you hear the shape of a drum?

Consider the Dirichlet problem in a domain $\Omega \subset \mathbb{R}^2$.

$$\begin{cases} \Delta u + \lambda^2 u = 0 \\ u|_{\partial\Omega} = 0. \end{cases}$$

$\Delta(\Omega) := \{0 < \lambda_1 \le \lambda_2 \le \cdots\}$ — Laplace spectrum.

Example 1 Let $\Omega_C = [0, \pi] \times [0, \pi] \ni (x, y)$. For any pair $k, m \in \mathbb{Z}_+ \setminus 0$ let

$$u(x, y) = \sin kx \cdot \sin my \quad \text{and} \quad \lambda = \sqrt{k^2 + m^2}.$$

The Laplace spectrum $\Delta(\Omega_C) = \cup_{k,m \in \mathbb{Z}_+ \setminus 0} \sqrt{k^2 + m^2}$.

Question (M. Kac'66) Does $\Delta(\Omega)$ determine Ω up to isometry?

Weyl law (H. Weyl'11) $N(\lambda) := \#$ eigenvalues (w multiplicity) in $(0, \lambda^2]$, then

$$\lim_{\lambda \to \infty} \lambda^{-1} N(\lambda) = (4\pi)^{-1} \text{Area } (\Omega).$$

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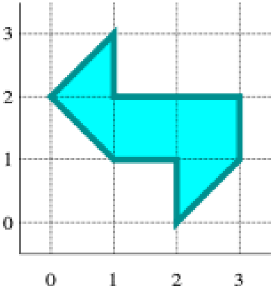
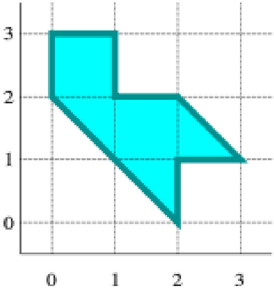
Gordon–Webb-Wolpert'92





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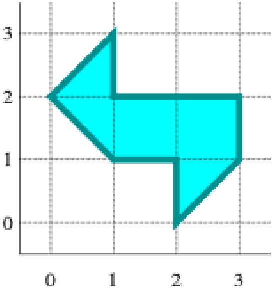

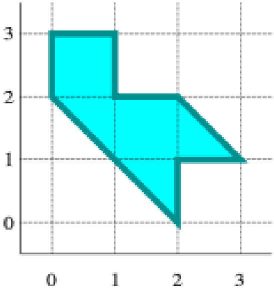

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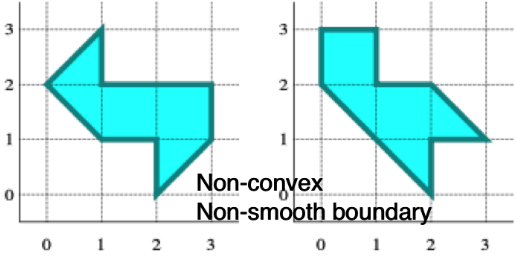

Consider domains with a smooth or an analytic boundary!

Osgood-Phillips-Sarnak A C^∞ isospectral set is compact.

Conjecture (Sarnak’90) A C^∞ isospectr. set consists of isolated points.

Hezari-Zeldich, Popov-Topalov Analytic deformations of ellipses.
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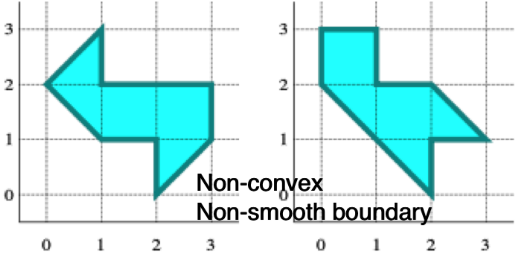

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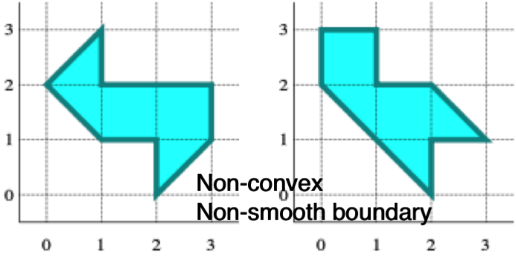

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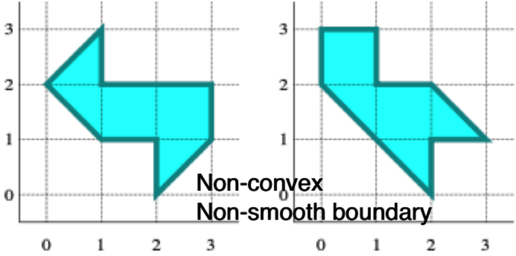

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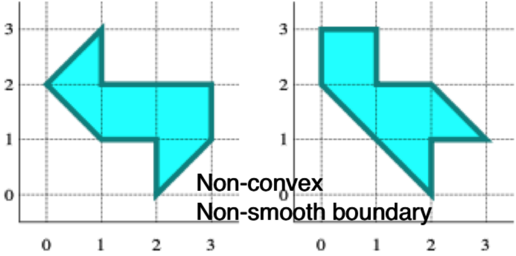

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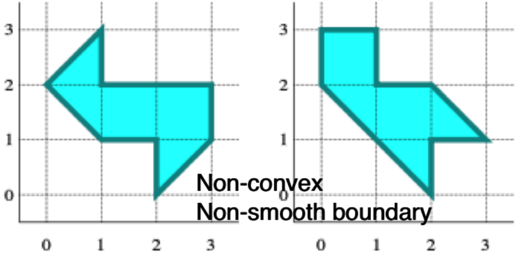

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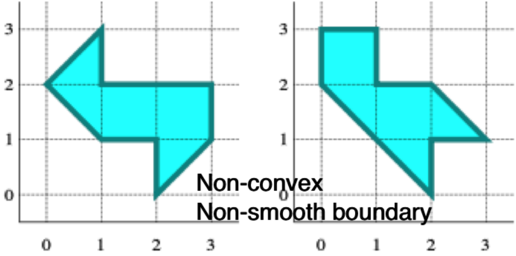

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Can you hear the shape of a Riemannian manifold?

Let (M, g) be a Riemannian compact manifold. Consider the spectrum of the Laplace-Beltrami operator $\Delta(M, g)$.

Question Does $\Delta(M, g)$ determine (M, g) up to an isometry?

Sunada, Vingeras* $\exists$ isospectral sets of arbitrary finite cardinality.

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Call Ω *spectrally rigid* (SR) if any smooth isopectral deformation $\{\Omega_t\}_t$ is an isometry, i.e. $\Delta(\Omega_t) \equiv \Delta(\Omega_0)$.

Conjecture (Sarnak'90) Any planar domain is SR.

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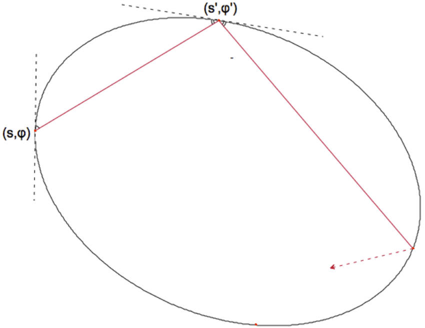
Conjecture (Sarnak'90) Any planar domain is SR.

Length spectrum

Let $\Omega \subset \mathbb{R}^2$ be a strictly convex domain. Define

$$\textit{the length spectrum} \quad \mathcal{L}(\Omega) := \cup_P L(P) \ \cup \mathbb{N}\, L(\partial\Omega),$$

$L(P)$ – perimeter of a periodic orbit, $\cup$ – over all per orbits.



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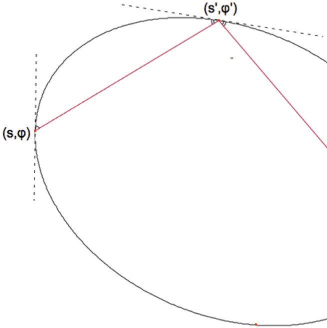

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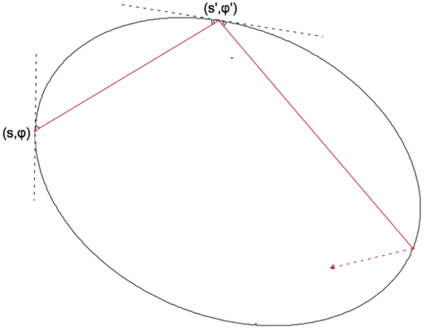

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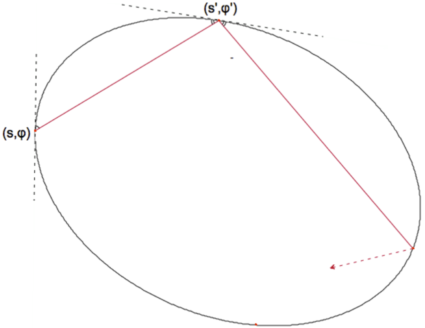

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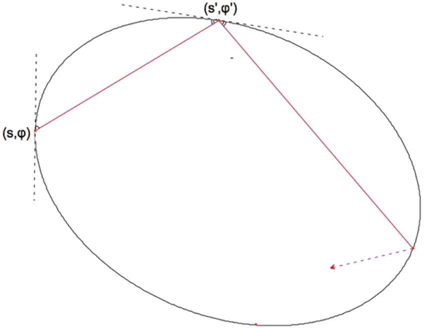

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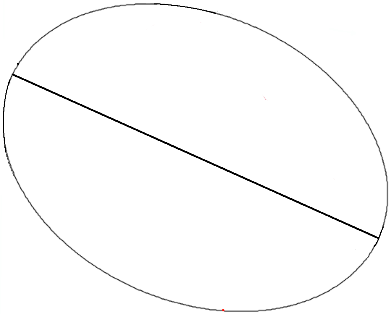

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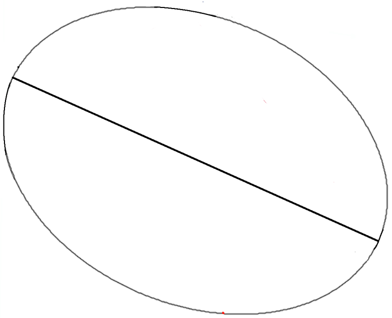

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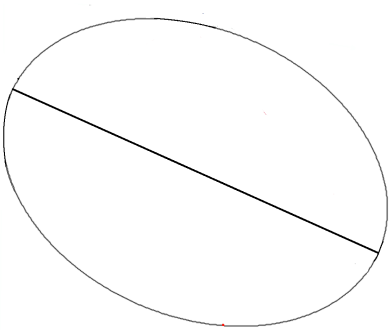

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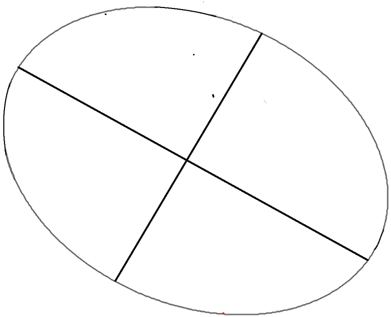

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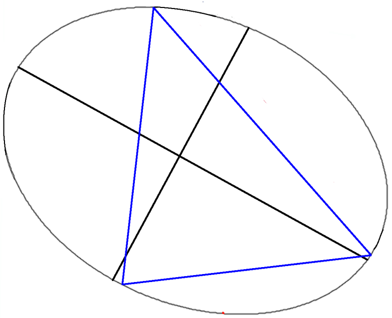

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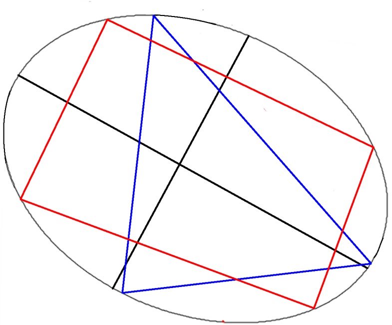

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$$w(t) = \mathrm{Re} \sum_{\lambda_j \in \Delta(\Omega)} \exp(i\lambda_j t)$$

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Can hear an axis-symmetric drum!

Call Ω *dynamically spectrally rigid* (DSR) if any smooth isopectral deformation $\{\Omega_t\}_t \subset \mathcal{S}^r$ is an isometry, i.e. $\mathcal{L}(\Omega_t) \equiv \mathcal{L}(\Omega_0)$.

Theorem 1 *De Simoi-K-Wei'16* A axis-symmetric domain near the circle is DSR.

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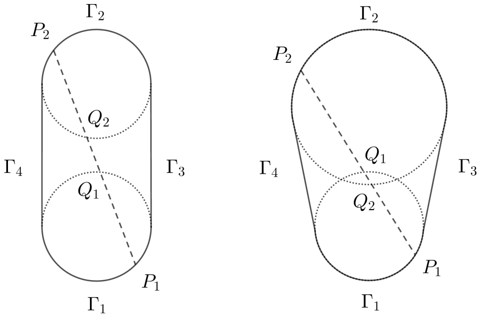

Theorem 1 *J. Chen-K-H. Zhang' 19* A p.a. Bunimovich stadium is DSR. In addition, a p.a. Bunimovich squash-like stadium is DSR.

Similar to Hezari-Zeldich.

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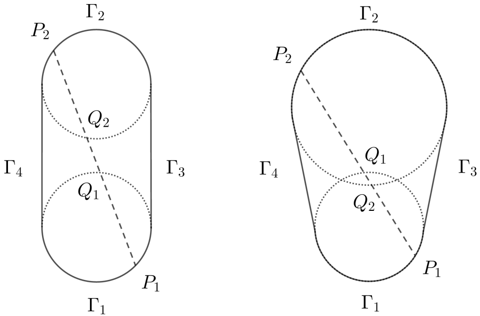

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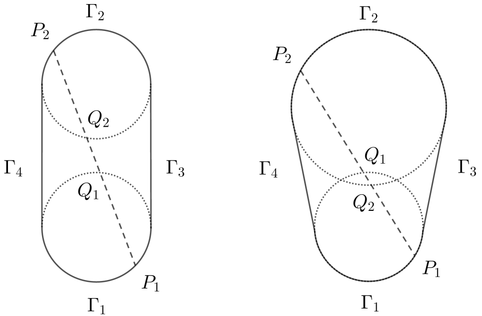

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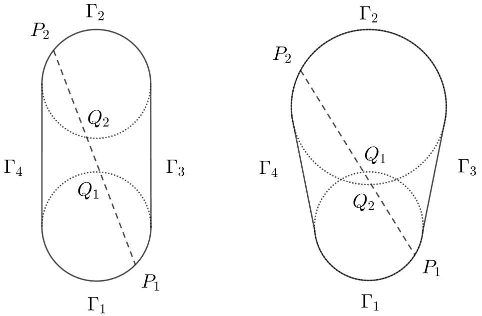

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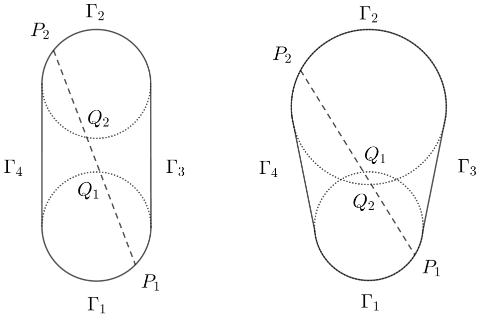

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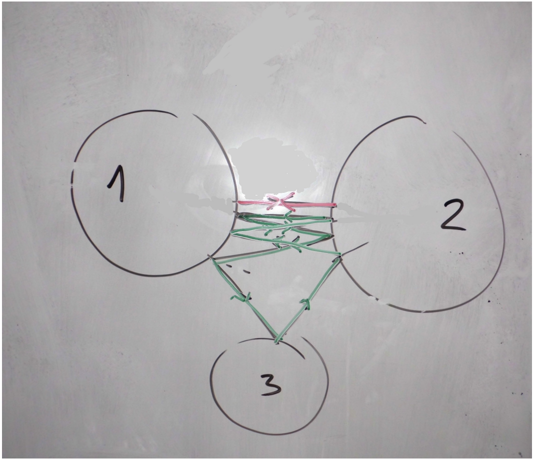

Balint=De Simoi-K-Leguil Marked Length Spectrum determins an analytic three disk system with $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetries.

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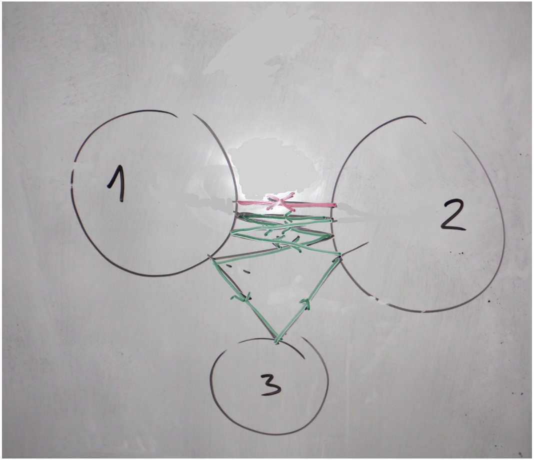

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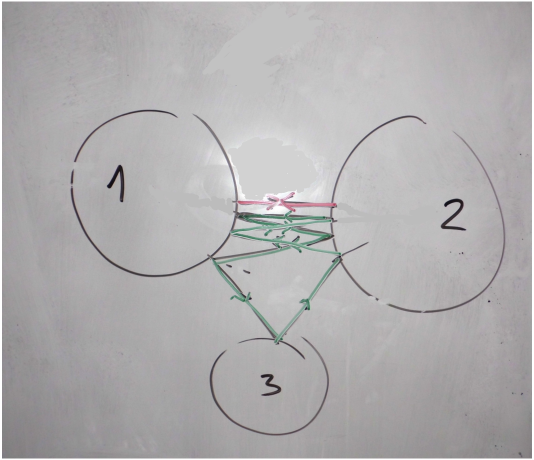

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Marked Length spectrum

Let (S, g) be a negatively curved compact surface.

Call the union of minimal geodesics in each homotopy class γ

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Guillemin-Kazhdan'80 any (S, g) is spectrally rigid.

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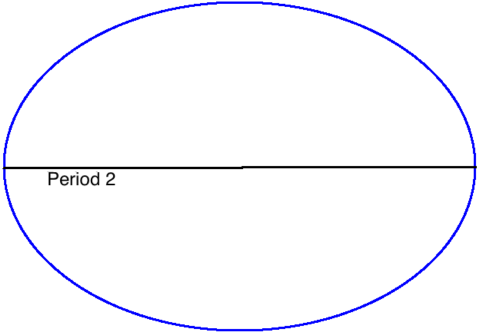
Ideas of proof of Dynamical Spectral Rigidity

- 'Skeleton' of the dynamics. Birkhoff proved

Lemma

For any convex domain Ω and any $q > 1$ there is a periodic orbit of period q, given by inscribed q-gons and denoted $S_q = S_q(\Omega)$. If Ω is axis-symmetric, then S_q can be chosen axis-symmetric.



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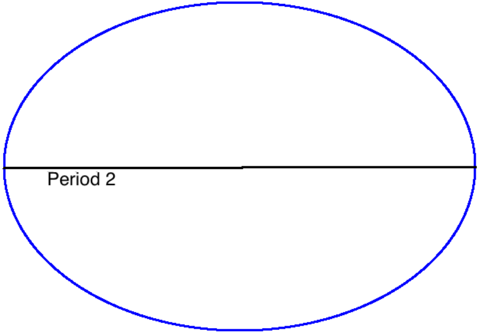

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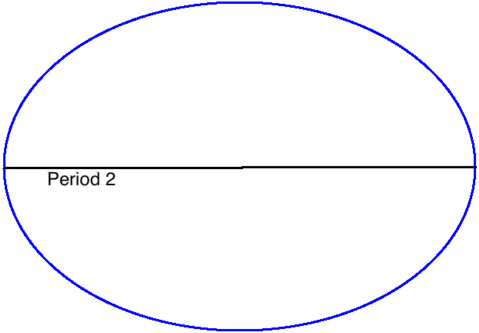

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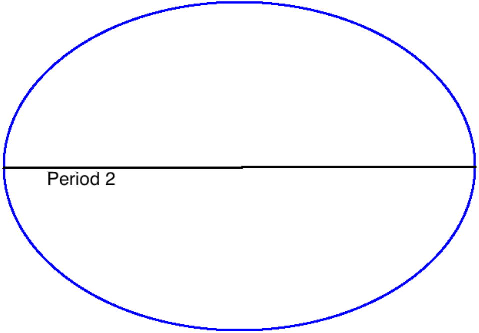

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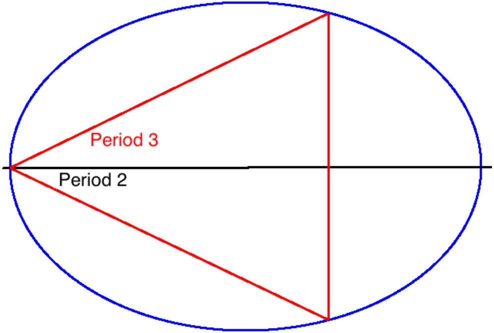

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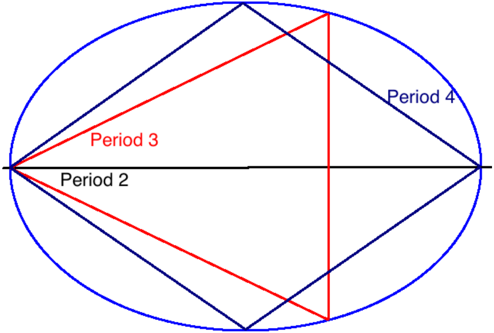

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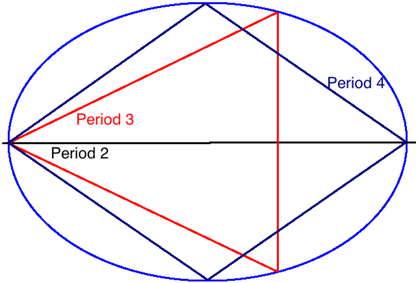

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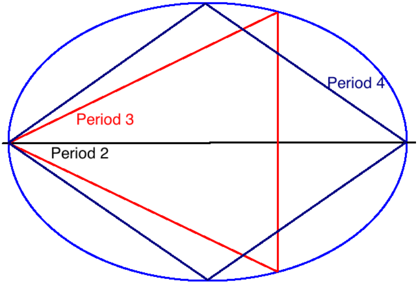

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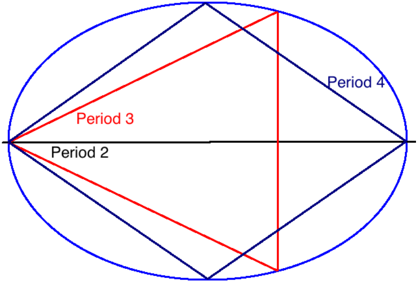

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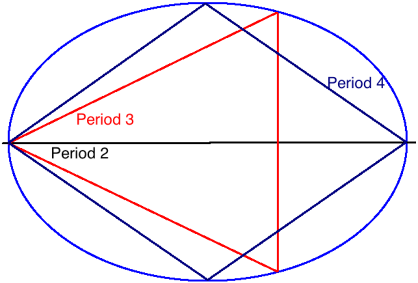

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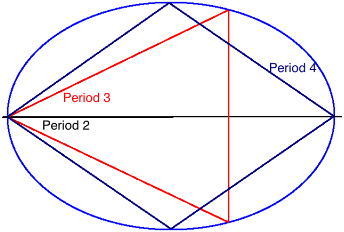

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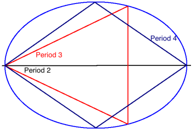
If $\mathcal{L}_\Omega$ is injective, then Ω is DSR.



Linearized Isospectral Operator for the circle

Consider an isospectral deformation $\{\Omega_t\}_t \subset \mathcal{S}^r$, of the circle. In polar coordinates $(r, s) \in \mathbb{R}_+ \times \mathbb{T}$

$$\partial\Omega_t = \{r = 1 + tn(s) + O(t^2)\}, \qquad n \in S^r(\mathbb{T}).$$

Then

$$\ell_q(n) = \sum_{k=1}^{q} n(\frac{k}{q}) = 0.$$

Lemma

Let $n(s) = \sum_{k\in\mathbb{Z}_+} n_k \cos ks$ be the Fourier expansion. Then $\ell_q(n) = 0$ implies $n_{kq} = 0$ for $k \geq 1$.

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The Linearized Isospectral Operator $\mathcal{L}_{\Omega_0}$ is upper triangular with units on the diagonal.

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- Let $S_q = (x_q^{(k)}, \varphi_q^{(k)}),\ q > 1$ be symmetric maximal q-gons. P_q be its perimeter.
- Marvizi-Melroze There are numbers $\{c_k\}_{k\geq 1}$ such that

$$P_q \sim c_0 + \frac{c_1}{q^2} + \frac{c_2}{q^4} + \frac{c_3}{q^6} + \cdots,$$

where for curvature $\rho(s)$ we have

$$c_1 = -2\int \rho^{2/3}(s)ds$$

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- Conjecture (Sarnak'90) Any planar domain is spertrally rigidity
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