Resonant dynamics of trojan exoplanets I: overview of resonant structure and diffusion

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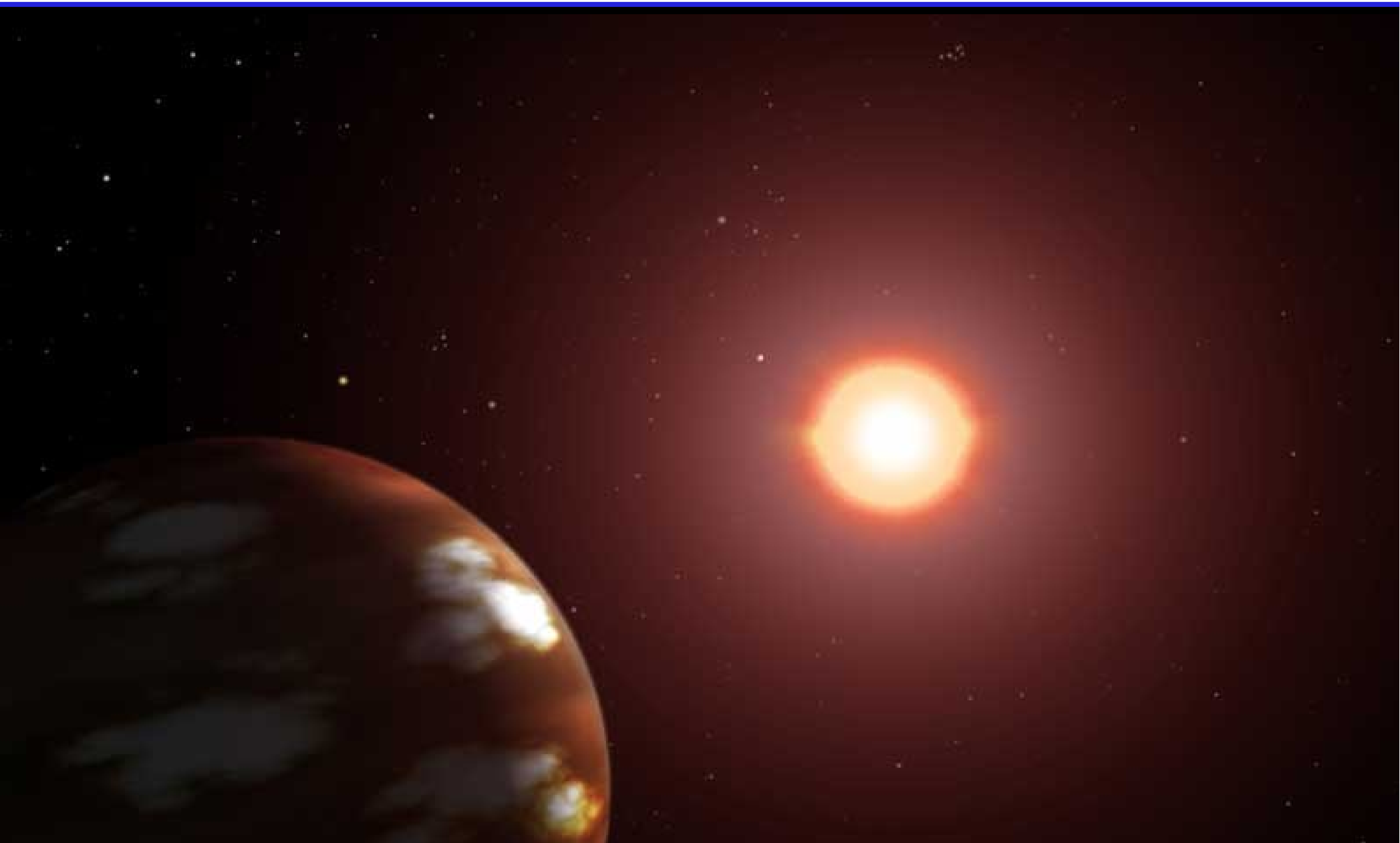
Contents

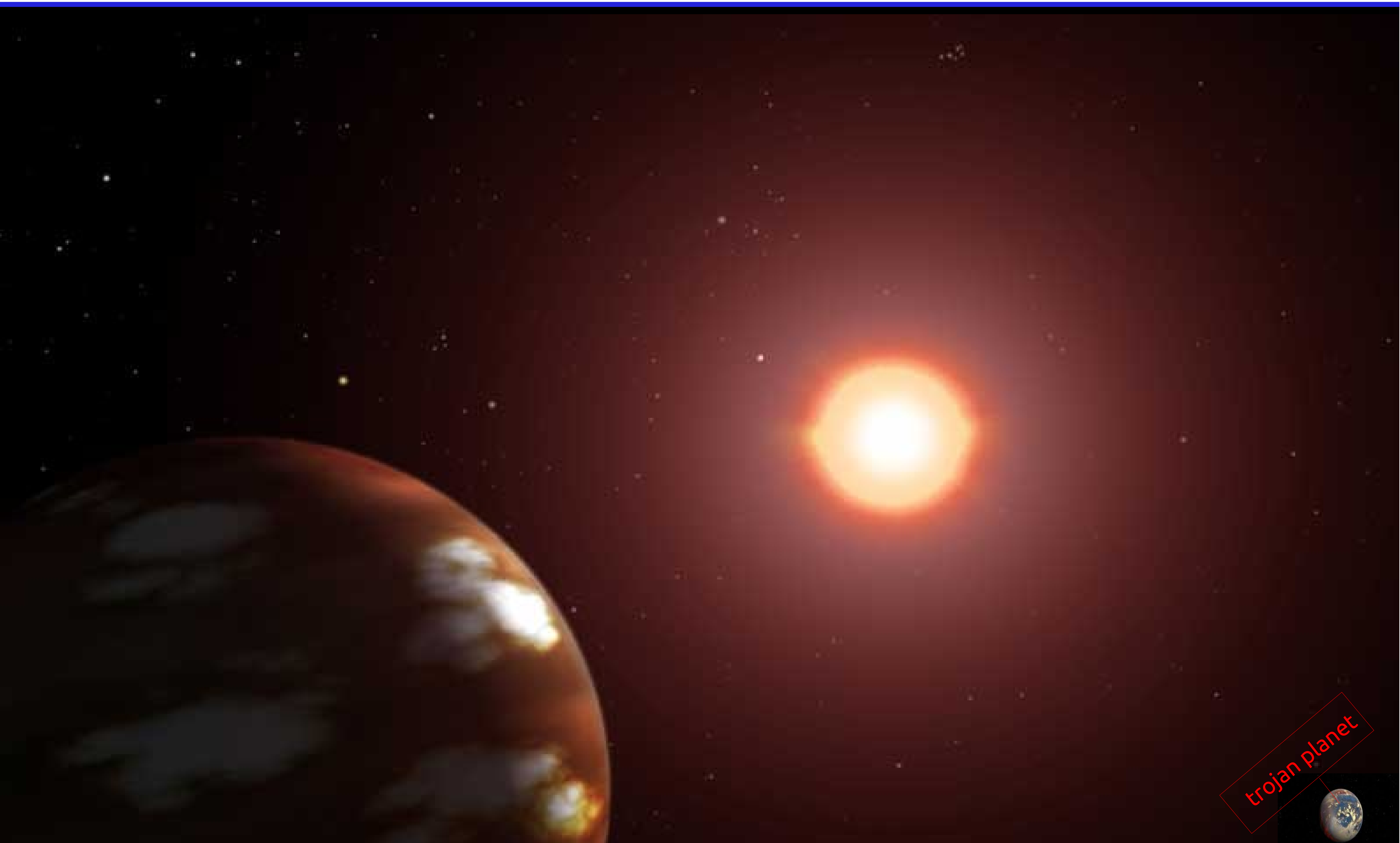
Introduction

Motivation



Introduction

Motivation



Construction of the hamiltonian

Starting point: pERTBP and further generalizations

$$H = \frac{p^2}{2} - \frac{1}{r} - \mu\left(\frac{1}{\Delta} - \frac{\mathbf{r}\cdot\mathbf{r}'}{r'^3} - \frac{1}{r}\right)$$

Modified Delaunay variables

$(x, y)\,(\lambda, \omega)$

$$x = \sqrt{a} - 1, \quad y = \sqrt{a}\left(\sqrt{1-e^2} - 1\right)$$

$$H = -\frac{1}{2(1+x)^2} + I_3 - \mu R(\lambda, \omega, x, y, \lambda'; \omega', e')$$

$g', g_j, j = 1, \ldots, S \qquad \omega' = \phi' + G(\phi', \phi_j) \; e' = e'_0 + F(\phi', \phi_j)$

$I', \phi' = g't, \, I_j, \phi_j = g_j t$

Complete Hamiltonian

$$H = -\frac{1}{2(1+x)^2} + I_3 + g'I' + \sum_{j=1}^{S} g_j I_j - \mu R(\lambda, \omega, x, y, \lambda', \phi'; e'_0) - \mu\mathcal{R}_2 - \sum_{j=1}^{S} \mu_j \mathcal{R}_j$$

Construction of the hamiltonian

Forced equilibrium

$$\mathcal{S}_1 = (\lambda - \lambda')X + \lambda' J_3 + (\omega - \phi')J_2 + \phi' P' + \sum_{j=1}^{S} \phi_j P_j$$

$$\tau = \lambda - \lambda', \quad q_3 = \lambda' \quad \beta = \omega - \phi', \quad \theta' = \phi', \quad \theta_j = \phi_j, \; j = 1, ..., S$$
$$x = X, \quad I_3 = J_3 - X, \quad y = J_2, \quad I' = P' - J_2, \quad I_j = P_j$$

$$H = < H > + H_1$$

$$< H > = -\frac{1}{2(1+x)^2} - x + J_3 - g'y - \mu < R > (\tau, \beta, x, y; e_0') \qquad < R > = \frac{1}{2\pi}\int_0^{2\pi} R d\lambda', \quad \tilde{R} = R - < R >$$

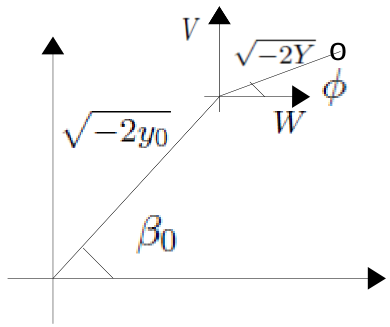
$$H_1 = g'P' + \sum_{j=1}^{S} g_j I_j - \mu\tilde{R}(\tau, \beta, x, y, \lambda', \phi'; e_0') - \sum_{j=1}^{S} \mu_j R_j(x, y, \beta, \phi', \phi_1, ..., \phi_s) - \mu\mathcal{R}_2(x, y, \tau, \beta, \phi', \phi_1, ..., \phi_s)$$

Forced equilibrium position

$$(\tau_0, \beta_0, x_0, y_0) = (\pi/3, \pi/3, 0, \sqrt{1 - e_0'^2} - 1) + O(g')$$

Construction of the hamiltonian

Expansion around the forced equilibrium



$$v = x - x_0, \quad u = \tau - \tau_0, \quad Y = -(W^2+V^2)/2, \quad \phi = \arctan(V/W)$$ where

$$V = \sqrt{-2y}\sin\beta - \sqrt{-2y_0}\sin\beta_0, \qquad W = \sqrt{-2y}\cos\beta - \sqrt{-2y_0}\cos\beta_0$$

$$H = \underbrace{-\frac{1}{2(1+v)^2} - v + J_3 - g'Y - \mu\mathcal{F}^{(0)}(u,\lambda'-\phi,v,Y;e_0')}_{H_b} - \mu\mathcal{F}^{(1)}(u,\phi,\lambda',v,Y;e_0')$$

$$+ \; g'P' - \mu\mathcal{F}^{(2)}(u,\phi,\lambda',v,Y,\phi';e_0') \; + \sum_{j=1}^{S} g_j I_j - \sum_{j=1}^{S}\mu_j\mathcal{F}_j(u,\phi,v,Y,\phi,\phi',\phi_j,\omega_{0j},e_0',e_{0j})$$

$$\mathcal{S}_2(u,\lambda',\phi,Y_1,Y_s,Y_p) = uY_1 + (\lambda'-\phi)Y_f + \phi Y_p$$

$$\phi_1 = \frac{\partial\mathcal{S}}{\partial Y_1} = u, \quad \phi_f = \frac{\partial\mathcal{S}}{\partial Y_f} = \lambda'-\phi, \quad \phi_p = \frac{\partial\mathcal{S}}{\partial Y_p} = \phi,$$

$$v = \frac{\partial\mathcal{S}}{\partial u} = Y_1, \quad J_3 = \frac{\partial\mathcal{S}}{\partial\lambda'} = Y_f, \quad Y = \frac{\partial\mathcal{S}}{\partial\phi} = Y_p - Y_f$$

$$H_b = -\frac{1}{2(1+v)^2} - v + (1+g')Y_f - g'Y_p - \mu\mathcal{F}^{(0)}(u,\phi_f,v,Y_p-Y_f;e_0')$$

Construction of the hamiltonian

Considerations about H_b

$$H_b \rightarrow e_{p,0} = \sqrt{V^2 + W^2} = \sqrt{-2Y}$$

$$H_b \rightarrow e_p = \sqrt{-2Y_p}$$

$$\omega_f \equiv \dot{\phi}_f = \frac{\partial H_b}{\partial Y_f} = 1 - 27\mu/8 + g' + \ldots, \qquad g \equiv \dot{\phi} = \frac{\partial H_b}{\partial Y_p} = 27\mu/8 - g' + \ldots$$

$$\overline{\mathcal{F}^{(0)}} = \frac{1}{2\pi}\int_0^{2\pi} \mathcal{F}^{(0)} d\phi_f$$

$$\overline{H_b}(u, v; Y_f, Y_p, e_0') = -\frac{1}{2(1+v)^2} - v + (1+g')Y_f - g'Y_p - \mu\overline{\mathcal{F}^{(0)}}(u, v, Y_p - Y_f, e_0')$$

$$\frac{\partial\overline{\mathcal{F}^{(0)}}}{\partial u_0} = \frac{\partial\overline{\mathcal{F}^{(0)}}}{\partial v_0} = 0 \longrightarrow J_s = \frac{1}{2\pi}\int_C (v - v_0)d(u - u_0) \longrightarrow \omega_s = \dot{\phi}_s = \sqrt{\frac{27\mu}{4}} + \ldots$$

Construction of the hamiltonian

Classification of resonances

$$m_f\omega_f + m_s\omega_s + mg + m'g' + m_1g_1 + \ldots + m_Sg_S = 0$$

secondary resonances 1:n

$$m_f = 1,\ m_s < 0,\ m = 0, \qquad \omega_f - n\omega_s = 0 \qquad n = -m_s$$

$$|m| + |m'| + |m_1| + \ldots + |m_S| > 0$$

$$|m_f| + |n| > 0$$ transverse

$$|m_f| + |n| = 0$$ secular

Construction of the hamiltonian

Planar ERTBP

$S = 0,\ g' = 0,\ e' = e'_0 = \text{const.}$ then $\beta \equiv \omega$, and $v \equiv x$

$$H_{ell} = -\frac{1}{2(1+x)^2} - x + Y_f - \mu\left(\mathcal{F}^{(0)}(u, \phi_f, x, Y_p - Y_f, e') + \mathcal{F}^{(1)}(u, \phi_f, \phi, x, Y_p - Y_f, e')\right)$$

shift with respect to L_4

$$u_0 = \frac{29\sqrt{3}}{24} e_p^2 + \ldots$$

$$\overline{H}_{b,ell} = -\frac{1}{2} + Y_f - \mu\left(\frac{27}{8} + \ldots\right)\frac{e_{p,0}^2}{2} - \frac{3}{2}x^2 + \ldots - \mu\left(\frac{9}{8} + \frac{63e'^2}{16} + \frac{129e_{p,0}^2}{64} + \ldots\right)\delta u^2 + \ldots$$

with $\delta u = u - u_0$

$$m_f\left(1 - \frac{27\mu}{8} + \ldots\right) = n\sqrt{6\mu\left(\frac{9}{8} + \frac{63e'^2}{16} + \frac{129e_p^2}{64} + \ldots\right)}$$

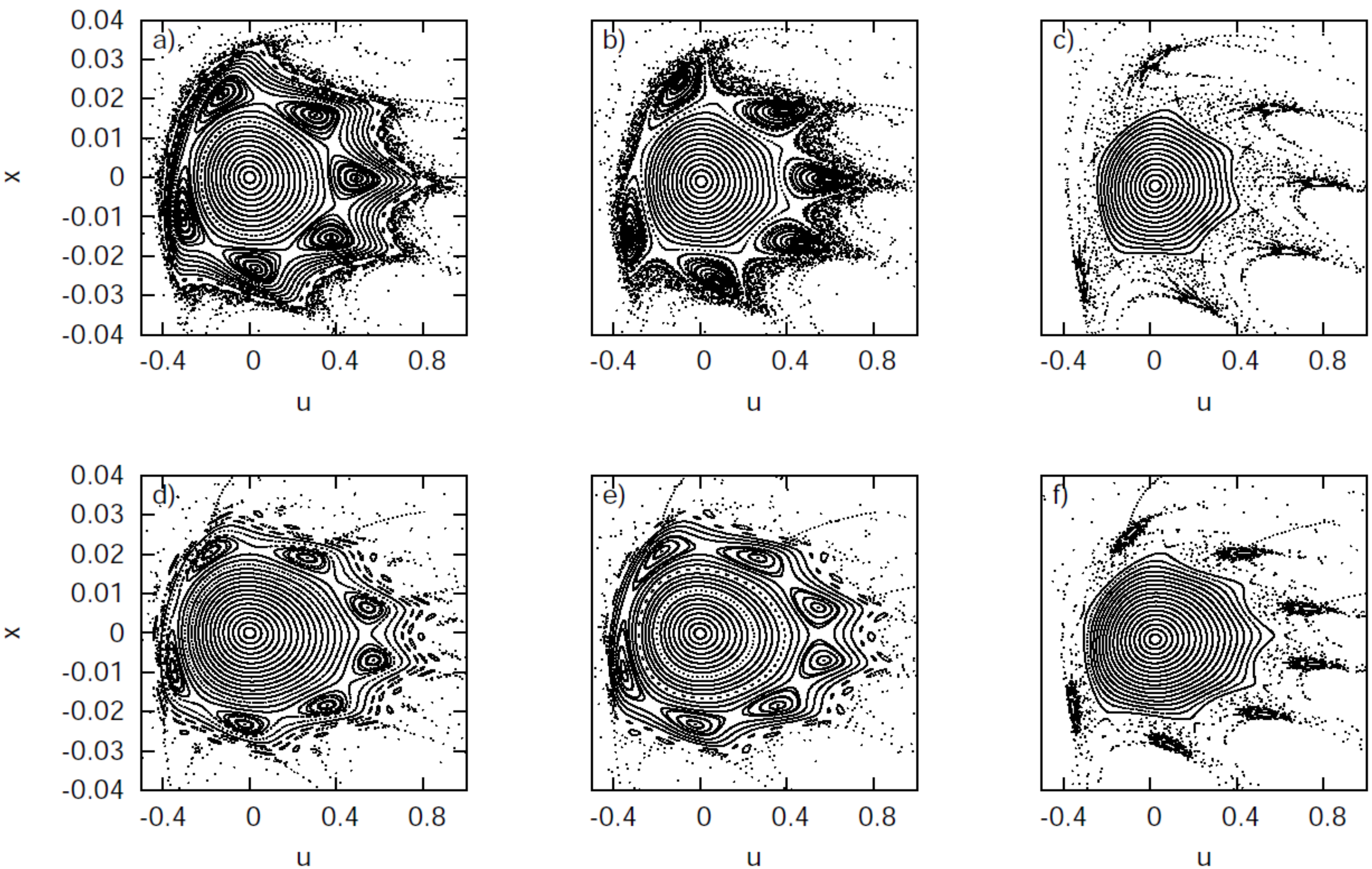
Numerical Experiments

Parametric Study

Phase portraits for e' = 0 (CRTBP)

pericenter crossing condition
35 initial conditions along the line $x = B(u-u_0)$

$0.001 \leq \mu \leq 0.01$ $\Delta\mu = 0.001$
$0.0 \leq \Delta u \leq 1.0$ $\Delta(\Delta u) \sim 0.03$



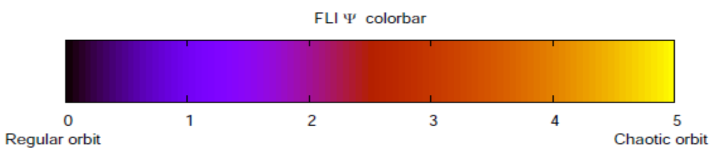
$\mu = 0.0041$
a) $e_p = 0.0001$
b) $e_p = 0.06$
c) $e_p = 0.1$

$\mu = 0.0031$
d) $e_p = 0.0001$
e) $e_p = 0.05$
f) $e_p = 0.1$

Parametric Study

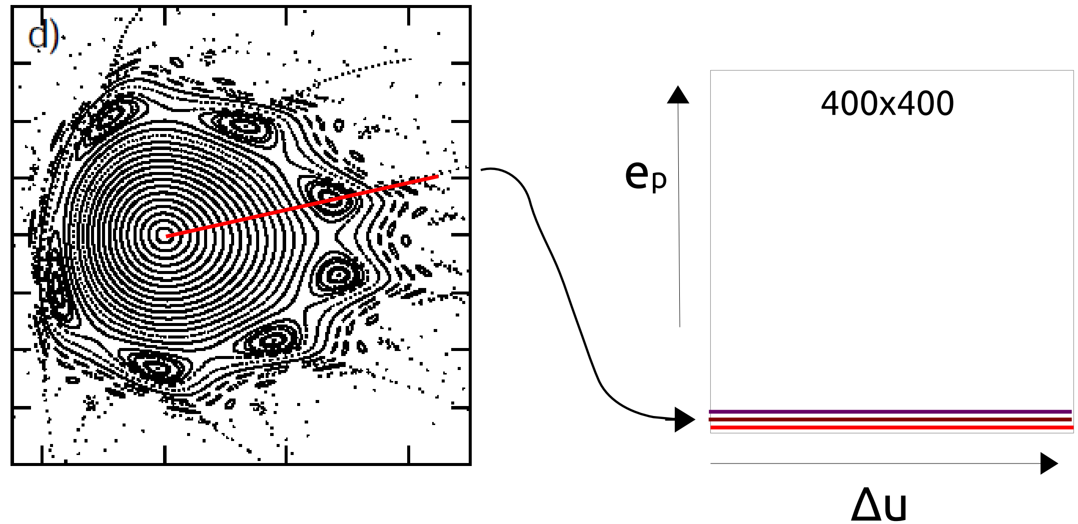
FLI stability maps

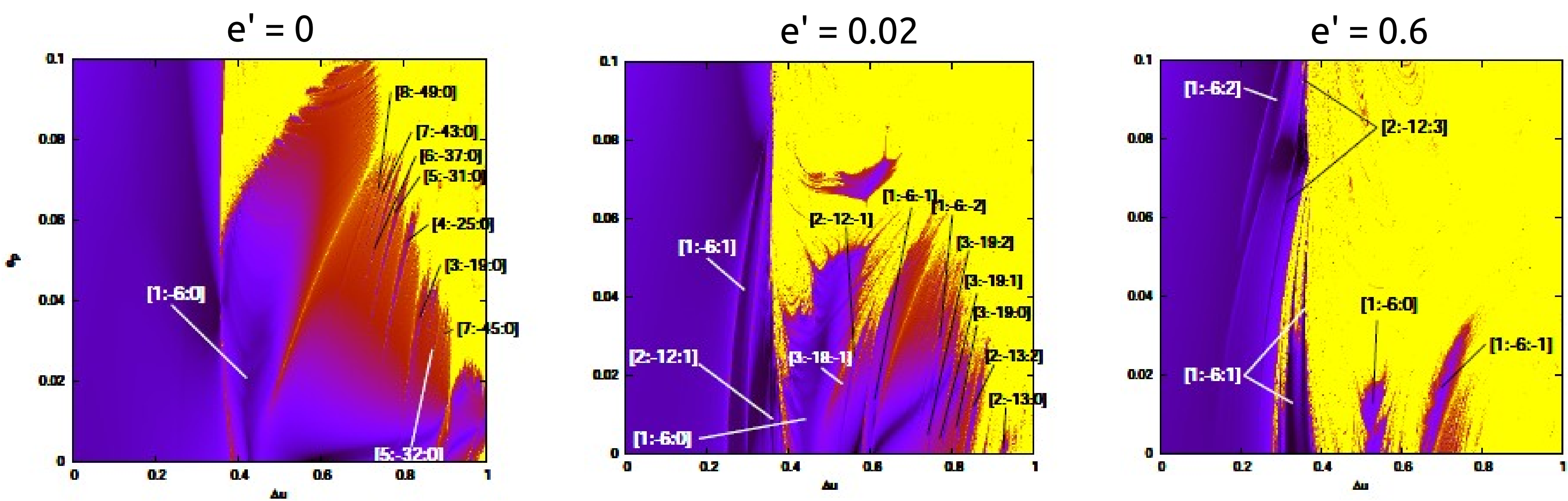
$$\Psi(t) = \sup_t \log_{10}(\|\xi\|),$$



$0 \leq \Delta u \leq 1$ $\quad 0 \leq e_p \leq 0.1$

$\Delta t = 1/300$ $\quad T \sim 1000$ periods



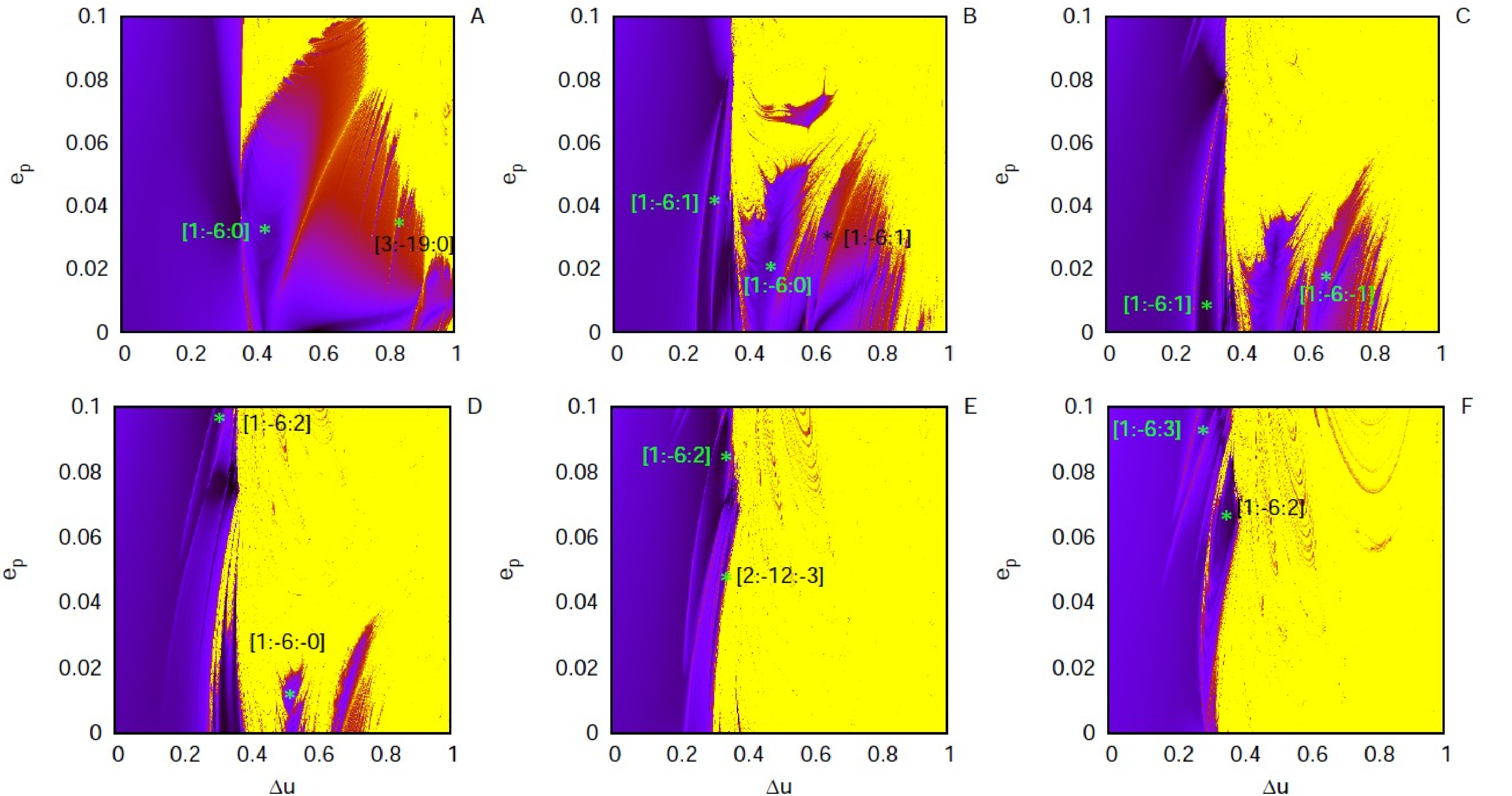


Parametric Study

Survey of resonances

μ = 0.0041 (1:6)

A) e' = 0, B) e' = 0.02, C) e' = 0.04, D) e' = 0.06, E) e' = 0.08, F) e' = 0.1

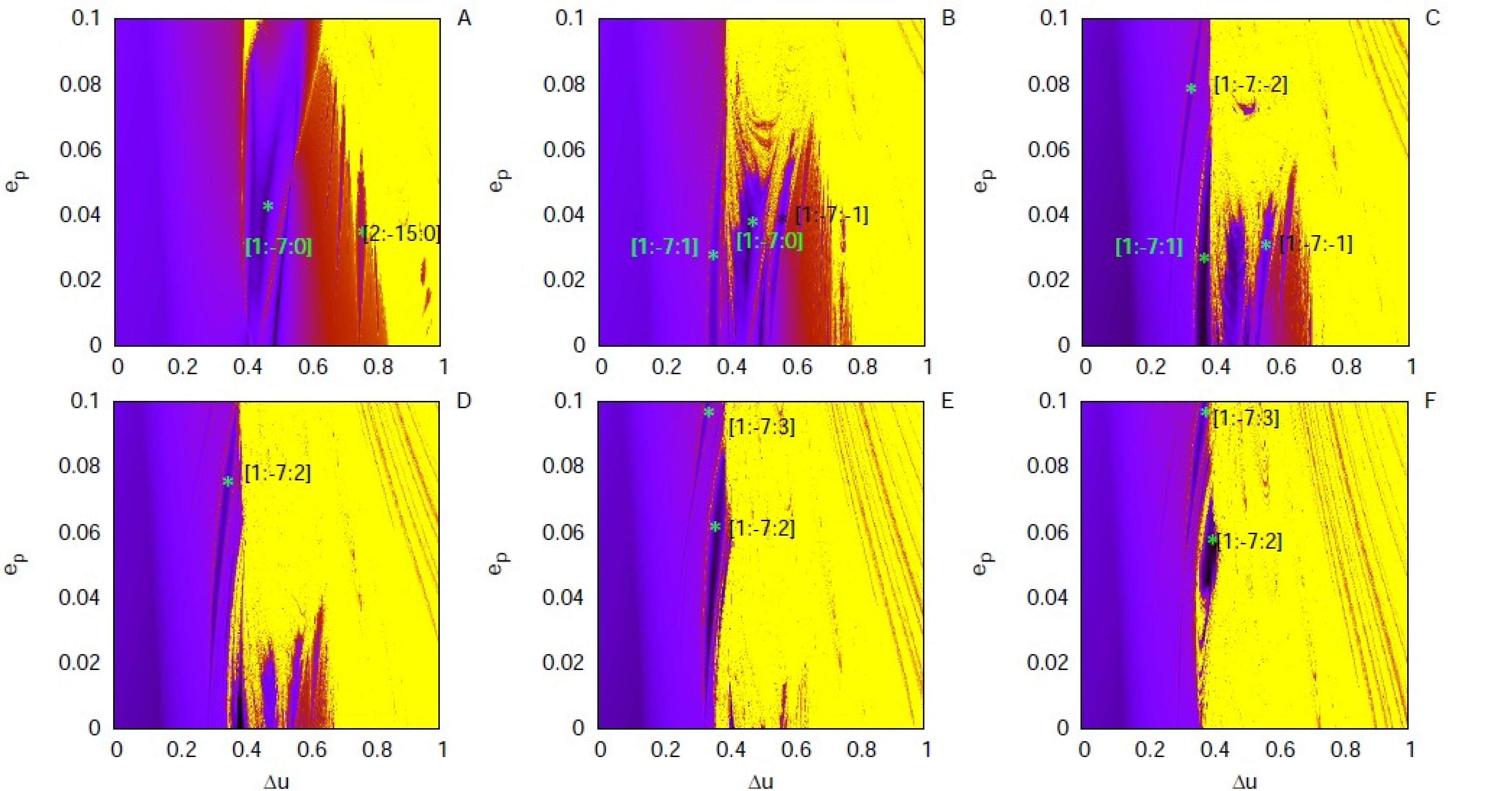


Parametric Study

Survey of resonances - Another example

μ = 0.0012 (1:12), 0.0014 (1:11), 0.0016 (1:10), 0.0021 (1:9), 0.0024 (1:8), 0.0031 (1:7), 0.0041 (1:6) and 0.0056 (1:5)

A) e' = 0, B) e' = 0.02, C) e' = 0.04, D) e' = 0.06, E) e' = 0.08, F) e' = 0.1

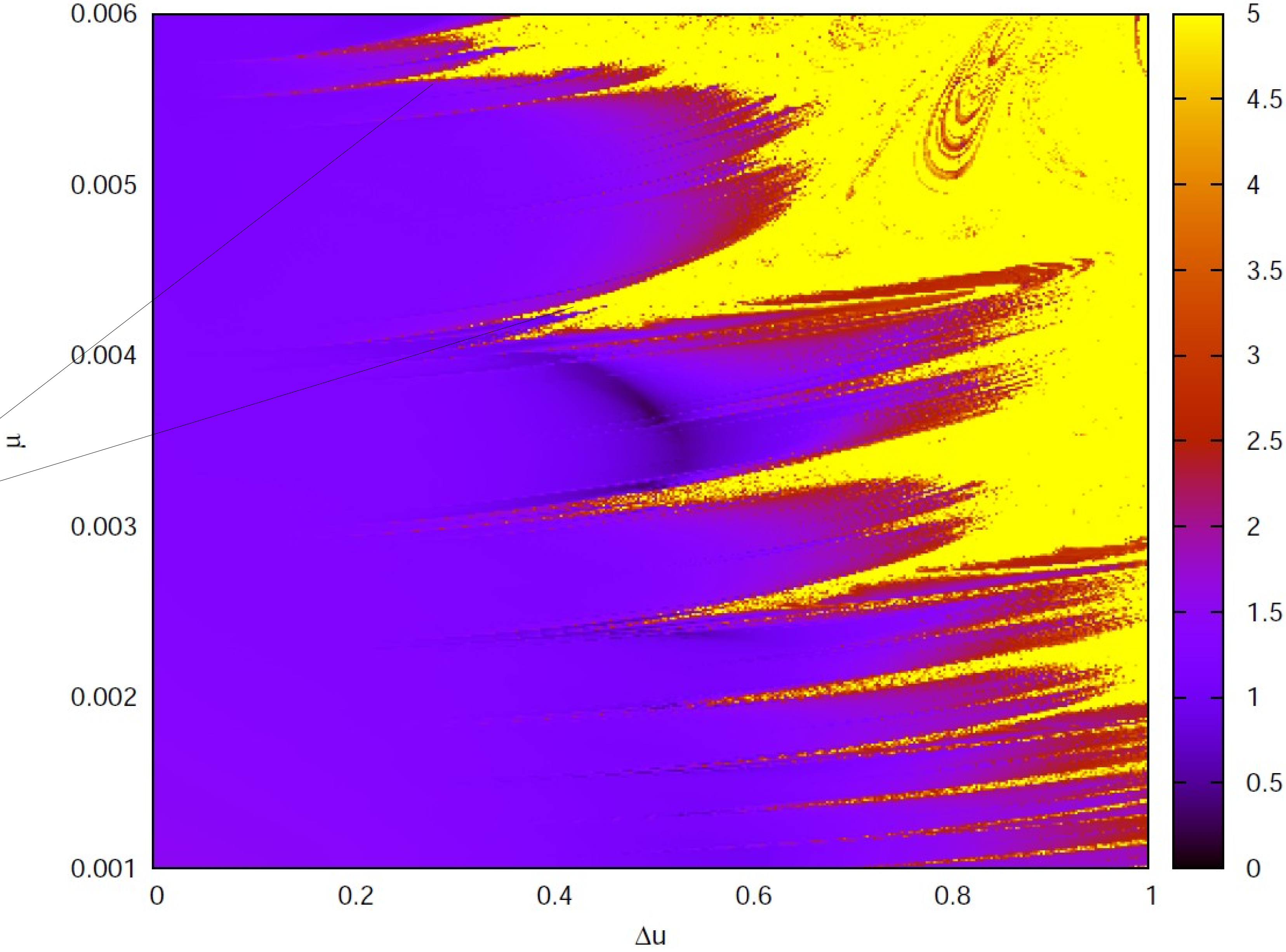


Parametric Study

Dependence on μ

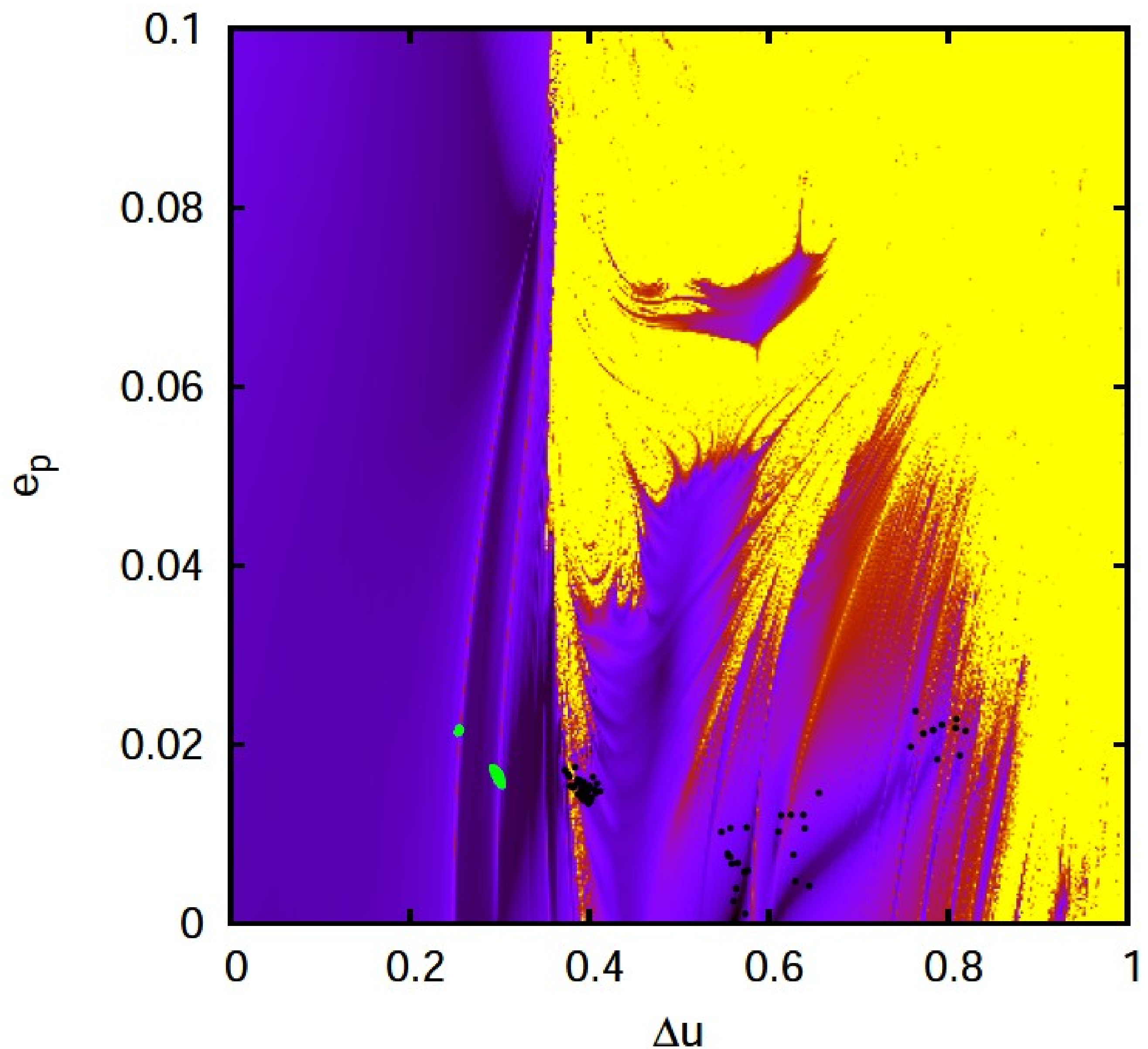
400x400
ep = e' = 0.02

Resonances



Chaotic Diffusion

Kinds of diffusion



Parameters

$e' = 0.02$
$\mu = 0.0041$
$e_p = 0.01625$

Initial Conditions

$x = 0$
$\varphi = \pi/3$
$Y_f = 0$

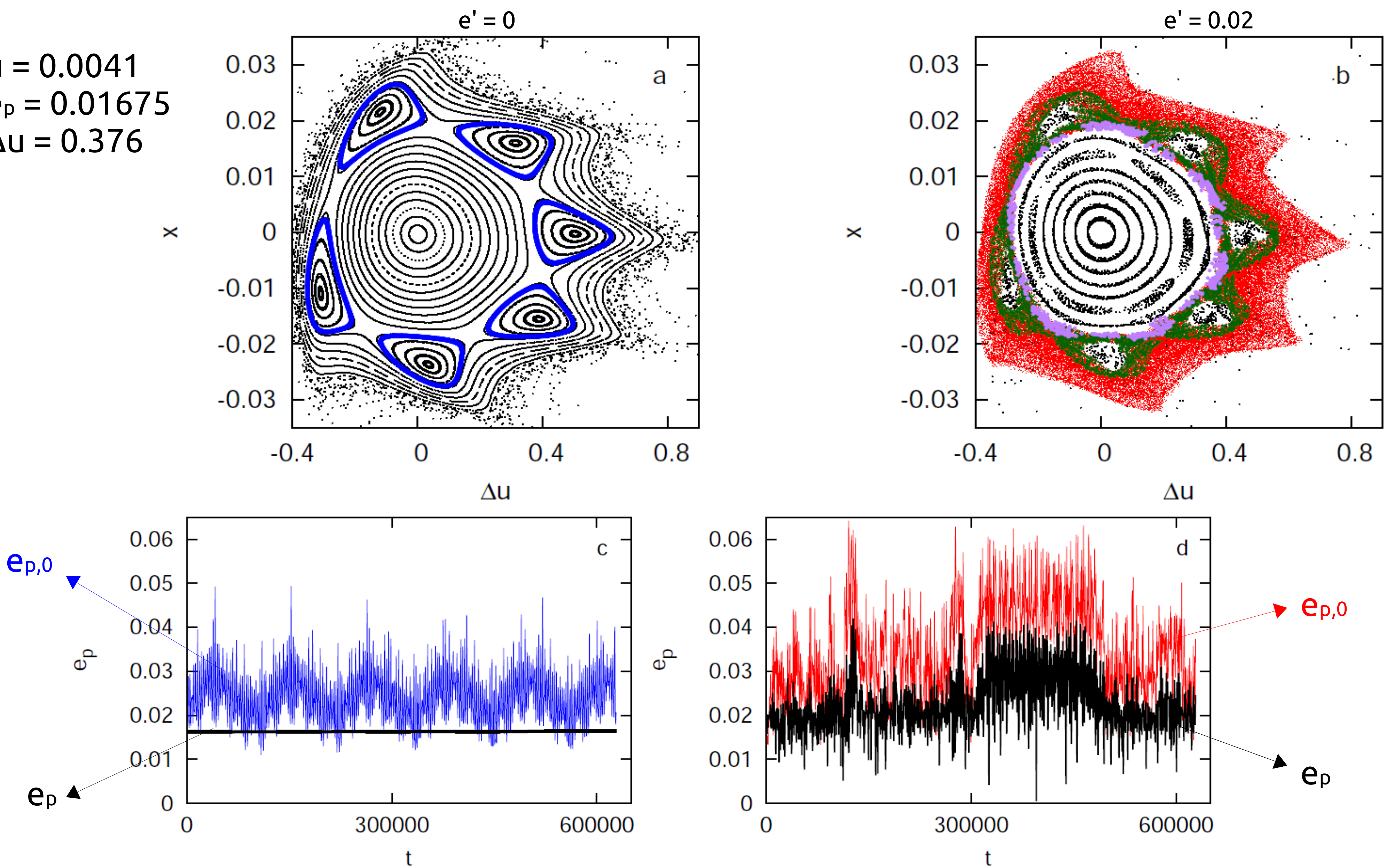
- (green) $\Delta u = 0.299$
- (black) $\Delta u = 0.376$

Integration 10^5 periods

Chaotic Diffusion

Main paradigm of diffusion: modulational

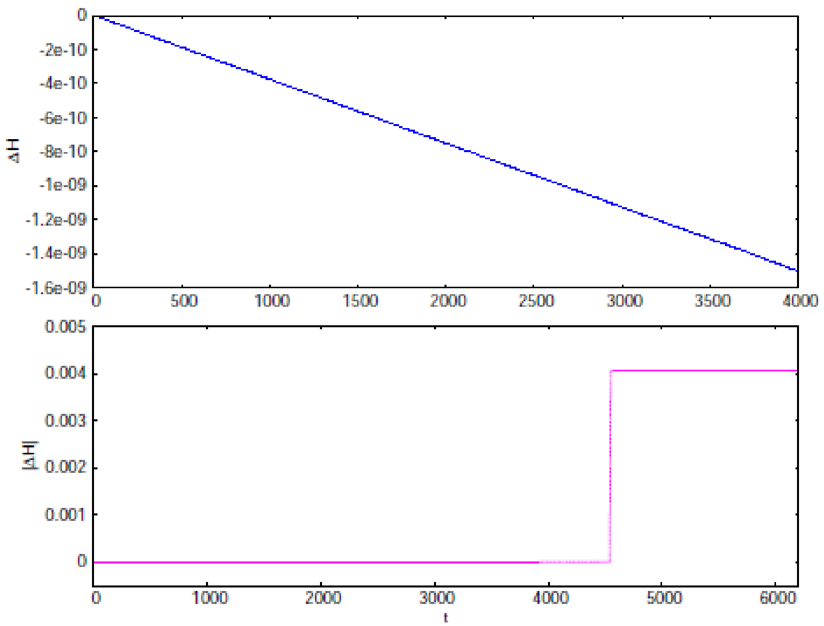
μ = 0.0041
e_p = 0.01675
Δu = 0.376



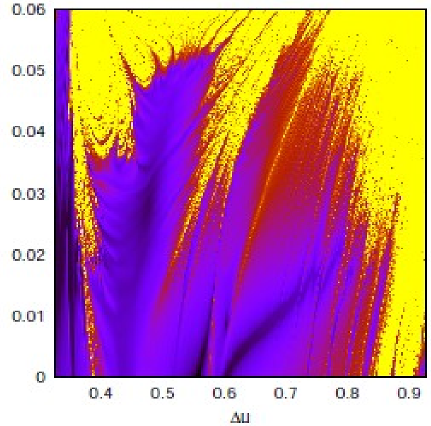
Statistics of Orbits

Classification of orbits

- regular orbits $\Psi(T) < \log_{10}(\frac{N}{10})$
- escaping orbits $\Delta H > 10^{-3}$
- transition orbits



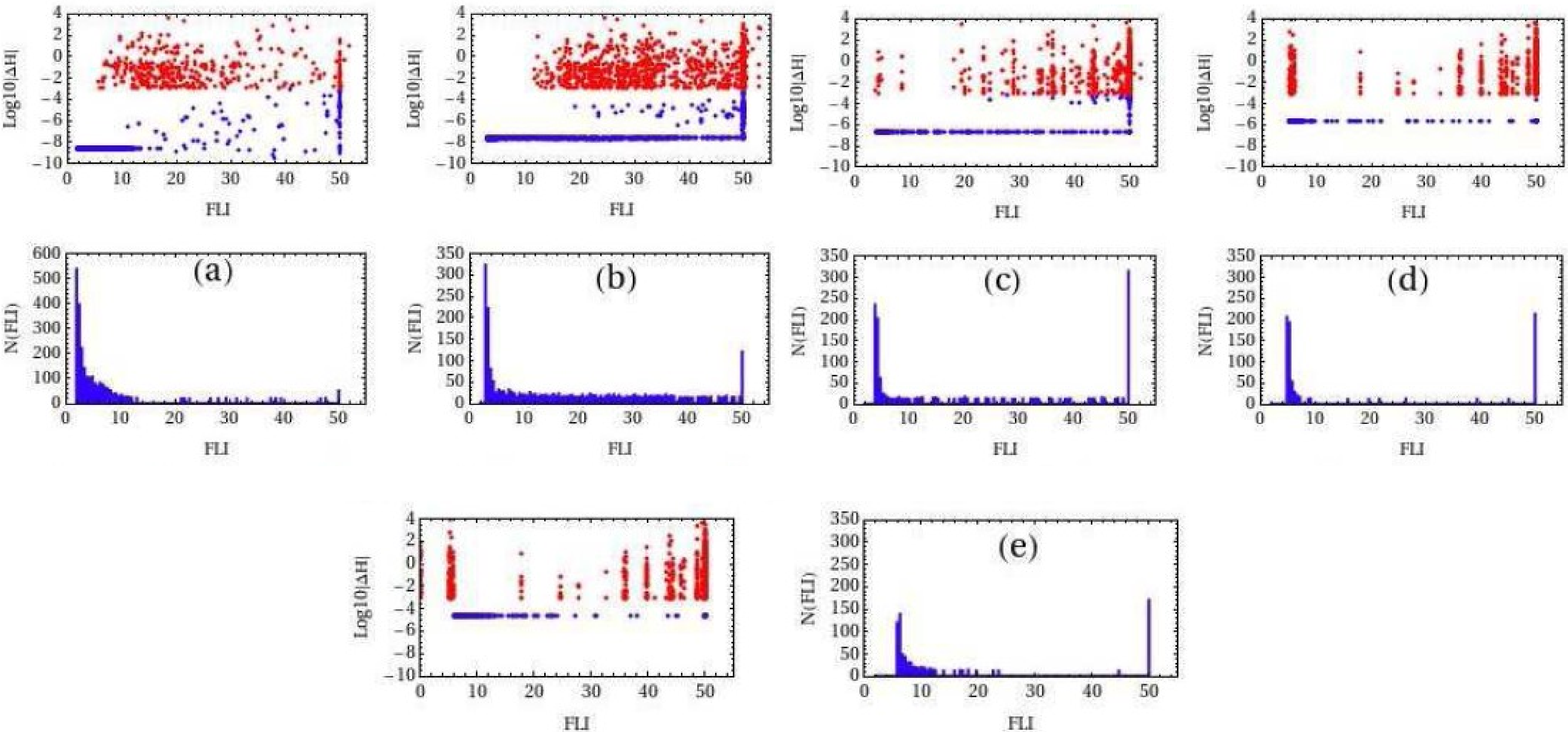
Snapshots at T = $10^3, 10^4, 10^5, 10^6, 10^7$ periods



Snapshot	(N. of periods)	Regular	Transition	Escaping
1	10^3	1220 (33.8%)	2027 (56.3%)	353 (9.9%)
2	10^4	1263 (35%)	1388 (38.5%)	949 (26.5%)
3	10^5	1296 (36%)	966 (26.8%)	1338 (37.2%)
4	10^6	1299 (36.1%)	699 (19.4%)	1602 (44.5%)
5	10^7	1309 (36.3%)	603 (16.8%)	1688 (46.9%)

Statistics of Orbits

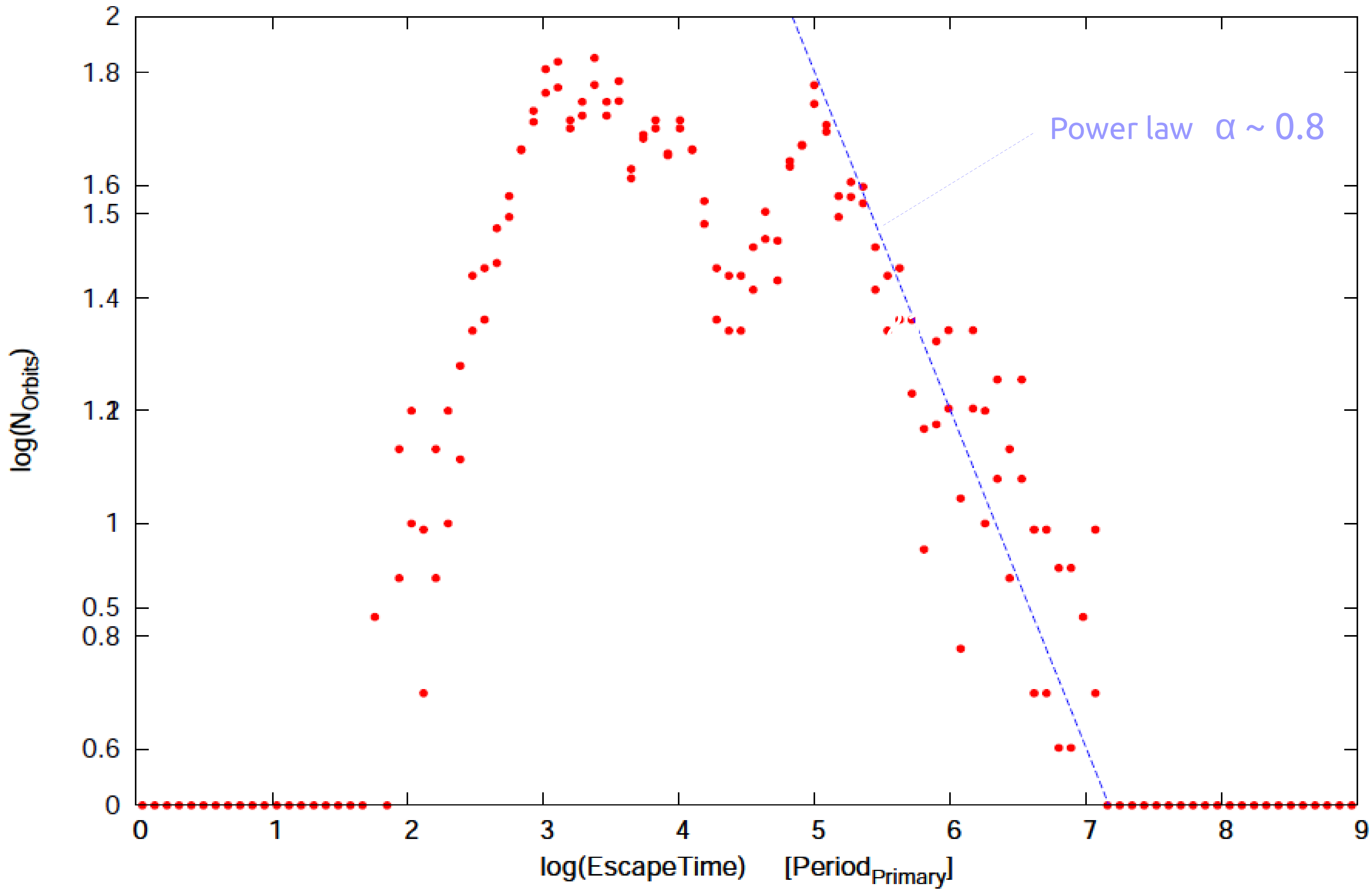
Stastistical results



(a) $T = 10^3$, (b) $T = 10^4$, (c) $T = 10^5$, (d) $T = 10^6$, (e) $T = 10^7$

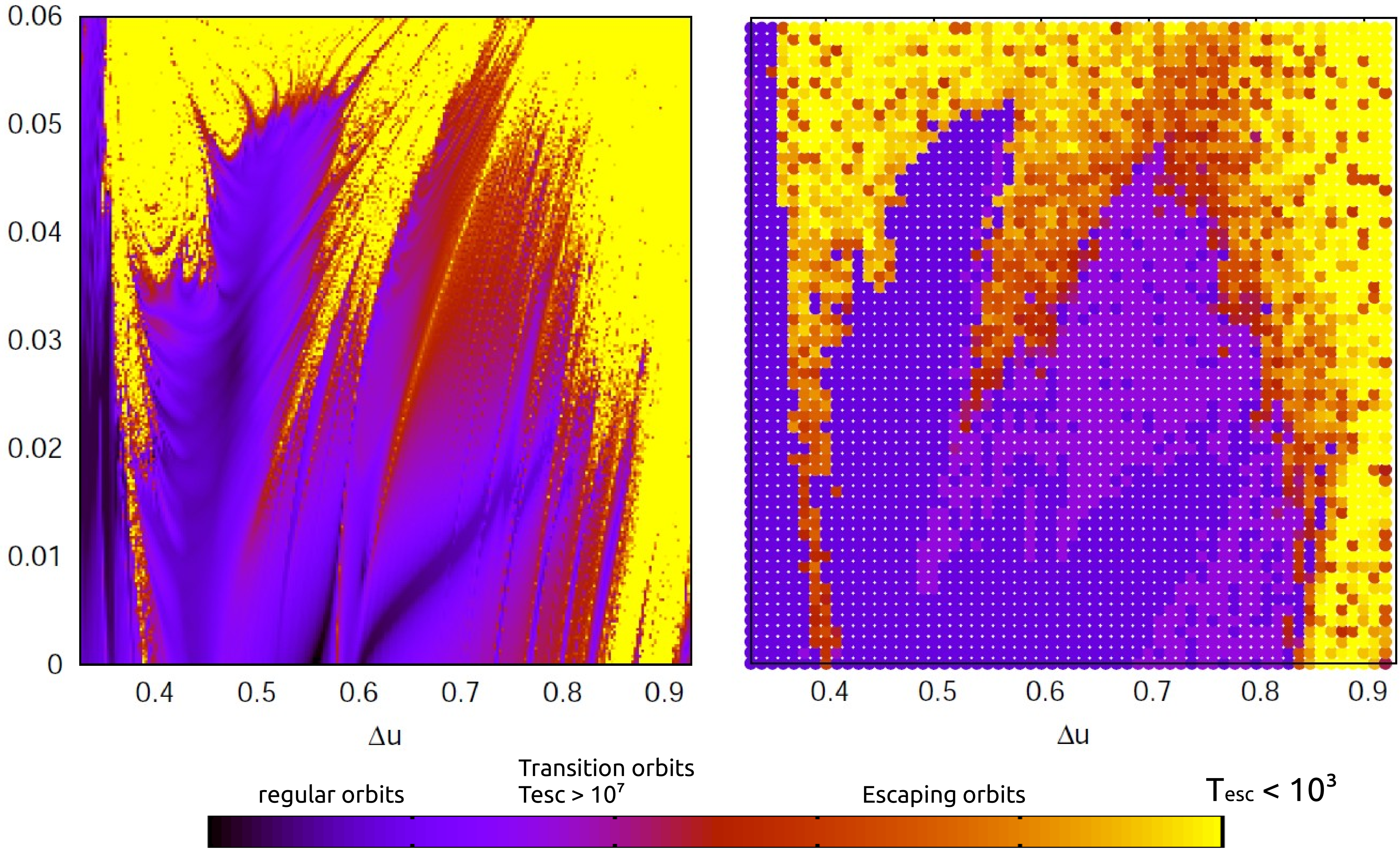
Statistics of Orbits

Stastistical results



Statistics of Orbits

Comparision between FLI and escaping times



Conclusions

- Hamiltonian Formalism in modified Delaunay variables
  - secular effects, due to one or more planets
  - hierarchy of Hamiltonian models corresponding to different levels of perturbation
  - resonant proper elements
  - characterisation of resonances

- Visualization of the resonant web - dependence on physical parameters
  - phase portraits
  - FLI maps

- Chaotic diffusion & statistics of escapes
  - different paradigms and rates of chaotic diffusion
  - statistical study of an ensemble of orbits in the resonant domain
    - two characteristic peaks in the escaping times distribution
    - correlation between the escaping times and the structure of the resonant web

Thanks for your attention!
Questions?

Secular dynamics adding more planets

$g', g_j, j = 1, \ldots, S$ frequencies of the leading terms in the quasi-periodic representation of the oscillations of the planets' eccentricity vectors

$$e' \exp i\omega' = e'_0 \exp i(\omega'_0 + g't) + \sum_{k=1}^{s} A_k \exp i(\omega'_{k0} + g_k t)$$

$$e_j \exp i\omega_j = B_{j0} \exp i(\omega_{j0} + g't) + \sum_{k=1}^{s} B_{kj} \exp i(\omega'_{kj} + g_i t)$$

$A_k,\ B_{kj},$ with $k = 1, \ldots, s,$ and B_0 the amplitudes of oscillation of the ecc. vector

$e'_0 > \sum_{k=1}^{s} A_k$ → $e' = e'_0 + F$, $\omega' = g't + G$

F and G trigonometric on $\phi' = g't,\ \phi_j = g_j t,\ j = 1, \ldots, s,$

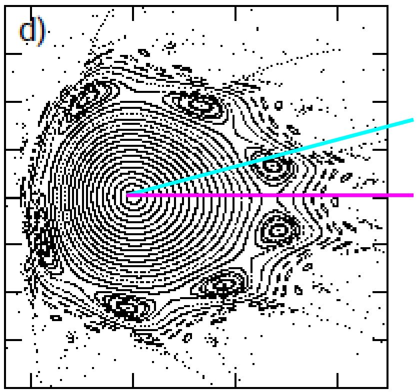
$I', \phi' = g't,\ I_j, \phi_j = g_j t$ $\qquad e' = e'_0 + F(\phi', \phi_j),\ \omega' = \phi' + G(\phi', \phi_j)$

$$H = -\frac{1}{2(1+x)^2} + I_3 + g'I' + \sum_{j=1}^{S} g_j I_j - \mu R(\lambda, \omega, x, y, \lambda', \phi'; e'_0) - \mu \mathcal{R}_2 - \sum_{j=1}^{S} \mu_j \mathcal{R}_j$$

Correlation between the proper libration and Δu

$$J_s = \frac{1}{2\pi}\int_C (v - v_0)d(u - u_0)$$ Action labels libration motion around the forced eq. point

We define Δu in the following way: for given e_p, we compute the position of the fixed point. We then consider all the invariant curves around the eq. point ($x = 0, u = u_0$) of the 1 d.o.f. $\overline{H}_b$ We also take the line $x = B(u-u_0)$. We call up the point where the invariant curve intersects the line. We finally define $\Delta u = (u_p-u_0)$. Up to quadratic terms in Δu, one has



$$J_s = \frac{3B^2/2 + \mu\left(9/8 + 63e'^2/16 + 129e_p^2/64\right)}{\left[6\mu\left(9/8 + 63e'^2/16 + 129e_p^2/64\right)\right]^{1/2}}\Delta u^2 + \mathcal{O}(\Delta u^4)$$

In general, for $B \neq 0$, $\Delta u \neq D_p$ (half-width of the oscillation of the variable u along the invariant curve of $\overline{H}_b$ corresponding to the action variable J_s), which is the common definition of the proper libration. Instead, one has

$$D_p = \left[\frac{3B^2/2 + \mu\left(9/8 + 63e'^2/16 + 129e_p^2/64\right)}{\mu\left(9/8 + 63e'^2/16 + 129e_p^2/64\right)}\right]^{1/2}\Delta u + \mathcal{O}(\Delta u^2)$$