Minimizing the optimality residual for algebraic Riccati equations

Alice Cortinovis[1], Federico Poloni[2]

[1] University of Pisa; Scuola Normale Superiore di Pisa. alice.cortinovis@sns.it
[2] Department of Computer Science, University of Pisa. federico.poloni@unipi.it

We briefly recall how we can find the minimizer of a linear quadratic optimal control problem of the form

$$\min_{(u,x)} \left\{ \mathcal{J}(u,x) = \int_0^{+\infty} (x^T Q x + u^T R u) \mathrm{d}t \text{ subject to } \dot{x} = Ax + Bu, x(0) = x_0 \right\}$$

by solving the associated continuous-time algebraic Riccati equation $A^T X + XA + Q - XBR^{-1}B^T X = 0$.

Denoting by $(u(t), x(t), X)$ the exact solutions and by $(\tilde{u}(t), \tilde{x}(t), \tilde{X})$ the computed solutions, we call *optimality residual* the difference $\mathcal{J}(\tilde{u}, \tilde{x}) - \mathcal{J}(u, x)$. We obtain a relation between the residual of the Riccati equation and the optimality residual, and we derive a way to compute an approximation of the latter.

We recall that algebraic Riccati equations can be solved by using a Schur decomposition to find the stable invariant subspace of a suitable Hamiltonian matrix $\mathcal{H}$. We analyze the behaviour of the optimality residual when a symplectic change of basis is applied to $\mathcal{H}$ before computing the Schur decomposition and we present some heuristics for finding a "good" change of basis.