Analisi Matematica I

Limiti di funzioni e continuità

Esercizio 1. Calcolare i seguenti limiti, verificandoli, poi, con la definizione.

(1) $\lim_{x\to 2} \sqrt{|x-2|}+3$

(2) $\lim_{x\to 3} \log(1+|x-1|)$

(3) $\lim_{x\to 1} \frac{1}{|x-1|}$

(4) $\lim_{x\to +\infty} \left(2+\frac{3}{x^2-1}\right)$

(5) $\lim_{x\to -\infty} \left(2+\frac{3}{x^2-1}\right)$

Esercizio 2. Calcolare i limiti delle seguenti funzioni sia per $x\to 0^+$ sia per $x\to +\infty$.

(1) $\frac{\sqrt{1+x}-1}{x(x^2+1)}$

(2) $\frac{x(\sqrt{1+x}-1)}{x^2+1}$

(3) $\frac{\sqrt{1+x}-e^x}{x^2(1+3^x)}$

(4) $\frac{(\sqrt{1+x}-e^x)^2}{x(1+3^x)}$

(5) $\frac{(\sqrt{1+x}-1)(e^x-1)}{x^2(1+2^x)}$

(6) $\frac{\log(1+x^2)+x}{x(1+x^2)}$

(7) $\frac{\log(1+x^2)+x\log x}{x(1+\log x)}$

(8) $\frac{\log(1+x^2)+x^2}{x\log x}$

(9) $\frac{\log(1+x^2)+x^2}{x^2\log x}$

(10) $\frac{\log(1+x^2)+x^2}{x^3\log x}$

(11) $\frac{\log(1+x)}{e^x-\sqrt{1+x}}$

(12) $\frac{(\sqrt{1+x}-1)^2}{\log(x^2+1)}$

(13) $\frac{(1+\sqrt[3]{x})^2-e^x}{x^2+\log(1+x)}$

(14) $\log x \cdot \log\Big(1+\dfrac{1}{x}\Big)$

(15) $\dfrac{(\log x)^2}{\log(1+x)}$

(16) $\dfrac{(\log x)^2 \log(1+x^2)}{\log(1+x)}$

(17) $\sqrt[3]{\sqrt{x+1}-1} \cdot \log x$

(18) $\sqrt{x}\Big(\log x + \log\Big(1+\dfrac{1}{x}\Big)\Big)$

(19) $\dfrac{e^{-x}-1+\sin(x^2)}{|\log x| + \log\Big(1+\frac{1}{x}\Big)}$

(20) $\dfrac{\sin(x^2+x\sqrt{x})+x^3}{x^{3/2}+x^2}$

(21) $\dfrac{(e^x - \sqrt{|\cos x|}\,)\cdot \operatorname{arctg} x}{(x^2+|\sin x|)^2}$

(22) $\dfrac{\sqrt[5]{1-\cos x}\cdot \sin x \cdot \log x}{\sqrt[6]{x}(e^x-1)}$

Esercizio 3. Calcolare i limiti delle seguenti funzioni per $x \to x_0$. In caso di funzioni infinite/infinitesime, determinarne l'ordine, se esiste.

(1) $f(x) = \dfrac{e^{x^2-1}-1}{\sqrt{\log x}} \qquad x \to 1^+$

(2) $f(x) = \dfrac{(e^{x^2-1}-1)\log(x-1)}{\sqrt{\log x}} \qquad x \to 1^+$

(3) $f(x) = \dfrac{(\sqrt{x}-1)^3}{\sin(x-1)} \qquad x \to 1$

(4) $f(x) = \dfrac{(\sqrt{x}-1)^3 \log(x-1)}{\sin(x-1)} \qquad x \to 1^+$

(5) $f(x) = \dfrac{(\sqrt{x}-1)e^{\frac{1}{1-x}}}{\sin(x-1)} \qquad x \to 1^+$

(6) $f(x) = \log(1+e^x-e^2) \qquad x \to 2$

(7) $f(x) = \dfrac{\log(1+e^x-e^2)}{\sqrt{2x}-2} \qquad x \to 2$

(8) $f(x) = \dfrac{\log(\cos x + x^3)}{x\sin x + x^5} \qquad x \to 0$

Esercizio 4. Determinare gli eventuali asintoti orizzontali od obliqui delle funzioni che seguono.

(1) $f(x) = \dfrac{2x^2+3x^{3/2}+3}{3(x+5)^2+\sqrt{x}} \qquad x \to +\infty$

(2) $f(x) = \dfrac{2x^3 + 3x^{3/2} + 3}{3(x+5)^2 + \sqrt{x}} \qquad x \to +\infty$

(3) $f(x) = \log\Big(\dfrac{2x^3 + 3x^{3/2} + 3}{3(x+5)^2 + \sqrt{x}}\Big) \qquad x \to +\infty$

(4) $f(x) = e^{\sqrt{x^2+5x-4}-x} \qquad x \to +\infty$

(5) $f(x) = e^{\frac{1}{3x^2+7}} + 2x \qquad x \to +\infty$

(6) $f(x) = \log(2e^{x+1} + 3x^{10}) \qquad x \to +\infty$

(7) $f(x) = \log(x^{12}e^x + |x|) \qquad x \to -\infty$

(8) $f(x) = \log(x^{12}e^x + 3e^{-x}) \qquad x \to -\infty$

(9) $f(x) = \log(x^{12}e^{-x} + 1) \qquad x \to -\infty$

(10) $f(x) = e^{\frac{1}{3x^2+7}} + 2|x| \qquad x \to -\infty$

Esercizio 5. Determinare, se esiste, l'ordine di infinito delle seguenti funzioni.

(1) $f(x) = \dfrac{\log x - \log(x^3+7)}{\log(5x)} \, \dfrac{\sqrt{x+1}-1}{\sqrt{x^2+1}-x} \qquad x \to +\infty$

(2) $f(x) = \dfrac{3x^2 \operatorname{tg}\left(\frac{1}{x}\right) + x^3 \log\left(\frac{x+1}{x}\right)}{(\operatorname{arctg} x)(\sqrt{x^4+3x} - x^2)} \qquad x \to +\infty$

(3) $f(x) = x^4 + x \sin x - e^{1/x} \qquad x \to +\infty$

(4) $f(x) = \log\Big(\dfrac{x^2 + x^3}{1 + x + x^2}\Big) + 3x^6 \qquad x \to +\infty$

(5) $f(x) = \Big(1 - \cos\Big(\dfrac{1}{x}\Big)\Big)(1 + x^3) \qquad x \to +\infty$

(6) $f(x) = \log(1 + 2x^4 + e^{2x}) \qquad x \to +\infty$

(7) $f(x) = \dfrac{3^{1/x} - 1}{\operatorname{arctg}\left(\frac{1}{x^4}\right)} \qquad x \to +\infty$

(8) $f(x) = \dfrac{\log(x^4 + 4x^3)}{x^{1/x} - 1} \qquad x \to +\infty$

(9) $f(x) = \dfrac{x}{|x-1|} \qquad x \to 1$

(10) $f(x) = \dfrac{x}{|x^2-1|} \qquad x \to 1$

(11) $f(x) = \dfrac{x}{\sqrt{x^2-1}} \qquad x \to 1^+$

(12) $f(x) = \dfrac{\sin x \log(9+x)}{e^{x^3} - \cos x} \qquad x \to 0$

(13) $f(x) = (\operatorname{tg} x)^{\log x} \qquad x \to 0^+$

(14) $f(x) = \dfrac{\arcsin(x^2 + x\log x)}{(5x - \log x)(x^2 + e^{-1/x})} \qquad x \to 0^+$

(15) $f(x) = \dfrac{e^{x^2} - e^x}{1 - \cos x} \qquad x \to 0^+$

(16) $f(x) = \dfrac{e^x - e^{-x}}{x^2} \qquad x \to 0$

(17) $f(x) = (1 + x)^{1/x^2} \qquad x \to 0^+$

(18) $f(x) = \left(\dfrac{e^x + 1}{e^x - 1}\right)^{e^{2x}} \qquad x \to 0^+$

(19) $f(x) = \dfrac{\frac{1}{x^5} \log\left(1 + x^4 \sin \frac{1}{x}\right) + \frac{2}{x^2} \log x}{\operatorname{tg} x + \arcsin x^2} \qquad x \to 0$

Esercizio 6. Determinare, se esiste, l'ordine di infinitesimo delle seguenti funzioni.

(1) $f(x) = \dfrac{\sin^2 x}{x} \qquad x \to 0$

(2) $f(x) = \dfrac{x}{\cos x} \qquad x \to 0$

(3) $f(x) = \dfrac{x}{x+1} \sin x \qquad x \to 0$

(4) $f(x) = \dfrac{1 - \cos x}{x} \qquad x \to 0$

(5) $f(x) = \dfrac{\sin x}{3\sqrt{x}} \qquad x \to 0^+$

(6) $f(x) = \dfrac{x^2}{|x|} \qquad x \to 0$

(7) $f(x) = \dfrac{\log(1 + x^3)}{x} \qquad x \to 0$

(8) $f(x) = \dfrac{\log(1 + x^2)}{e^x - 1} \qquad x \to 0$

(9) $f(x) = \dfrac{x^2 + \operatorname{arctg} x}{x \log x + e^{x^2} - 1} \qquad x \to 0^+$

(10) $f(x) = \dfrac{e^x - \cos x}{\operatorname{arctg} \sqrt{x} + \sin x} \qquad x \to 0^+$

(11) $f(x) = 1 - |\log x|^{\frac{1}{|\log x|}} \qquad x \to 0^+$

(12) $f(x) = (x - 1) \log x \qquad x \to 1$

(13) $f(x) = \dfrac{\sin \sqrt{x - 2} + (x - 2)^2}{\sqrt[4]{x - 2} + 1 - \cos \pi x} \qquad x \to 2^+$

(14) $f(x) = \dfrac{\sin x - 1}{2x - \pi} \qquad x \to \pi/2$

(15) $f(x) = (\sqrt{x + 1} - \sqrt{x - 1}) \sin\left(\dfrac{1}{x}\right) \qquad x \to +\infty$

(16) $f(x) = \dfrac{\operatorname{arctg}\left(\frac{1}{x}\right)}{x^4 + 3\sqrt{x}} \qquad x \to +\infty$

(17) $f(x) = \cos(\sqrt{x} - \sqrt{x+1}) - 1 \qquad x \to +\infty$

(18) $f(x) = \dfrac{5x^2 - 1}{x^3 + 1} \cdot \dfrac{1 - e^{\cos(1/x)}}{\operatorname{arctg}(x \log x)} \qquad x \to +\infty$

Esercizio 7. Calcolare il limite per $n \to +\infty$ delle seguenti successioni

(1) $a_n = \sin\left(2\pi\sqrt{n^2 + \sqrt{n}}\right)$

(2) $a_n = n \sin\left(\dfrac{n^2(n-1) - 3}{n}\pi\right).$

(3) $a_n = (2^n - n!) \sin\left(\dfrac{\pi}{2} + \operatorname{arctg}(n!)\right).$

(4) $a_n = (3^n - n!) \cos\left(\pi + \operatorname{arctg}(n!)\right).$

(5) $a_n = \dfrac{n^{n+1} + 2 \cdot n!}{(n-1)^n + 2n^n} \operatorname{tg}\left(\dfrac{2\pi}{n}\right).$

(6) $a_n = \dfrac{e^{n + \log n} + n \sin(n!)}{e^{n+1} + n^2} \operatorname{tg}\left(\dfrac{2\pi}{n}\right).$

(7) $a_n = \cos n \cdot \log\left(1 + \dfrac{1}{n}\right)$

(8) $a_n = \dfrac{\log(1 + e^n)}{n}$

(9) $a_n = \log\left(1 + \dfrac{\pi^n}{n!}\right)$

(10) $a_n = \dfrac{\log(3^n + \pi^n)}{2n}$

(11) $a_n = \dfrac{\log(4^n + \pi^n)}{2n + \log n}$

(12) $a_n = \dfrac{(3 + n! + \pi^{3n})(n + \sqrt{n})}{(2 \cdot n! + 3)\log(1 + n^n)}$

(13) $a_n = \log(n + \sqrt{n^2 + 1}) - \log n$

(14) $a_n = \log(n - \sqrt{n^2 - 1}) + \log n$

(15) $a_n = \dfrac{\log(n^2 + 1) - \log n}{\log(2n)}\left(n^2 + n^{3/2}\log n - 2\sqrt{n}\right)$

(16) $a_n = \dfrac{\log((n+6)!) - \log(n! + n^7)}{\log(6n^8 + \sin\frac{n\pi}{2})}$

(17) $a_n = 2^{n+1} - 2^{\sqrt{n^2 - 1}}$

(18) $a_n = \sqrt[n]{n^3}$

(19) $a_n = \sqrt[2n+1]{-n}$

(20) $a_n = \sqrt[n]{n^2 + 3}$

(21) $a_n = \sqrt[n]{2^n + n^2}$

(22) $a_n = \left(\sqrt{n+\sqrt{n}} - \sqrt{n}\right) \sqrt[n]{2^{n+1}+n^2}$

(23) $a_n = \left(\dfrac{2n+1}{3n+4}\right)^{n/3}$

(24) $a_n = \left(\dfrac{2n+1}{3n+4}\right)^{3/n}$

(25) $a_n = \left(\dfrac{3n+4}{2n+1}\right)^{n}$

(26) $a_n = (\sqrt[n]{2}-1)^n$

(27) $a_n = \left(1+\dfrac{1}{n!}\right)^{n^n}$

(28) $a_n = \left(1+\dfrac{1}{n^n}\right)^{n!}$

(29) $a_n = \dfrac{(2n+1)^n + n^n}{(2n+2)^n - n^n}$

(30) $a_n = n^2(2n+\sqrt{n}\,)^{1/n} - \cos(n^3)$

(31) $a_n = \left(4n^n - (n+1)^n\right)^{1/n}$

(32) $a_n = \left((n+1)^{n+1} - n^{n+1}\right)^{1/n}$

(33) $a_n = \dfrac{(n^2-1)^n \, \sin(\frac{\pi n+1}{4n-1})}{(n-2)^{2n} + \operatorname{arctg}(n! - 2^n)}$

(34) $a_n = \dfrac{(3n^{2n^2} - (n^2+1)^{n^2})^2 + 4n^{n^2}}{(n^2+5)^{2n^2} + 6(n^2)^{n^2+n}}$

(35) $a_n = \dfrac{((n+1)!)^{2n}((7\cdot n!)^{3/n!}-1)}{n^{2n}\log((n!)^6)((n!)^{2n-1}+n^2)}.$

(36) $a_n = \dfrac{\log((n!)^2)((n!)^{3n-1}+n^3)}{(n!)^{3n}((8\cdot n!)^{2/n!}-1)}.$

(37) $a_n = \dfrac{n^{3n}\log((n!)^2)(n!)^{3n-1}}{(((n+1)!)^{3n}+n^3)((8\cdot n!)^{2/n!}-1)}.$

(38) $a_n = \dfrac{\log(n!)}{n\log n}$

(39) $a_n = ((n-1)!)^{1/n^3}$

(40) $a_n = \dfrac{((n+1)!)^{1/n^3} - ((n-1)!)^{1/n^3}}{\log(n^4+n)\,\operatorname{tg}(\frac{1}{2n^3})}.$