Analisi Matematica I
Limiti di successioni

Esercizio 1. Verificare, usando la definizione, i seguenti limiti di successioni

(1) $\lim_{n\to\infty} \frac{2n+3}{n+1} = 2$

(2) $\lim_{n\to\infty} \frac{n^2+3n}{n^2+1} = 1$

(3) $\lim_{n\to\infty} \sqrt{\frac{2n+3}{2n+1}} = 1$

(4) $\lim_{n\to\infty} \frac{4\sqrt{n}-3}{\sqrt{n}+1} = 4$

(5) $\lim_{n\to\infty} \sqrt{n+1} - \sqrt{n} = 0$

(6) $\lim_{n\to\infty} \sqrt{n^2+n+1} = +\infty$

(7) $\lim_{n\to\infty} \frac{n^2+1}{n+1} = +\infty$

(8) $\lim_{n\to\infty} \log_2 \frac{1}{n} = -\infty$

(9) $\lim_{n\to\infty} \sqrt{n} - n = -\infty$

Esercizio 2. Calcolare il limite per $n \to \infty$ delle seguenti successioni

(1) $a_n = \frac{n^3+2n^2-\sqrt{n}}{n^2+3n-1}$

(2) $a_n = \frac{n^3+2n^2-\sqrt{n}}{n^2+3n-1}\left(\frac{1}{n}-\frac{2}{n^2}\right)$

(3) $a_n = \frac{n^2+3n^{3/2}+\sqrt{n+3}+1}{5n^3+\sqrt[3]{n+7}}$

(4) $a_n = \frac{n^2+3n^{3/2}+\sqrt{n+3}+1}{5n^2+\sqrt[3]{n+7}}$

(5) $a_n = \frac{n^2+3n^{3/2}+\sqrt{n+3}+1}{5n^{1/3}+\sqrt[3]{n+7}}$

(6) $a_n = \left(\frac{2n^2+3+\sqrt{n}}{n^2+1}-2\right)(n^{4/3}+2n+1)$

(7) $a_n = \left(\frac{2n^2+3+\sqrt{n}}{n^2+1}-2\right)^2(n^{3/2}+7n^2+\pi)$

(8) $a_n = \sqrt{\frac{2n^2+3+\sqrt{n}}{n^2+1}-2}\;(7n+2)$

(9) $a_n = \Big(\dfrac{2n^2+3+\sqrt{n}}{n^2+1} - 2\Big)\sqrt{7n+2}$

(10) $a_n = \sqrt{n+1} - \sqrt{n}$

(11) $a_n = \sqrt{n+1} - n$

(12) $a_n = \sqrt{n^2+1} - n$

(13) $a_n = \sqrt{n^2+n+1} - \sqrt{n}$

(14) $a_n = \sqrt{n^2+n+1} - n$

(15) $a_n = \sqrt{n^2+\sqrt{n+1}} - n$

(16) $a_n = (\sqrt{n^2+\sqrt{n+1}} - n)^2$

(17) $a_n = (\sqrt{n^2+\sqrt{n+1}} - n)^2\sqrt{n+1}$

(18) $a_n = \sqrt{n^2+n^3+1} - n$

(19) $a_n = \sqrt{n^2+n^{1/3}+1} - n$

(20) $a_n = \sqrt{n^2+n^3+1} - n^{3/2}$

(21) $a_n = \sqrt{n^3+n+1} - n^{3/2}$

(22) $a_n = \dfrac{n^2+3n^{3/2}+\sqrt{n+3}+1}{5n^{1/3}+\sqrt[3]{n+7}}(\sqrt{n^3+n+1} - n^{3/2})$

(23) $a_n = \dfrac{n^2+3n^{3/2}+\sqrt{n+3}+1}{5n^{1/3}+\sqrt[3]{n+7}} + \sqrt{n^3+n+1} - n^{3/2}$

(24) $a_n = \Big(\dfrac{n^2+3n^{3/2}+\sqrt{n+3}+1}{5n^{1/3}+\sqrt[3]{n+7}}\Big)^2 + \sqrt{n^3+n+1} - n^{3/2}$

(25) $a_n = \sqrt[3]{n^3+2n^2} - n$

(26) $a_n = \sqrt[3]{n^6-n^4+1} - n^2$

Esercizio 3. Calcolare il limite per $n \to \infty$ delle seguenti successioni

(1) $a_n = \dfrac{\sqrt{n+\sqrt{n+3}} - n5^{-\sqrt{n}} + 3}{\big(n^5+3\,\mathrm{arctg}(n!)+7\big)^{2/7}}$

(2) $a_n = \dfrac{\sqrt{n+\sqrt{20^n+1}} + 5\cdot 2^n\sqrt{n} + 2}{3^n + 8\cdot 5^{-n^2+n} + 1}$

(3) $a_n = \dfrac{n!}{(n+1)! - (n-1)!}$

(4) $a_n = \dfrac{n^{n-3}(n+3)! + n^{n-2}(n+2)!}{n!\cdot n^n}$

(5) $a_n = \dfrac{n!(2n + 3\cos n) - (n+1)!}{n!(2n - \log_3 n) + 2^{\log_3(n!)}}$

(6) $a_n = \dfrac{2n! + (2n)!}{n^n + 3n!}$

(7) $a_n = \dfrac{(\sqrt{n+1} + \sqrt{n}\,)n! + 3n^{51} + 5^{n+1}}{(n-1)!\,\big(4n + n^{1/3} + \sin(n^5+3)\big)^{3/2}}$

(8) $a_n = \dfrac{(\sqrt{n+1} - \sqrt{n}\,)n! + 3n^{51} + (n+1)5^{n+1}}{(n-1)!\,\sqrt{4n + 2n^2 + n^{3/2}\sin(n^5+3)}}$

(9) $a_n = \dfrac{n^n + 3^n}{2^{n\log_2 n}}$

(10) $a_n = \dfrac{n^n + n!}{2^{n^2}}$

(11) $a_n = \dfrac{n!(n^7 + 5n^2 + 1)2^n}{6^{n^2}}$

(12) $a_n = \dfrac{5^{n\log_5 n} - 5^n}{n^{\log_5 n} + n^{n + \log_5 n}}$

(13) $a_n = \dfrac{n! \cdot 3^{(n+1)!} + 5^{(n+1)!}}{\big((n+1)!\big)^2}$

(14) $a_n = \dfrac{n! \cdot 7^{n!} - 5^{(n+1)!}}{\big((n+1)!\big)^2 + 32^{n^2} + 1}$

(15) $a_n = \dfrac{n! \cdot 7^{n(n+1)} + 4^{(n+1)!}}{(n+1)^n}$

(16) $a_n = \dfrac{(n!)^n \cdot 7^{n(n+1)} + 4^{(n+1)!}}{\big((n-1)!\big)^n}$

(17) $a_n = \dfrac{(n! \cdot 7^n)^{n+1} - 4^{(n+1)!} + 2n^n}{n^{(n-1)!}}$

(18) $a_n = \dfrac{(n-3)!\,n^n - (n+1)!\,n^{n-4}}{2(n-4)!\,(n^n - n!\log_4 n)}$

Esercizio 4. Determinare l'ordine di infinito/infinitesimo delle seguenti successioni

(1) $a_n = \log_{12} n + \sqrt[12]{n}$

(2) $a_n = n^{150} + \left(\dfrac{3}{2}\right)^n$

(3) $a_n = n^{1/2}(1 + n^{1/4})$

(4) $a_n = \dfrac{1}{\sqrt{n+1} - \sqrt{n}}$

(5) $a_n = \dfrac{n!}{(n-1)!} - 3$

(6) $a_n = \dfrac{(n+1)! - (n-1)!}{n!}$

(7) $a_n = \dfrac{5^{2n \log_5 n} - 3^{n \log_3 (n^2)} + n^4}{7^{n^2 \log_7 (n^2)} - 4^{2n^2 \log_4 n} + n^2}$

(8) $a_n = \dfrac{\sqrt{n + \sqrt{n+3}} - n^3 4^{-\sqrt{n}} + 5}{\left(n^5 - 4 \operatorname{arctg}(n^n) + 12\right)^{1/16}}$

(9) $a_n = \dfrac{2n!\,(n^n - n!\log_2 n)}{(n-3)!\,n^n - (n+1)!\,n^{n-4}}$

(10) $a_n = \dfrac{((n+1)!)^2\, 5^{n \log_5 n} - (n!)^2\, 3^{(n-1)\log_3 n}}{((n-1)!)^2\, 4^{(n-2)\log_4 n} - ((n-2)!)^2\, 6^{(n+1)\log_6 n}}$

(11) $a_n = \dfrac{1}{\sqrt{n}} - \left(\dfrac{2}{3}\right)^n$

(12) $a_n = \dfrac{\sqrt{n} + n^{7/3}}{n^5 + n^3 + 8}$

(13) $a_n = 2 - \dfrac{2n^2}{n^2 + n}$

(14) $a_n = \dfrac{n!}{(n+1)! - (n-1)!}$

(15) $a_n = \dfrac{(n^2+1)(7^{n!} + 3^{n^2})}{(7^{n!} + 9^{n^2+n})(n^3 + \log_4 n)}$

(16) $a_n = \dfrac{n^{2(n+4)}\, n! - (n-1)!\, n^{2n+7}}{(n+1)!\, n^{2n+9} + (n^2)^n\, (n+2)!}$

Esercizio 5. Calcolare il limite per $n \to \infty$ delle seguenti successioni

(1) $a_n = \left(1 + \dfrac{1}{2n}\right)^n$

(2) $a_n = \left(1 + \dfrac{1}{2n+1}\right)^n$

(3) $a_n = \left(1 + \dfrac{1}{n}\right)^{2n}$

(4) $a_n = \dfrac{(n+1)^n}{n^n + 3}$

(5) $a_n = \dfrac{(n+1)^n}{n^n + n^2}$

(6) $a_n = \dfrac{(2n+1)^n}{(2n)^n + n^4}$

(7) $a_n = \dfrac{(2n)^n + 2^n}{(2n+1)^n}$

(8) $a_n = \dfrac{(2n+1)^n}{2n^n + 1}$

(9) $a_n = (n+1)^n - n!$

(10) $a_n = (n+1)^{n+1} - n^{n+1}$

(11) $a_n = \dfrac{(n+1)^{n+1} - n^{n+1}}{(n-1)^{n+1} - n!}$

(12) $a_n = (n+1)^{n!} - n^{2n}$

(13) $a_n = \dfrac{(n+1)^n + n!}{n^n + 5^n - n!}$

(14) $a_n = \dfrac{(2n+1)^n + n! + 1}{(2n+2)^n - n! + n^2}$

(15) $a_n = \dfrac{(2n+1)^n + (2n)^n}{(2n+2)^n - (2n+1)^n}$

(16) $a_n = \dfrac{n^{n-3} + (n-3)^n}{6n^n + 7n^{n/2}}$

(17) $a_n = \dfrac{(3n)^n - n^{3n}}{(n-1)^{3n} + (n+3)!}$

(18) $a_n = \left(\sqrt{1+e^{-n}} - 1\right)\left(e^n - 3^n + n^2\right)$

(19) $a_n = \dfrac{(e^n+1)(n+\log n)}{(e^n+2^n)(2+\log n^6)}$

(20) $a_n = \dfrac{(n^2+5n+7)e^{1/n}}{(n+1)(\sqrt{n+3} - \sqrt{n})}$

(21) $a_n = \sqrt[n]{2n}$

(22) $a_n = \sqrt[n]{n^3}$

(23) $a_n = \sqrt[2n+1]{-n}$

(24) $a_n = \sqrt[n]{n^2+3}$

(25) $a_n = \sqrt[n]{2^n + n^2}$

(26) $a_n = \left(\sqrt{n+\sqrt{n}} - \sqrt{n}\right)\sqrt[n]{2^{n+1} + n^2}$

(27) $a_n = \dfrac{(n^2+5n+7)e^{1/n}}{(n+1)(\sqrt{n+3} - \sqrt{n})}$

(28) $a_n = n^2(2n + \sqrt{n}\,)^{1/n} - \cos(n^3)$

(29) $a_n = \left(4n^n - (n+1)^n\right)^{1/n}$

(30) $a_n = \big((n+1)^{n+1} - n^{n+1}\big)^{1/n}$

Esercizio 6. Calcolare il limite per $n \to \infty$ delle seguenti successioni, usando la formula di Stirling

(1) $a_n = \dfrac{n!}{n^{n/2}}$

(2) $a_n = \sqrt[n]{n!}$

(3) $a_n = \dfrac{\sqrt[n]{n!}}{n}$

(4) $a_n = \dfrac{\sqrt[2n]{n!}}{n}$

(5) $a_n = \dfrac{1}{n}\sqrt[n]{\dfrac{(2n)!}{n!}}$

(6) $a_n = \sqrt[n]{\dbinom{2n}{n}}$

(7) $a_n = \sqrt[n]{\dbinom{4n}{2n}}$