Analisi Matematica II
Serie numeriche

Esercizio 1. Determinare se le serie seguenti convergono (assolutamente o semplicemente)

(1) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}-\sqrt{n}}$

(2) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n-1}}{n}$

(3) $\displaystyle\sum_{n=1}^{\infty} \frac{(\sqrt{n}+1)^n\left(\sqrt{n^n+n^{-5}}-\sqrt{n^n-n^{-6}}\right)}{\sqrt{n+n^{-1}}-\sqrt{n-n^{-2}}}$

(4) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \sin\frac{1}{\log n}$

(5) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{2+n^2\log n}{n^3}$

(6) $\displaystyle\sum_{n=1}^{\infty} \frac{n!+n^n}{\binom{2n}{n}}$

(7) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n-2^{n\log n}}{\binom{3n}{n}}$

(8) $\displaystyle\sum_{n=1}^{\infty} \left(1-n^2\sin^2\frac{1}{n}\right)$

(9) $\displaystyle\sum_{n=1}^{\infty} \left(1-\frac{2}{n}\right)^{n^2}$

(10) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\cos\frac{3}{n}}{\cos\frac{2}{n}}\right)^{n^3}$

Esercizio 2. Trovare i valori di $\beta \in \mathbb{R}$ per cui risultano convergenti le seguenti serie

(1) $\displaystyle\sum_{n=1}^{+\infty} \frac{\log(1+n^2)}{(n+3)^\beta}$

(2) $\displaystyle\sum_{n=1}^{+\infty} \frac{\log(1+2^n)}{(n+3)^\beta}$

(3) $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2+\log(1+n^2)}{(1+n^3)^\beta}$

(4) $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2\log(1+n^2)}{(1+n^3)^\beta}$

(5) $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2+\log(1+2^n)}{(1+n^3)^\beta}$

(6) $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2\log(1+2^n)}{(1+n^3)^\beta}$

(7) $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2}{(1+n^3)^\beta + \log(1+n^2)}$

(8) $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2}{(1+n^3)^\beta + \log(1+2^n)}$

(9) $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2}{(1+n^3)^\beta \log(1+n^2)}$

(10) $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2}{(1+n^3)^\beta \log(1+2^n)}$

(11) $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2}{(1+n^3)(\log(1+n^2))^\beta}$

(12) $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2}{(1+n^3)(\log(1+2^n))^\beta}$

(13) $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2}{(1+n^3)^\beta \log(1+\frac{1}{n^2})}$

(14) $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2}{(1+n^3)(\log(1+\frac{1}{n^2}))^\beta}$

(15) $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2}{(1+n^3)^\beta \log(1+2^{-n})}$

(16) $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2}{(1+n^3)(\log(1+2^{-n}))^\beta}$

(17) $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{n^2(n^\beta+1)}$

Esercizio 3. Trovare i valori di $x \in \mathbb{R}$ per cui risultano convergenti (semplicemente o assolutamente) le seguenti serie

(1) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n-1}}{n(2n-1)}$

(2) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n \log n}{1+\sqrt{n}}$

(3) $\displaystyle\sum_{n=2}^{\infty} \frac{(x-\frac{1}{2})^n}{\log n}$

(4) $\displaystyle\sum_{n=1}^{\infty} (-1)^n (n+1) x^n$

(5) $\displaystyle\sum_{n=1}^{\infty} \frac{(3-x)^n}{3^n\sqrt{n^2-1}}$

(6) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2(x+1)^n}{\sqrt{n^2-1}+3^n}$

(7) $\sum_{n=1}^{\infty} n\left(\frac{2}{3}\right)^n\left(x-\frac{3}{2}\right)^n$

(8) $\sum_{n=1}^{\infty} \frac{2^{n+1}}{e^{nx}}$

(9) $\sum_{n=1}^{\infty} \frac{2^n(\sin x)^n}{n}$

(10) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n^2-1}}{(x-1)^n}$

(11) $\sum_{n=1}^{\infty}(n^2+2)\left(\frac{x+1}{x-1}\right)^n$

(12) $\sum_{n=1}^{\infty}(4-3^x)^n \ \operatorname{tg}\left(\frac{\sqrt{n}+2}{n+n^2}\right)$

(13) $\sum_{n=1}^{\infty} \frac{x^n}{1+nx^2}$

(14) $\sum_{n=1}^{\infty}\left(1-\frac{x}{n}\right)^{n^2}$

(15) $\sum_{n=2}^{\infty} \frac{1}{(\log n)^x}$

(16) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^x(n^2+1)}}$

(17) $\sum_{n=1}^{\infty} \frac{(1-x)^n}{n^x}$

(18) $\sum_{n=1}^{\infty} \frac{n^{2x}-1}{n^3+1}$

(19) $\sum_{n=1}^{\infty} \frac{n^{x-2}+n^{4-x^2}}{n^2}$

(20) $\sum_{n=1}^{\infty}\left(1-\sqrt[n]{2}\cos\sqrt{\frac{x}{n}}\right)$