Esercizio 8. Calcolare i seguenti limiti.

(1) $\lim_{x\to 0} \frac{e^{-x^2} - \cos(\sqrt{2}x)}{x^3 \sin x}$

(2) $\lim_{x\to 0} \frac{\cos(\sqrt{2}x^2) - e^{-x^4}}{x^5(\arcsin x)^3}$

(3) $\lim_{x\to 0} \frac{x^2 - \sin^2 x}{x^3(e^x - \cos x)}$

(4) $\lim_{x\to 0} \frac{x^2(\arcsin x - x)}{\sin^3 x - x^3}$

(5) $\lim_{x\to 0} \frac{\sin^2 x - \log(1+x^2)}{x^2 - x \arcsin x}$

(6) $\lim_{x\to 0} \frac{\log(1+x^3) - \sin^3 x}{x^2 \, \mathrm{arctg}(x^3)}$

(7) $\lim_{x\to 0} \frac{e^{x^2} - 1 - x\log(1+x)}{\mathrm{tg}^2 x \, \log(1+x)}$

(8) $\lim_{x\to 0} \frac{\sin(x^2) - (\log(1+x))^2 - x^3}{x^2 \, \mathrm{arctg}(x^2)}$

(9) $\lim_{x\to 0} \frac{\sin(6x) - 6x\, e^{-6x^2}}{x^3(\mathrm{arctg}\, x)^2}$

(10) $\lim_{x\to 0} \frac{\log(1+x) - x}{\sqrt{\cos x} - 1}$

(11) $\lim_{x\to 0} \frac{\mathrm{arctg}(\log(1+x^2)) - x^2}{x^4}$

(12) $\lim_{x\to 0} \frac{\sin x - x - \log(\cos x)}{x \sin x}$

(13) $\lim_{x\to 0} \frac{(\sin^2 x - \log(\cos x))\log(1+\sin x)}{\sin^2 x \sin(2x)}$

(14) $\lim_{x\to 0} \frac{e^x - e^{\sin x}}{\mathrm{tg}\, x - x}$

(15) $\lim_{x\to +\infty} \left(3x^2 \log\left(1+\frac{1}{x}\right) - 2x + (x^2+x)\log\left(1-\frac{1}{x}\right)\right)$

(16) $\lim_{x\to +\infty} x^4\left(1 - \cos\frac{2}{x} - x\log\left(1+\frac{2}{x^3}\right) + \frac{3}{x^5}\right)$

(17) $\lim_{x\to 1} \frac{e^x - 4e^{\sqrt{x}} + 3e^{\sqrt[3]{x}}}{\log x - x + 1}$

(18) $\lim_{x\to \pi} \frac{(\pi - x)^2 - 8 + 8\sin(x/2)}{4\cos^2(x/2) - (\pi - x)^2}$

(19) $\lim_{x\to 0} \frac{\log(1+x^2) - \sinh(x^2)}{x^2(\mathrm{arctg}\, x)^2}$

(20) $\lim_{x\to 0} \dfrac{\log(1+x^2) - x\sinh(x) + \frac{2}{3}x^4}{(\operatorname{arctg} x - x)^2}$

(21) $\lim_{x\to 0} \dfrac{\sqrt{1+x^2} - \cosh(x)}{(e^{x^2} - \cos x)^2}$

Esercizio 9. Calcolare il limite per $n \to \infty$ delle seguenti successioni.

(1) $a_n = \left(\sqrt{n^2+n} - n\right)\left(3n^2 \log\left(1 + \dfrac{1}{n}\right) - 2n + (n^2+n)\log\left(1 - \dfrac{1}{n}\right)\right)$

(2) $a_n = n^4\left(1 - \cos\dfrac{2}{n} - n\log\left(1 + \dfrac{2}{n^3}\right) + \dfrac{3}{n^5}\right)$

(3) $a_n = n^3 \sqrt[n]{\dfrac{n+1}{n}} - n^3 - n$

(4) $a_n = n^3 \sqrt{\dfrac{n+1}{n}} - n^3 - \dfrac{1}{2}n^2 + \dfrac{1}{8}n$

(5) $a_n = \dfrac{(\operatorname{arctg}\frac{1}{n} - \frac{1}{n})(\sin\frac{1}{n} + e^{-n})}{e^{-\frac{1}{2n^2}} - \cos\frac{1}{n}}$

(6) $a_n = \dfrac{(1+\frac{2}{n})^{n^2} + n^{\sqrt{n}}}{e^{2n} - 3^{n \operatorname{arctg} n}}$

(7) $a_n = \dfrac{(n+\sqrt{n})^n + (n+2)^n + 3n!}{(n+4)^n(e^{\sqrt{n+1}} + 2^{\sqrt{n}})}$

(8) $a_n = \dfrac{n^{n+1}(n^{\sqrt{n+1}-\sqrt{n}} - 1)(\sqrt{n+1} + 1)}{(n+\log n)^n(1+\log n)}$

(9) $a_n = \dfrac{3n! + (en)^n + (5\sqrt[3]{n+2})^n}{\left((n+1)(1+\frac{1}{n})^n + 1\right)^n}$

(10) $a_n = \dfrac{(n - \log n + 2)^n(n + \log\log n)^n}{(n^2 + 5\log n)^n - (n^2+2)^n}$

(11) $a_n = \dfrac{(2 + n^{1/n})^n(\sqrt[3]{\sqrt{n}+2} - \sqrt[3]{\sqrt{n}+1}\,)}{(1 + 2\pi^{1/n})^n + (1 + \log n)^{\log n}}$

(12) $a_n = \dfrac{(2 + 8^{1/n})^n - 2\cdot 3^n}{(3n^{1/n} + \frac{1}{n})^n(\sqrt[n]{n^2+2n} - n^{2/n})}$

(13) $a_n = \dfrac{\left(e^{3/n^2} - \cos(\frac{1}{2n})\right)^{n^2}}{\left(e^{2/n^2} - \cos(\frac{3}{2n})\right)^{n^2}}$