Linear Algebra Issues in Wireless Communications

Day 3

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Content of Day#3

- Matrix problems in radio resource management
- Criteria for user scheduling and its matrix formalization
- Robustness & accuracy
- The simplest one doesn't mean the fastest one

Resource Allocation: First Definition

Problem Formulation (imprecise):

Select beamforming to maximize "system utility"

Means: Allocate power to users and in spatial dimensions

Satisfy: Physical, regulatory & economic constraints

Some Assumptions:

Linear transmission and reception

Perfect synchronization (whenever needed)

Flat-fading channels (e.g., using OFDM)

Perfect channel knowledge

Ideal transceiver hardware

Centralized optimization

Mathematical Model

Definitions (PHY):

$\mathbf{H}_q^k \in \mathbb{C}^{N_r \times N_t}$ - DL channel matrix of UE #q to grouping area (cluster) k;

$\mathbf{W}_q^k \in \mathbb{C}^{N_t \times N_r}$ - precoding matrix for cluster k;

$\mathbf{Y}_q^k \in \mathbb{C}^{N_r \times 1}$ - received signal by user q in cluster k.

Let us define number of users Q_k scheduled in cluster k, then we can formulate system model

$$\mathbf{Y}_q^k = \mathbf{H}_q^k \sum_{i=1}^{Q_k} \mathbf{W}_i^k \mathbf{X}_i^k + \sum_{\substack{j=1\ldots K \\ j \neq k}} \mathbf{H}_q^j \sum_{i=1}^{Q_k} \mathbf{W}_i^j \mathbf{X}_i^j + \boldsymbol{w}_q^k,$$

where

$\sum_{\substack{j=1\ldots K \\ j \neq k}} \mathbf{H}_q^j \sum_{i=1}^{Q_k} \mathbf{W}_i^j \mathbf{X}_i^j$ - interference from other clusters, $\boldsymbol{w}_q^k$ – thermal noise of user terminal.

Key points

How to select subset of intra-cell users ?

How to select subset of inter-cluster users ?

$$\mathbf{Y}_q^k = \mathbf{H}_q^k \sum_{i=1}^{Q_k} \mathbf{W}_i^k \mathbf{X}_i^k + \sum_{\substack{j=1\ldots K \\ j\neq k}} \mathbf{H}_q^j \sum_{i=1}^{Q_k} \mathbf{W}_i^j \mathbf{X}_i^j + \mathbf{w}_q^k$$

How to define optimal weighting to increase spectrum efficiency ?

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How to Measure User Performance?

Mean Square Error (MSE)

- Difference: transmitted and received signal
- Easy to Analyze
- Far from User Perspective?

Bit/Symbol Error Probability (BEP/SEP)

- Probability of error (for given data rate)
- Intuitive interpretation
- Complicated & ignores channel coding

Information Rate

- Bits per "channel use"
- Mutual information: perfect and long coding
- Anyway closest to reality?

$$g_k(\mathrm{SINR}_k) = \log_2(1 + \mathrm{SINR}_k)$$

$$\mathrm{MSE}_k = M - \frac{\mathrm{SINR}_k}{1+\mathrm{SINR}_k}$$

M data streams are intended for transmission to user k

All improves with SINR:

$$\frac{\text{Signal}}{\text{Interference + Noise}}$$

Generic Measure User Performance

Covariance matrix of interference+noise:

$$\mathbf{R}_{uu}^{(q,k)} = \sum_{\substack{j=1...K \\ j\neq k}} \sum_{i=1}^{Q_k} \mathbf{H}_q^j \mathbf{W}_i^j \mathbf{R}_{xx}^{(i,j)} \left(\mathbf{H}_q^j \mathbf{W}_i^j\right)^H + \sigma^2 \mathbf{I}$$

For grouping users the maximum rate of each user is

$$r_q = \log_2 \frac{\left| \mathbf{I} + \tilde{\mathbf{H}}_q^k \left(\sum_{i=1}^{Q_k} \mathbf{R}_{uu}^{(i,k)} \right) \left(\tilde{\mathbf{H}}_q^k \right)^H \right|}{\left| \mathbf{I} + \tilde{\mathbf{H}}_q^k \left(\sum_{\substack{i=1 \\ i\neq q}}^{Q_k} \mathbf{R}_{uu}^{(i,k)} \right) \left(\tilde{\mathbf{H}}_q^k \right)^H \right|}.$$

$$g = \frac{\mathrm{r}_i'(\mathrm{t})}{\overline{\mathrm{r}}_i(\mathrm{t})} + \frac{\mathrm{r}_j'(\mathrm{t})}{\overline{\mathrm{r}}_j(\mathrm{t})} + \frac{\mathrm{r}_k'(\mathrm{t})}{\overline{\mathrm{r}}_k(\mathrm{t})} - \left(\frac{\mathrm{r}_i(\mathrm{t})}{\overline{\mathrm{r}}_i(\mathrm{t})} + \frac{\mathrm{r}_j(\mathrm{t})}{\overline{\mathrm{r}}_j(\mathrm{t})} \right)$$

Then (MAC), proportionally fair (PF) algorithm is used to maximize total wireless network throughput while at the same time allowing all users at least a minimal level of service.

Problem Formulation

- General Formulation of Resource Allocation:

$$\underset{\textit{subject to } SINR_q}{\text{maximize}} \left\{ g_1, g_2, \ldots, g_{Q_k} \right\}$$

- Multi-Objective Optimization Problem
  - Generally impossible to maximize for all users!
  - Must divide power and cause inter-user interference

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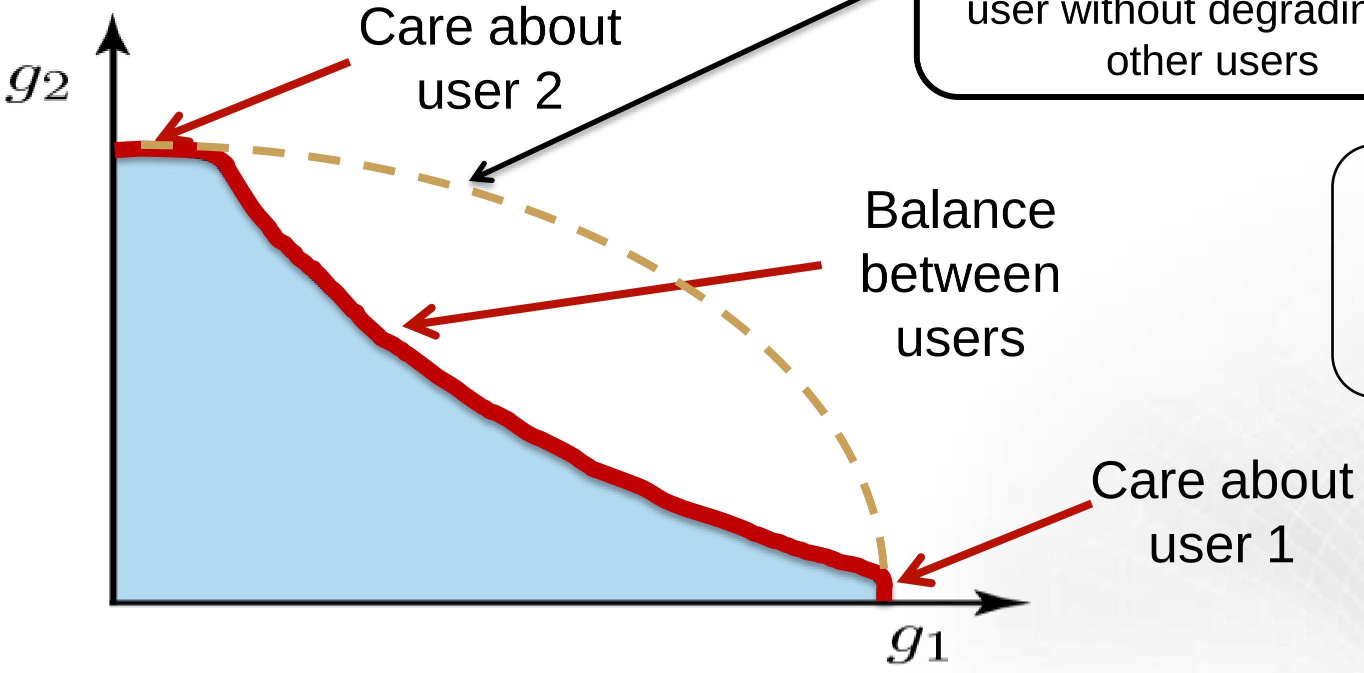
Performance Region



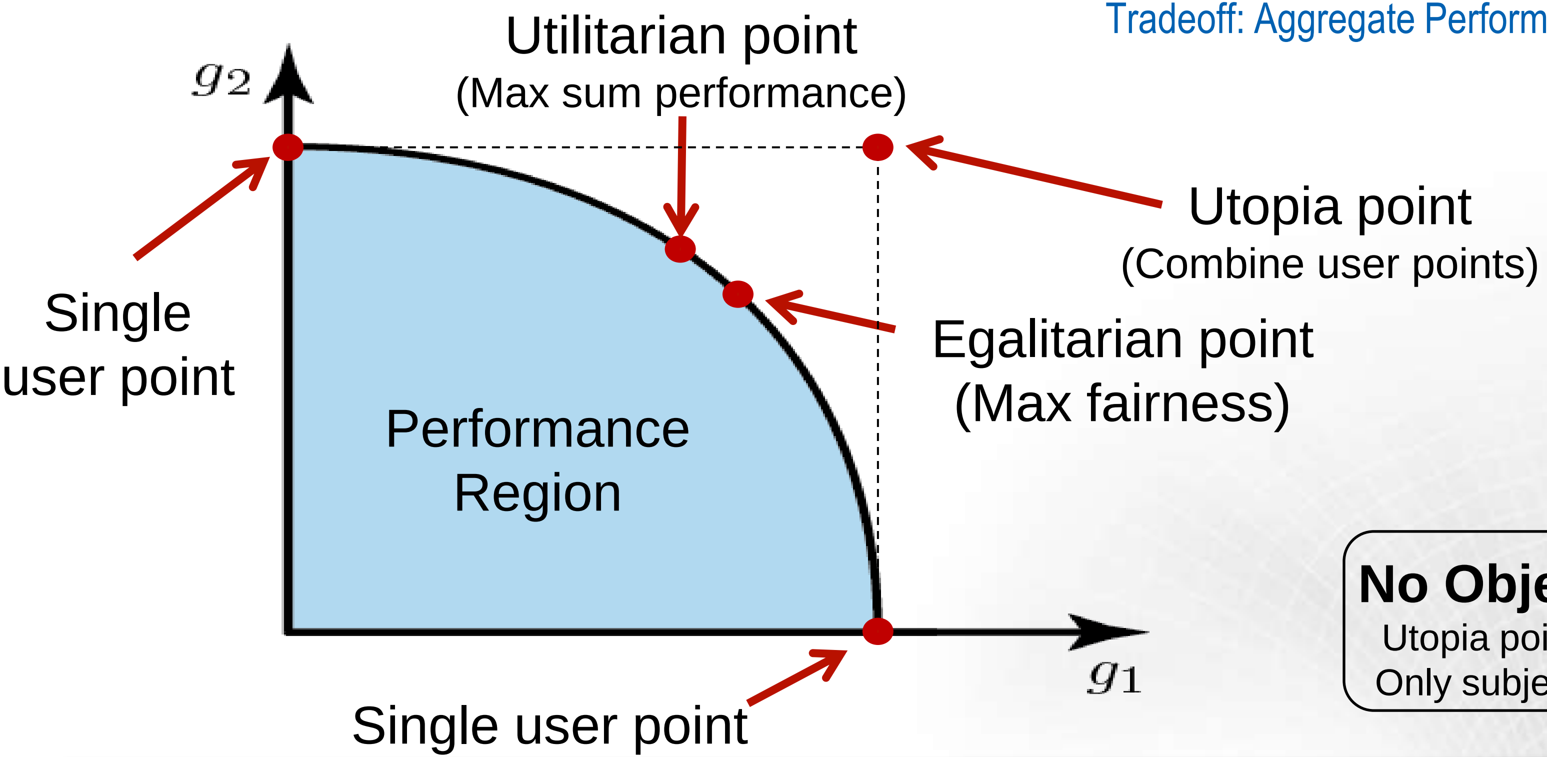
Pareto Boundary
Cannot improve for any user without degrading for other users

Other Names
Rate Region
Capacity Region
MSE Region, etc.

Performance vs. Fairness

Which Pareto Optimal Point to Choose?

Tradeoff: Aggregate Performance vs. Fairness



No Objective Answer
Utopia point outside of region
Only subjective answers exist!

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Subjective Approach

System Designer Selects Utility Function

Put different weights to move between extremes

Examples:

Sum performance: $f(\mathbf{g}) = \sum_k g_k$

Proportional fairness: $f(\mathbf{g}) = \prod_k g_k$

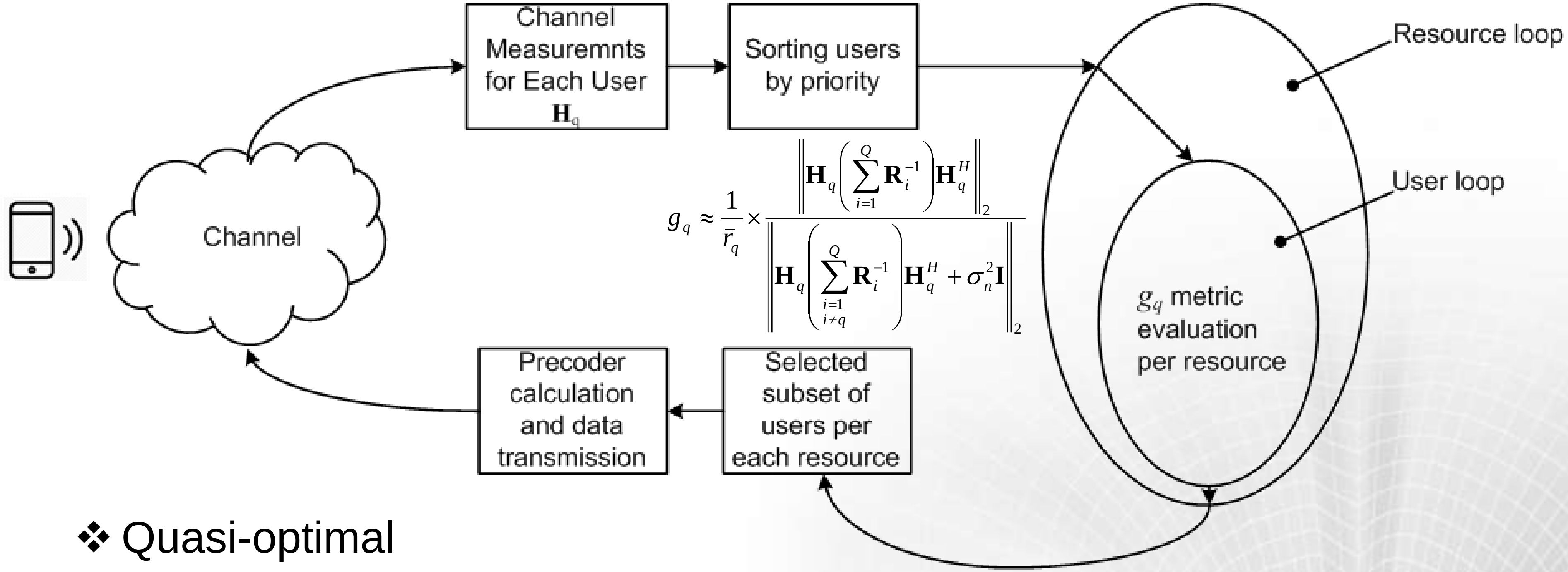
Harmonic mean: $f(\mathbf{g}) = K_r \left(\sum_k g_k^{-1}\right)^{-1}$

Max-min fairness: $f(\mathbf{g}) = \min_k g_k$

Known as *A Priori* Approach
Select utility function before optimization

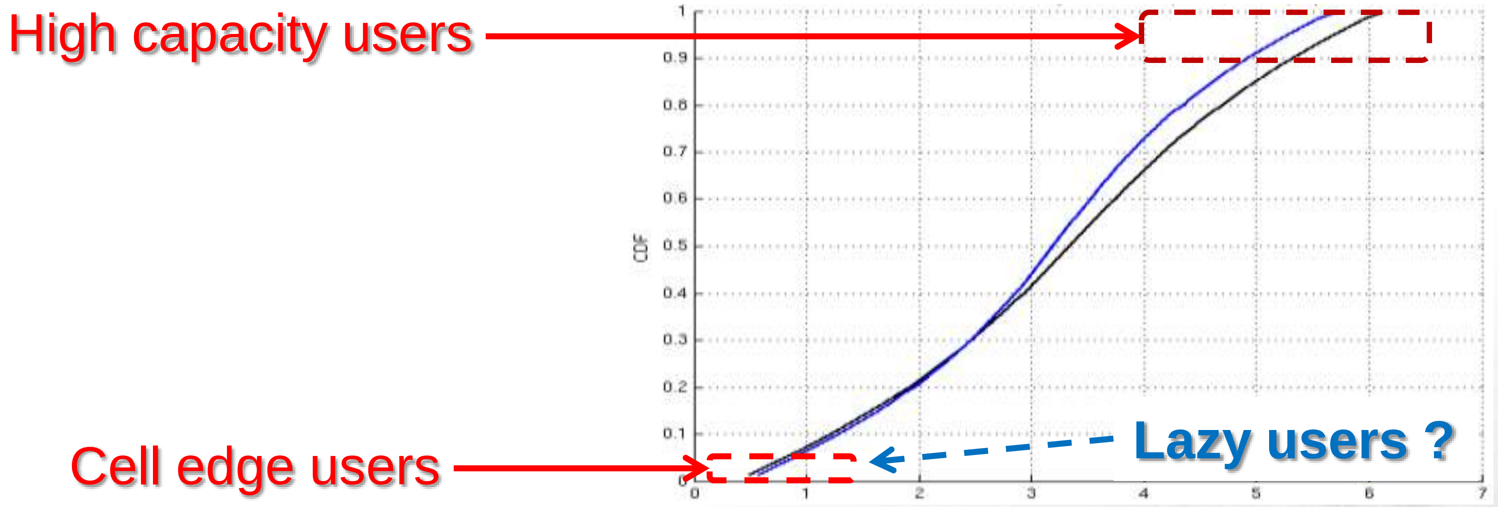
Deal with FULL BUFFER TRAFFIC

Greedy Algorithm



- ❖ Quasi-optimal
- ❖ High complexity

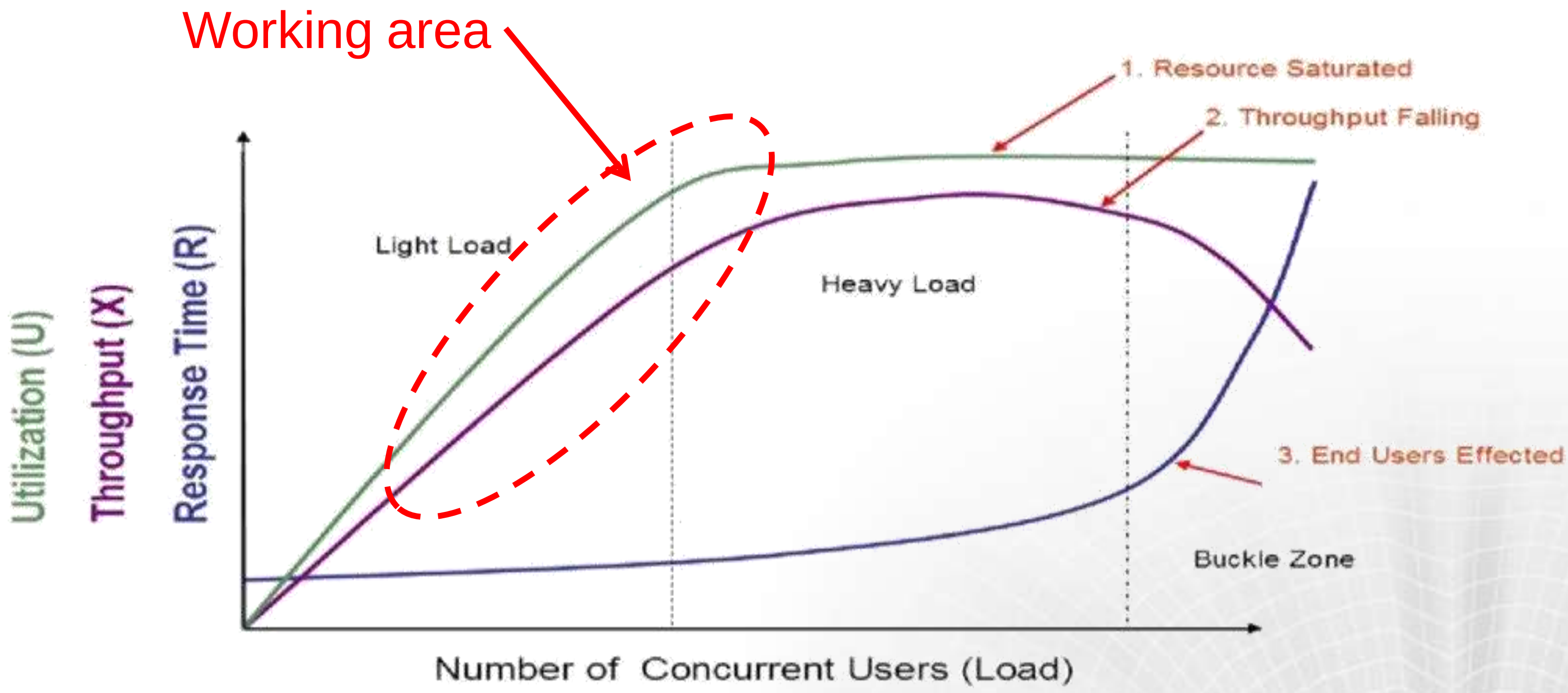
Legacy PF Scheduling vs. QoS Aware Scheduling



In real network, traffic is not based on

FULL BUFFER MODEL

Legacy PF Scheduling vs. QoS Aware Scheduling

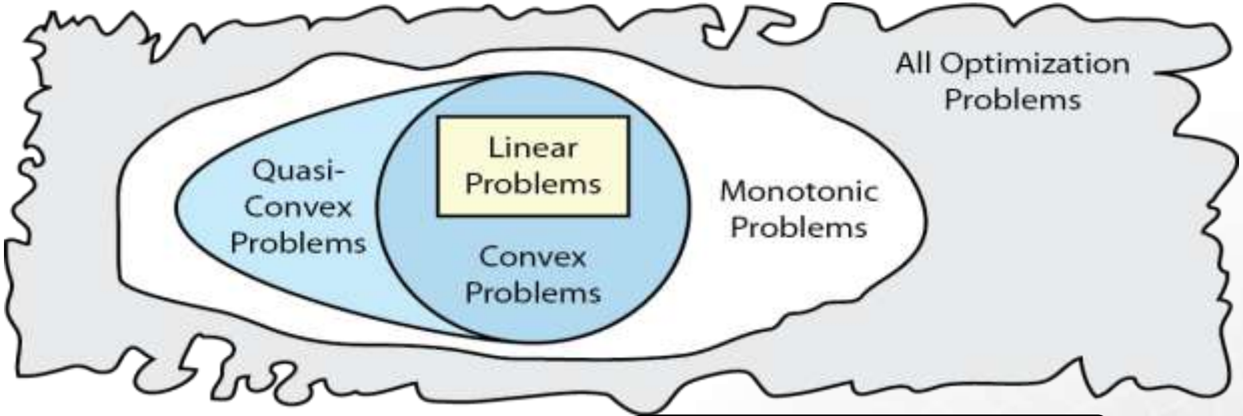


Complexity of Single-Objective Optimization Problems

NP: Non-deterministic Polynomial-time

Classes of Optimization Problems

Different scaling with number of parameters and constraints



Main Classes

Convex: Polynomial time solution ← Practically solvable

Monotonic: Exponential time solution ← Approximations needed

Arbitrary: More than exponential time ← Hard to even approximate

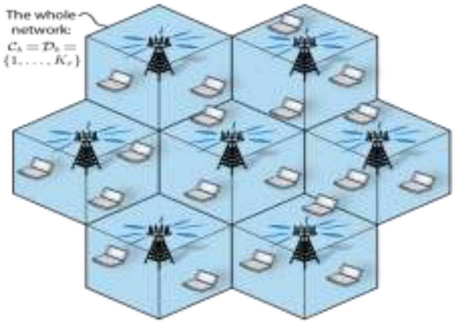
Summary: Complexity of Resource Allocation

Complexity Analysis for Any Dynamic Cooperation Clusters

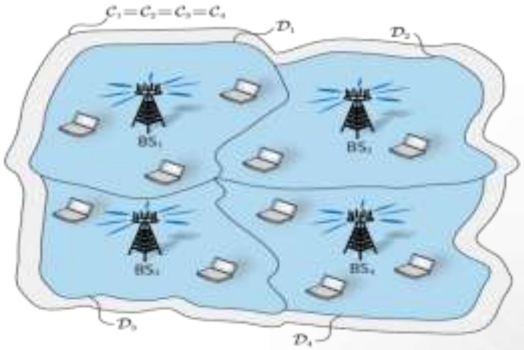
Same optimization algorithms!

Extra characteristics can sometime simplify

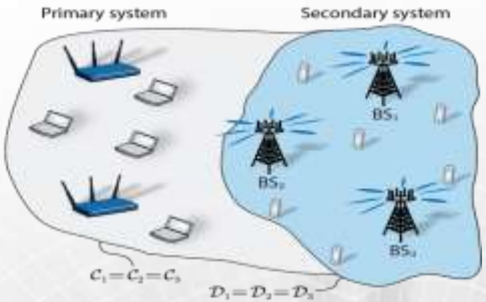
Multi-antenna transmission: More complex, higher performance



Ideal Joint Transmission



Coordinated Beamforming



Underlay Cognitive Radio

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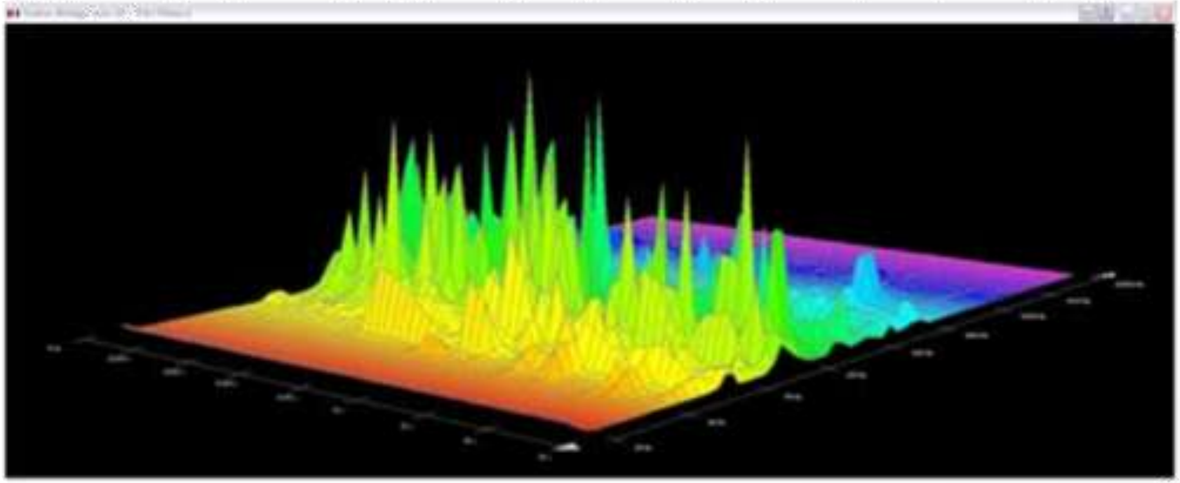
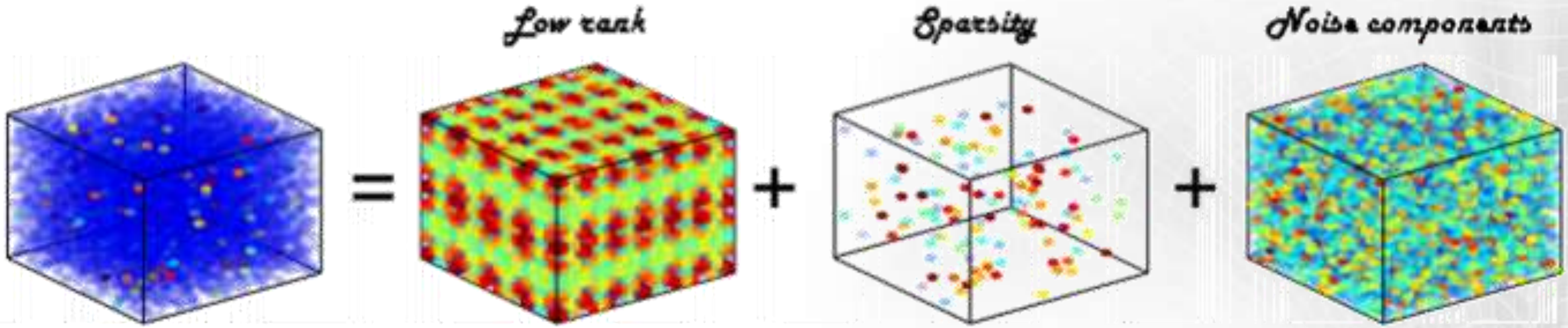
Remarks

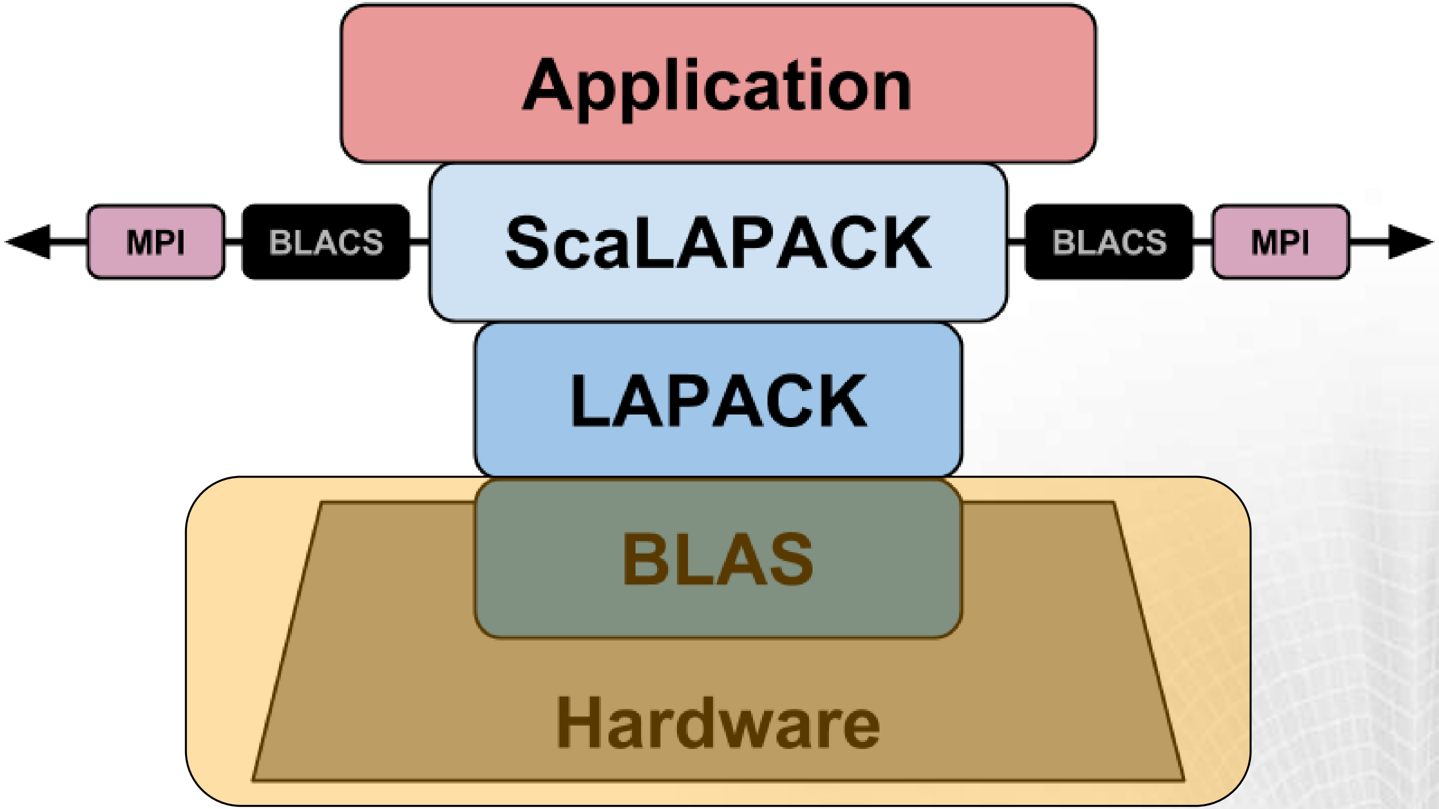
I. Search algorithms is naïve approach, but have huge complexity
II. Numerical properties of channel matrix are not fully clear for researchers (the simplest approximation is Wishart Matrix)
III. There is no well adjusted metric, which can link QoS and Channel/Precoder parameters
IV. Multi-user & multi-traffic scenarios didn’t consider several years ago at all
V. No PF theory for multi-dimensional case: all PF theory was developed for Single User MIMO and applied as it (!) for Multi-User.

3D tensor model as spatial channel extension





The simplest one doesn't mean the fastest one



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Thanks for your attention!