

Principali sviluppi di Taylor centrati in $x_0 = 0$

$$\llcorner (1+x)^a = \sum_{k=0}^n \binom{a}{k} x^k + o(x^n) = 1 + ax + \frac{a(a-1)}{2!} x^2 + \frac{a(a-1)(a-2)}{3!} x^3 + o(x^3)$$

$$\llcorner (1-x)^{-1} = 1+x+x^2+x^3+x^4+o(x^4) \quad \llcorner (1-x)^{-2} = 1+2x+3x^2+4x^3+5x^4+o(x^4)$$

$$\llcorner (1+x)^{1/2} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + o(x^3) \quad \llcorner (1+x)^{-1/2} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + o(x^3)$$

$$\llcorner (1+x)^{1/3} = 1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5x^3}{81} + o(x^3) \quad \llcorner (1+x)^{-1/3} = 1 - \frac{x}{3} + \frac{2x^2}{9} - \frac{14x^3}{81} + o(x^3)$$

$$\llcorner e^x = \sum_{k=0}^n \frac{x^k}{k!} + o(x^n) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + o(x^5)$$

$$\llcorner \log(1+x) = \sum_{k=1}^n \frac{(-1)^{k-1} x^k}{k} + o(x^n) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + o(x^5)$$

$$\llcorner \sin(x) = \sum_{k=0}^n \frac{(-1)^k x^{2k+1}}{(2k+1)!} + o(x^{2n+2}) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + o(x^8)$$

$$\llcorner \sinh(x) = \sum_{k=0}^n \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2}) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + o(x^8)$$

$$\llcorner \cos(x) = \sum_{k=0}^n \frac{(-1)^k x^{2k}}{(2k)!} + o(x^{2n+1}) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + o(x^7)$$

$$\llcorner \cosh(x) = \sum_{k=0}^n \frac{x^{2k}}{(2k)!} + o(x^{2n+1}) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + o(x^7)$$

$$\llcorner \tan(x) = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + o(x^8) \quad \llcorner \tanh(x) = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + o(x^8)$$

$$\llcorner \arctan(x) = \sum_{k=0}^n \frac{(-1)^k x^{2k+1}}{2k+1} + o(x^{2n+2}) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + o(x^8)$$

$$\llcorner \operatorname{arctanh}(x) = \frac{1}{2} \log \left(\frac{1+x}{1-x} \right) = \sum_{k=0}^n \frac{x^{2k+1}}{2k+1} + o(x^{2n+2}) = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + o(x^8)$$

$$\llcorner \arcsin(x) = x + \frac{x^3}{6} + \frac{3x^5}{40} + \frac{5x^7}{112} + o(x^8) \quad \llcorner \arccos(x) = \frac{\pi}{2} - \arcsin(x)$$

$$\llcorner (1+x)^x = 1 + x^2 - \frac{x^3}{2} + \frac{5x^4}{6} + o(x^4) \quad \llcorner (1+x)^{1/x} = e - \frac{ex}{2} + \frac{11ex^2}{24} - \frac{7ex^3}{16} + o(x^3)$$