BSc Engineering Sciences – A. Y. 2018/19

Written exam of the course Mathematical Analysis 2

August 29, 2019

Last name: .... First name: ....
Matriculation: ....

Solve the following problems, motivating in detail the answers.

1. (1) Compute the derivative, with respect to t, of the function

$$f(t) = \int_{t^2}^{t^3} \frac{\sin u}{u}\, du.$$

(2) Let $f \in C^2(\mathbb{R}^2)$ be a solution of the first order linear partial differential equation

$$3\frac{\partial f}{\partial t} + 2\frac{\partial f}{\partial x} = 0.$$

Find $c \in \mathbb{R}$ such that f is also a solution of the one dimensional wave equation

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2}.$$

Solution.

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2. Find the extremal values of the function $f(x, y, z) = x^2 + y^2 + z^2$ on the line L defined by two equations $x + y + z = 1$ and $x - z = 2$.
Solution.

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3. Let C be the curve $\{(x, y) : xy = 1, 1 \le x \le 3\}$ in $\mathbb{R}^2$. Find a parametrization $\boldsymbol{\alpha}(t)$ of C starting at $(1, 1)$ and ending at $(3, \frac{1}{3})$, and compute the line integral

$$\int_C \boldsymbol{f} \cdot d\boldsymbol{\alpha},$$

where $\boldsymbol{f}(x, y) = (y, -x^4)$ is a vector field in $\mathbb{R}^2$.

Solution.

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4. Compute the integral

$$\iiint_T dxdydz\,(z+1)\sqrt{\frac{x^2+y^2}{4-x^2-y^2}}$$

where

$$T := \{(x,y,z) \in \mathbb{R}^3 \,:\, x^2+y^2-2y \leq 0,\, x^2+y^2+z^2 \leq 4\}.$$

Solution.

Matriculation: ....

5. Let $\boldsymbol{F}(x, y, z) = (xy, e^{-y^2}, yz)$ be a vector field on $\mathbb{R}^3$ and

$$S = \{(x, y, z) : x^2 + z^2 = 9,\ 0 \leq x,\ 0 \leq y \leq 2\}$$

be a surface in $\mathbb{R}^3$. Compute the surface integral

$$\iint\limits_S \boldsymbol{F} \cdot \boldsymbol{n}\, dS,$$

where $\boldsymbol{n}$ is a unit normal vector on S with positive x-component.
Solution.