BSc Engineering Sciences – A. Y. 2018/19
Written exam of the course Mathematical Analysis 2
July 10, 2019

Last name: .... First name: ....
Matriculation: ....

Solve the following problems, motivating in detail the answers.

1. Find a power series solution $y(x)$ of the differential equation

$$(1 + x^2)y''(x) - xy'(x) - 3y(x) = x$$

subject to the intial conditions $y(0) = y'(0) = 1$, and determine its radius of convergence.
Solution.

2. Find all the stationary points of the following scalar field, defined on $\mathbb{R}^2$,

$$f(x,y) = e^{x^2+y^2}\left(x+y-\frac{5}{2}\right)$$

and classify them into relative minima, maxima and saddle points.

Solution.

3. Let C be the curve $\{(x, y) : x^2 + (y - 1)^2 = 1, x \geq 0\}$ in $\mathbb{R}^2$. Find a parametrization $\boldsymbol{\alpha}(t)$ of C starting at $(0, 0)$ and ending at $(0, 2)$, and compute the line integral

$$\int_C \boldsymbol{f} \cdot d\boldsymbol{\alpha},$$

where $\boldsymbol{f}(x, y) = (y, x^2)$ is a vector field in $\mathbb{R}^2$.

Solution.

4. Find the volume of the set $D \subset \mathbb{R}^3$ which is contained inside the cylinder of equation $x^2 + y^2 = 1$ and bounded by the surfaces of equation $z = x^2 + y^2 - 2$ and $x + y + z = 4$.

Solution.

5. Let $\boldsymbol{F}(x, y, z) = (x^2 + x - 2xy^2, y^3 + 4yz^2, x^2z)$ be a vector field on $\mathbb{R}^3$, S be the surface of the ellipsoid:

$$S := \{(x, y, z) : (x+1)^2 + y^2 + 4z^2 = 4\},$$

and $\boldsymbol{n}$ the outgoing normal unit vector on S at each point of S.

Compute the surface integral

$$\iint_S \boldsymbol{F} \cdot \boldsymbol{n} \, dS.$$

Solution.