BSc Engineering Sciences – A. Y. 2018/19

Written exam of the course Mathematical Analysis 2

February 14, 2019

Last name: .... First name: ....

Matriculation: ....

Solve the following problems, motivating in detail the answers.

1. Find the Taylor series expansion, with intial point $x_0 = 1$, of the function

$$f(x) = \frac{x}{(x-2)(x^2-2x+2)},$$

determine its radius of convergence r, and study the convergence for $x = \pm r$.

Solution.

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2.

(1) Find the extremal values of the function $f(x, y, z) = x + 2y + 2z$ on the surface S defined by $x^2 + y^2 + z^2 = 1$.

(2) Let $g(x, y)$ the function implicitly defined by $x^2 + y^2 + g(x, y)^2 = 1, g(x, y) > 0$ and $h(x, y) = x + 2y + 2g(x, y)$. Compute $\frac{\partial h}{\partial x}(x_0, y_0), \frac{\partial h}{\partial y}(x_0, y_0)$, where (x_0, y_0, z_0) is the maximum of (1).

Solution.

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3.

(1) Let $c > 0$. Find the solution $f(x,t)$ of the partial differential equation

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2}$$

with the initial condition $f(x,0) = \dfrac{\sin x}{x^2+1}$, $\dfrac{\partial f}{\partial t}(x,0) = xe^{-x^2}$.

(2) Find $\alpha > 0$ for which the function $g(x,y,z) = \dfrac{1}{(x^2+y^2+z^2)^\alpha}$ satisfies the partial differential equation on $\mathbb{R}^3 \setminus \{(0,0,0)\}$:

$$\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2} = 0.$$

Solution.

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4. Compute the integral

$$\iiint_D z(x^2+y^2+z^2)e^{-(x^2+y^2)}\,dxdydz,$$

with $D = \{(x,y,z) \in \mathbb{R}^3 \,:\, x^2+y^2+z^2 \le 1,\ z \ge \sqrt{x^2+y^2}\}$.
Solution.

Matriculation: ....

5. Let $\boldsymbol{F}(x, y, z) = (x^3 - xy^2z, -xz^3 - xy^2z, y^3 - yz^2)$ be a vector field on $\mathbb{R}^3$, C be the circle

$$C = \{(x, y, z) : y^2 + z^2 = 1, x = 2\}.$$

Compute the line integral

$$\int_C \boldsymbol{F} \cdot d\boldsymbol{\alpha},$$

where $\boldsymbol{\alpha} = (2, \sin t, \cos t), t \in [0, 2\pi]$.
Solution.