BSc Engineering Sciences – A. Y. 2018/19

Written exam of the course Mathematical Analysis 2

January 29, 2019

Last name: .... First name: ....
Matriculation: ....

Solve the following problems, motivating in detail the answers.

1. Find a power series solution $y(x)$ of the differential equation

$$y''(x) + 2x\,y'(x) - 8y(x) = 0,$$

such that $y(0) = 1$, $y'(0) = -1$, and determine its radius of convergence.
Solution.

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2.

(1) Find all the stationary points of the following scalar field, defined on $\mathbb{R}^2$,

$$f(x, y) = x^3 + y^3 - 6xy$$

and classify them into relative minima, maxima and saddle points.

(2) Compute the derivative of the following function of x:

$$g(x) = \int_{-x^2}^{x^2} e^{t^2} dt.$$

Solution.

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3. Determine whether the following vector field on $\mathbb{R}^2$

$$\boldsymbol{f}(x,y) = \left(xe^{x^2+y^2} - y, ye^{x^2+y^2} + x\right)$$

is a gradient of some scalar field. Depending on this result,

- If $\boldsymbol{f}(x,y)$ is a gradient, find one of these scalar fields φ such that $\boldsymbol{f}(x,y) = \nabla\varphi(x,y)$.
- If $\boldsymbol{f}(x,y)$ is not a gradient, compute $\int_C \boldsymbol{f} \cdot d\boldsymbol{\alpha}$, where $\boldsymbol{\alpha}(t) = (\cos t, \sin t), t \in [0, 2\pi]$.

Solution.

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4. Compute the integral

$$\iiint_D \frac{x+y}{\sqrt{2}} dxdydz,$$

where $D := \{(x,y,z) \in \mathbb{R}^3 \; : \; \frac{x^2+y^2}{4} + z^2 \leq 1, y \geq 0\}$.
Solution.

Matriculation: ....

5. Let $\boldsymbol{F}(x,y,z) = (x(x^2+y^2+z^2), y(x^2+y^2+z^2), z(x^2+y^2+z^2))$ be a vector field on $\mathbb{R}^3$, S be the surface of the sphere with radius $a > 0$:

$$S := \{(x,y,z) : x^2+y^2+z^2 = a^2\},$$

and $\boldsymbol{n}$ the outgoing normal unit vector on S at each point of S.

Compute the surface integral

$$\iint_S \boldsymbol{F} \cdot \boldsymbol{n} \, dS.$$

Solution.