Futuro in Ricerca 2013 – Numerical Analysis

Design of Reliable, Exact, and Application-oriented techniques for geometric Modeling and numerical Simulation (DREAMS)

DREAMS Workshop

Department of Mathematics, University of Rome "Tor Vergata"
Via della Ricerca Scientifica, 00133 Rome
January 26 - 27, 2016

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Tuesday 26, 2016

14,00 - 14,45 *Graziano Gentili* (University of Florence)
14,45 - 15,15 *Duccio Mugnaini* (University of Insubria)
15,15 - 15,45 *Cesare Bracco* (University of Florence)

15,45 - 16,15 Coffee break

16,15 - 17,00 *Alessandro Reali* (University of Pavia)
17,00 - 17,30 *Carlo Garoni* (University of Rome "Tor Vergata")
17,30 - 18,00 *Fabio Roman* (University of Turin)

Wednesday 27, 2016

10,00 - 10,45 *Tom Lyche* (University of Oslo)
10,45 - 11,15 *Thomas Takacs* (University of Pavia)

11,15 - 11,45 Coffee break

11,45 - 12,30 *Alessandra Aimi* (University of Parma)
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List of abstracts

Tuesday 26, 2016 14,00 - 14,45

Polynomial curves with rational rotation-minimizing frames

Graziano Gentili (University of Florence)

Pythagorean–hodograph (PH) curves can be defined as those quaternion polynomial curves $\mathbf{r}(\xi)$ whose tangent vector is such that $\mathbf{r}'(\xi) = \mathcal{A}(\xi)\,\mathbf{i}\,\mathcal{A}^*(\xi)$ for some quaternion polynomial $\mathcal{A}(\xi)$. A frame $(\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3)$ on a space curve $\mathbf{r}(\xi)$ is called *rotation–minimizing* if it defines an orthonormal basis for $\mathbb{R}^3$ in which $\mathbf{f}_1 = \mathbf{r}'/|\mathbf{r}'|$ is the curve tangent, and the normal–plane vectors $\mathbf{f}_2$, $\mathbf{f}_3$ exhibit no instantaneous rotation about $\mathbf{f}_1$. Polynomial curves that admit *rational* rotation–minimizing frames (or RRMF curves) form a subset of the PH curves, and occupy a special position in the study of rigid body motions in spatial kinematics. RRMF curves $\mathbf{r}(\xi)$ turn out to be those PH curves whose tangent vector $\mathbf{r}'(\xi) = \mathcal{A}(\xi)\,\mathbf{i}\,\mathcal{A}^*(\xi)$ is characterized by the existence of a polynomial complex curve $\gamma(\xi)$ such that Equation (12) holds

$$\frac{\langle \mathcal{A}'\mathbf{i}, \mathcal{A}\rangle}{\langle \mathcal{A}, \mathcal{A}\rangle} \equiv \frac{\langle \gamma'\mathbf{i}, \gamma\rangle}{\langle \gamma, \gamma\rangle} \tag{12}$$

where $\langle\cdot,\cdot\rangle$ denotes the standard Euclidean scalar product of $\mathbb{R}^4$. By using the notion of *Fourier indicatrix* $\frac{\langle \mathcal{B}'\mathbf{i}, \mathcal{B}\rangle}{\langle \mathcal{B}, \mathcal{B}\rangle}$ of a quaternion polynomial curve $\mathcal{B}$, and introducing the notion of the *core* of $\mathcal{B}$, a comprehensive characterization of the complete space of RRMF curves is developed, that subsumes all previously known special cases. This novel characterization helps clarify the structure of the complete space of RRMF curves, distinguishes the spatial RRMF curves from trivial (planar) cases, and opens the way toward new construction algorithms.

Mathematical problems that stand at the crossroad of different disciplines and subjects may be particularly attractive, mainly because studying such problems gives the occasion of putting together different points of view and perspectives coming from researchers with distinct backgrounds and abilities. Rida T. Farouki, Graziano Gentili, Carlotta Giannelli, Alessandra Sestini, and Caterina Stoppato found such an occasion and produced together the work that is the object of this talk.

Tuesday 26, 2016 14,45 - 15,15

A novel curvature continuous path finding scheme with scene reconstruction by image segmentation

Duccio Mugnaini (University of Insubria)

We introduce a new global path finding strategy that only requires as input data an image of the workspace. As preliminary phase, we detect the obstacle silhouettes within the scene by a variant of the Mumford-Shah segmentation model starting from the given image. Subsequently, a navigability graph in terms of an adjacency matrix based on pixel values of the segmented image is considered. An established path finding algorithm, fine-tuned to the present context, is then applied to identify a feasible piecewise linear path with reasonable shape. Finally, in order to produce a curvature-continuous path, the vertices of the obtained polyline are interpolated by considering a G^2 PH quintic spline interpolation scheme. The smooth collision-free PH spline path is identified thanks to a suitable choice of the available tension parameters.

We report several examples in significant scenarios to compare PH-based solutions to those obtained using alternative settings and we show the benefit gained by the use of PH curves in terms of path fairness. This is a joint work with Marco Donatelli, Carlotta Giannelli, and Alessandra Sestini.

Tuesday 26, 2016 15,15 - 15,45

Tchebycheffian spline spaces over T-meshes

Cesare Bracco (University of Florence)

The activities of the project DREAMS include the study and the development of techniques based on several types of adaptive spline spaces, such as hierarchical spline spaces and spline spaces over T-meshes. The talk will focus on the generalization to the case of Tchebycheffian spline spaces of some well-known results for polynomial spline spaces over T-meshes: the elements of the resulting spaces are spline functions coinciding on each cell of the T-mesh with a function belonging to an extended Tchebycheff space, that is, a class of non-polynomial spaces whose properties make them suitable for design purposes. The analogies with the polynomial case and the difficulties in extending the theory to the Tchebycheffian case will be highlighted, in particular considering the study of the dimension.

Tuesday 26, 2016 16,15 - 17,00

Some advances and applications of isogeometric analysis

Alessandro Reali (University of Pavia and TUM-IAS)

Isogeometric Analysis (IGA) is a recent simulation framework, originally proposed by T. J. R. Hughes and coworkers in 2005, to bridge the gap between Computational Mechanics and Computer Aided Design (CAD). The basic IGA paradigm consists of adopting the same basis functions used for geometry representations in CAD systems - such as, e.g., Non-Uniform Rational B-Splines (NURBS) - for the approximation of field variables, in an isoparametric fashion. This leads to a cost-saving simplification of the typically expensive mesh generation and refinement processes required by standard finite element analysis. In addition, thanks to the high-regularity properties of its basis functions, IGA has shown a better accuracy per-degree-of-freedom and an enhanced robustness with respect to standard finite elements in a number of applications ranging from solids and structures to fluids, opening also the door to geometrically flexible discretizations of higher-order partial differential equations in primal form, as well as to highly efficient collocation methods.

This lecture aims at illustrating the unique potential of IGA through some convincing applications, mainly belonging to the field of solid and structural mechanics, where the superior results that can be provided by IGA with respect to standard finite elements are clearly pointed out. In particular, after a brief introduction about the IGA approximation properties of structural vibrations, the application to a real-life case, the so-called NASA "Aluminum Testbed Cylinder", is shown along with comparisons with experimental results. As a further example, the demanding explicit structural dynamics simulation of a patient-specific aortic valve, modeled by nonlinear hyper-elastic shells and involving large deformations and contact, is presented and carefully analyzed in terms of accuracy and efficiency. Such results are also complemented by an impressive example of fluid-structure interaction (FSI) simulation of an aortic valve prosthesis, based on an "immersed" IGA method. As a third representative case study, the bending behavior of a complex structure like a shape memory alloy stent is simulated in the large deformation regime, with particular attention to a correct modeling of buckling phenomena. Convincing preliminary results about the extension to the complete simulation of the stent implantation process are also shown. The lecture is finally concluded by a brief presentation of some further works and ideas in progress.

Tuesday 26, 2016 17,00 - 17,30

GLT sequences: theory and applications

Carlo Garoni (University of Rome "Tor Vergata")

When discretizing a linear PDE by a linear numerical method, the computation of the numerical solution reduces to solving a linear system. The size of this system grows when the discretization parameter n increases, i.e., when we refine the discretization mesh. We are then in the presence of a sequence of linear systems with increasing size. It is usually observed in practice that the corresponding sequence of discretization matrices enjoys an asymptotic spectral distribution, which is compactly described by a function, the so-called spectral symbol. The main tool for computing spectral symbols of PDE discretization matrices is the theory of Generalized Locally Toeplitz (GLT) sequences.

In this presentation, we will discuss the theory of GLT sequences. We will also outline its application to the computation of the spectral symbol of PDE discretization matrices.

Tuesday 26, 2016 17,30 - 18,00

Spectral analysis of matrices in IgA collocation methods with GB-splines

Fabio Roman (University of Turin)

Generalized splines are smooth piecewise functions with sections in spaces of the form

$$\mathbb{P}_p^{U,V} := \langle 1, t, \ldots, t^{p-2}, U(t), V(t) \rangle, \quad t \in [a, b], \quad 2 \leq p \in \mathbb{N}.$$

Besides classical polynomial splines, other interesting examples are trigonometric or hyperbolic generalized splines for which U, V are taken as $\cos(\alpha t), \sin(\alpha t)$, or $\cosh(\alpha t), \sinh(\alpha t)$, respectively. Under suitable conditions on U, V, the space $\mathbb{P}_p^{U,V}$ has the same structural properties of the space of polynomials of degree p. Similarly, generalized splines possess all the desirable properties of polynomial splines, including a representation in terms of basis functions that are a natural extension and possess all the nice properties of the polynomial B-splines. Such basis functions are referred to as GB-splines.

Thanks to the above properties, tensor-product GB-splines are an interesting problem-dependent alternative to tensor-product (polynomial) B-splines and NURBS in isogeometric analysis (IgA). The collocation formulation has been intensively employed in the IgA context, and isogeometric collocation methods based on GB-splines have been recently investigated.

Like for any discretization method, the IgA collocation paradigm requires to solve large linear systems. Information about the spectral properties of the related matrices is extremely important for the design of fast solvers for these linear systems, in order to pursue an applicative interest.

In this talk we focus on the linear systems arising in the numerical solution of second order elliptic problems by the IgA collocation approach based on trigonometric or hyperbolic GB-splines. In particular, we prove that the corresponding stiffness matrices possess an asymptotic eigenvalue distribution when the fineness parameters tend to zero. This asymptotic distribution is compactly described by a function, the so-called symbol of the given sequence of matrices. This is a joint work with Carla Manni and Hendrik Speleers.

Wednesday 27, 2016 10,00 - 10,45

Polynomial, trigonometric, and exponential simplex splines

Tom Lyche (University of Oslo)

We give an introduction to polynomial simplex splines, and present trigonometric and exponential versions of these functions.

Wednesday 27, 2016 10,45 - 11,15

Approximation properties of analysis-suitable G^1 multi-patch isogeometric spaces

Thomas Takacs (University of Pavia)

In this talk, we consider multi-patch domains that have a parametrization which is only C^0 at the patch interface. On such domains we study the h-refinement of C^1-continuous isogeometric spaces. These spaces in general do not have optimal approximation properties. The reason is that the C^1-continuity condition easily over-constrains the solution which is, in the worst case, fully locked to linears at the patch interface. However, numerical evidence shows that optimal convergence occurs for bilinear two-patch geometries and C^1 splines of polynomial degree at least 3. We introduce analysis-suitable G^1-continuous geometry parametrizations, a class of parametrizations that includes bilinears. We analyze the structure of C^1 isogeometric spaces and, by theoretical results and numerical testing, infer that analysis-suitable G^1 geometry parametrizations are the ones that allow optimal convergence of C^1 isogeometric spaces (under conditions on the continuity along the interface). Beyond analysis-suitable G^1 parametrizations optimal convergence is prevented.

Wednesday 27, 2016 11,45 - 12,30

IgA-SGBEM: an overview

Alessandra Aimi (University of Parma)

The Isogeometric Analysis approach (IgA), introduced by Hughes and collaborators, establishes a strict relation between the geometry of the problem domain and the approximate solution representation, giving surprising computational advantages. It has also brought a renewed interest for Boundary Element Methods (BEMs), since one has to discretize only the boundary of the problem domain and this can be done in an accurate way by powerful geometric modeling techniques.

Among BEMs, the Symmetric Galerkin version (SGBEM) is recognized as particularly suitable for mixed boundary value problems and for coupling with FEM. In this context, we have recently introduced the IgA concept into SGBEM, using classical B-splines to represent both the boundary and the approximate solution.

In this talk we will give an overview on IgA-SGBEM and discuss about an extension including NURBS and generalized B-splines. The computational advantages over standard and curvilienar SGBEMs, where the numerical solution is given by means of Lagrangian basis functions, will be underlined by means of several numerical results. This is a joint work with Mauro Diligenti, Maria Lucia Sampoli and Alessandra Sestini.

Wednesday 27, 2016 12,30 - 13,00

Hierarchical box splines in isogeometric analysis

Tadej Kanduc (INdAM c/o University of Florence)

Isogeometric analysis (IgA) is the recently established paradigm based on the finite element method that replaces simple building blocks in geometry and solution space with more complex and geometrically-oriented compounds. Box splines represent a promising tool to model complex geometry in the context of IgA. As an intermediate approach between B-splines and splines on triangulations, box spline incorporate several advantages from both the concepts.

Local refinement possibilities may be achieved by considering the hierarchal approach in connection with a nested sequence of box spline spaces. Since box splines do not posses special elements to impose the boundary conditions, a weak treatment of the boundary conditions for the numerical solution of partial differential equations is presented. Near the domain boundary, a special domain layer is used to enforce the boundary conditions in a weak sense. The thickness of the layer is adaptively defined in order to avoid unnecessary extra computations. Numerical examples show the optimal convergence rate of hierarchical box splines. This is a joint work with Carlotta Giannelli, Francesca Pelosi, and Hendrik Speleers.