1. Let A be a subring of a ring B, such that the set $B - A$ is closed under multiplication. Show that A is integrally closed in B.

2. Show that R is a Noetherian ring if and only if every non-empty set of ideals in R has a maximal element.

3. Let A be a Noetherian ring.
(a) Prove that every ideal I in A contains a power of its radical.
(b) Using (a), show that the nilradical in A is nilpotent.

4. Prove that if R is a Noetherian ring then the formal power series ring $R[[x]]$ is also Noetherian.

5. Let A be a Noetherian ring. Prove that the following are equivalent.
(i) A is Artinian.
(ii) $\mathrm{Spec}(A)$ is discrete and finite.
(iii) $\mathrm{Spec}(A)$ is discrete.

6. Let k be a field and A a finitely generated k-algebra. Prove that the following are equivalent.
(i) A is Artinian.
(ii) A is a finite k-algebra.