Efficient Algorithms for Low-Energy Bounded-Hop Broadcast in Ad-Hoc Wireless Networks
(Extended Abstract)

Christoph Ambühl[1⋆], Andrea E.F. Clementi[2], Miriam Di Ianni[2], Nissan Lev-Tov[3⋆⋆], Angelo Monti[4], David Peleg[3⋆⋆], Gianluca Rossi[2⋆⋆⋆], and Riccardo Silvestri[4]

[1] Instituto Dalle Molle di Studi sull'Intelligenza Artificiale
Galleria 2, 6928 Manno - SWITZERLAND
`christoph@idsia.ch`
[2] Dipartimento di Matematica, Università degli Studi di Roma"Tor Vergata"
Via della Ricerca Scientifica 1, 00133 Roma - ITALY
`{clementi,diianni,rossig}@mat.uniroma2.it`
[3] Department of Computer Science and Applied Mathematics,
The Weizmann Institute of Science Rehovot, 76100 - ISRAEL.
`{nissan.lev-tov,david.peleg}@weizmann.ac.il`
[4] Dipartimento di Informatica, Università degli Studi di Roma "La Sapienza"
Via Salaria 113, 00198 Roma - ITALY
`{monti,silver}@dsi.uniroma1.it`

Abstract. The paper studies the problem of computing a minimal energy cost range assignment in a *ad-hoc wireless network* which allows a station s to perform a broadcast operation in at most h hops. The general version of the problem (i.e., when transmission costs are arbitrary) is known to be `log`-APX hard even for $h = 2$. The current paper considers the well-studied real case in which n stations are located on the plane and the cost to transmit from station i to station j is proportional to the α-th power of the distance between station i and j, where α is any positive constant. A polynomial-time algorithm is presented for finding an optimal range assignment to perform a 2-hop broadcast from a given source station. The algorithm relies on dynamic programming and operates in (worst-case) time $O(n^7)$. Then, a polynomial-time approximation scheme (PTAS) is provided for the above problem for any fixed $h \geq 1$. For fixed $h \geq 1$ and $\epsilon > 0$, the PTAS has time complexity $O(n^\mu)$ where $\mu = O((\alpha 2^\alpha h^\alpha / \epsilon)^{\alpha^h})$.

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1 Introduction

Multi-hop wireless networks [13] require neither fixed, wired infrastructure nor predetermined interconnectivity. In particular, *ad hoc* networking [11] is the most popular type of multi-hop wireless networks because of its simplicity. An *ad-hoc* wireless network consists of a homogeneous system of radio stations connected by wireless links. In an ad hoc network, every station is assigned a transmission range. The overall range assignment determines a transmission (directed) graph since one station s can transmit to another station t if and only if t is within the transmission range of s. The transmission range of a station depends, in turn, on the energy power supplied to the station. In particular, the power P_s required by a station s to correctly transmit data to another station t must satisfy the inequality

$$\frac{P_s}{\mathtt{dist}(s,t)^\alpha} > \gamma \tag{1}$$

where $\mathtt{dist}(s,t)$ is the Euclidean distance between s and t, $\alpha \geq 1$ is the *distance-power gradient*, and $\gamma \geq 1$ is the *transmission quality* parameter. In an ideal environment (i.e., in empty space) it holds that $\alpha = 2$ but it may vary from 1 to more than 6 depending on the environment conditions at the location of the network (see [14]).

The fundamental problem underlying any phase of a dynamic resource allocation algorithm in ad-hoc wireless networks is the following. Find a transmission range assignment such that (1) the corresponding transmission graph satisfies a given connectivity property Π, and (2) the overall energy power required to deploy the assignment (according to Inequality (1)) is minimized (see for example [9, 12]). In [6], the reader may find an exhaustive survey on the previous results related to the above problem.

In this paper we address the case in which Π is defined as follows: *Given a set of stations and a specific source station s, the transmission graph has to contain a directed spanning tree rooted at s (a* branching *from s) of depth at most h*. The relevance of this case is due to the fact that any transmission graph satisfying the above property allows the source station to perform a *broadcast* operation in at most h hops. Broadcast is a task initiated by the source station which transmits a message to all stations in the wireless network. This task constitutes a major part of real life multi-hop radio networks [1, 2, 9].

Previous results The broadcast range assignment problem described above is a special case of the following optimization problem, called h-MINIMUM ENERGY CONSUMPTION BROADCAST SUBGRAPH (in short, h-MECBS). Given a weighted directed graph $G = (V, E)$ with $|V| = n$ and an edge weight function $w : E \rightarrow \mathbb{R}^+$, a *range assignment* for G is a function $r : V \rightarrow \mathbb{R}^+$; the *transmission (directed) graph* induced by G and r is defined as $G_r = (V, E')$ where

$$E' = \bigcup_{v \in V} \{(v, u) : (v, u) \in E \wedge w(v, u) \leq r(v)\}.$$

The h-MECBS problem is then defined as follows. Given a *source node* $s \in V$, find a range assignment r for G such that G_r contains a directed spanning tree of G rooted at s of depth at most h and $\mathsf{cost}(r) = \sum_{v \in V} r(v)$ is minimized.

The h-MECBS problem is NP-hard and, if P $\neq$ NP, it is not approximable within a sub-logarithmic factor, even when the problem is restricted to undirected graphs [10] and $h = 2$.

The intractability of the general version of h-MECBS does not necessarily imply the same hardness result for its restriction to wireless networks. In particular, let us consider, for any $d \geq 1$ and $\alpha \geq 1$, the family of graphs N_d^α, called *(d-dimensional) wireless networks*, defined as follows. A complete (undirected) graph G belongs to N_d^α if it can be embedded in a d-dimensional Euclidean space such that the weight of an edge is equal to the α-th power of the Euclidean distance between the two endpoints of the edge itself. The restriction of h-MECBS to graphs in N_d^α is then denoted by h-MECBS[N_d^α]. It is clear that the previously described broadcast range assignment problem in the ideal 2-dimensional environment corresponds to h-MECBS[N_2^2].

Observe that if $\alpha = 1$ (that is, the edge weights coincide with the Euclidean distances), then the optimal range assignment is simply obtained by assigning to s the distance from the node farthest from it and assigning 0 to all other nodes. We then have that, for any $d \geq 1$ and $h \geq 1$, h-MECBS[N_d^1] is solvable in polynomial time. Moreover, it has also been shown that, for any $\alpha \geq 1$, h-MECBS[N_1^α] is solvable in polynomial time [7].

It is also possible to prove that, for any $d \geq 2$, $\alpha > 1$ and $h = n - 1$, h-MECBS[N_d^α] is NP-hard (this version is referred to as the unbounded case). The proof of this result is an adaptation of the one given in [8] to prove the NP-hardness of computing a minimum range assignment that guarantees the strong connectivity of the corresponding transmission graph. This adaptation is described in [4]. In [3, 5] it is shown that, as for the unbounded case, whenever $\alpha \geq d$ the MST-based algorithm proposed in [9] achieves constant approximation. Given a graph $G \in \mathsf{N}_2^\alpha$ and a specified source node s, the MST-based algorithm first computes a minimum spanning tree T of G (observe that this computation does not depend on the value of α). Subsequently, it makes T directed by rooting it at s. Finally, the algorithm assigns to each vertex v the maximum among the weights of all edges of T outgoing from v.

Our results. Bounding the number of hops in message broadcasting on a wireless network is a crucial issue for the QoS of the network. We thus aim to provide efficient solutions for the h-MECBS[N_2^2] problem when h is a "small" constant (i.e., independent from the network size). In particular, we provide the first polynomial-time algorithm that solves the 2-MECBS[N_2^α] problem for any $\alpha \geq 0$. The algorithm use a crossed dynamic programming to get an optimal solution. The dynamic programming is far from being simple and requires $O(n^7)$ time to fill up the relative matrices.

Then, we derive a polynomial-time approximation scheme (PTAS) that works for the h-MECBS[N_2^α], for any fixed constant $h > 1$. For fixed $h \geq 1$ and $\epsilon > 0$, the PTAS has time complexity $O(n^\mu)$ where $\mu = O((\alpha 2^\alpha h^\alpha / \epsilon)^{\alpha^h})$.

2 A Polynomial-Time Algorithm for the 2-MECBS on the Plane

In this section we describe a polynomial time algorithm for the 2-MECBS problem on the Euclidean plane. The stations are represented by points in the Euclidean plane. Let $\texttt{cost}(c, p)$ be the cost required of station c in order to cover station p with minimum power. We only require $\texttt{cost}$ to be a positive function for which $\texttt{cost}(c, r_1) \leq \texttt{cost}(c, r_2)$ if $\texttt{dist}(c, r_1) \leq \texttt{dist}(c, r_2)$ holds. This also includes the cost function mentioned in the introduction.

The input of the algorithm are n points in the Euclidean plane, a specified source station s and a cost function $\texttt{cost} : \{1, \ldots, n\}^2 \mapsto \mathbb{R}$ with the above properties.

Our algorithm is based on a procedure that computes an optimal 2-broadcast range assignment for a fixed range of station s. Since the range of s in an optimal solution is defined by the farthest station f covered by the station s, we only have to invoke this procedure $n-1$ times and take the best solution in order to solve the 2-MECBS problem.

In the rest of this section, we describe the procedure for a fixed source range. Since the range of s is fixed, we can define $\mathcal{P}$ as the set of stations covered by s. Let $\overline{\mathcal{P}}$ be the set of stations not covered by s. Let us rename the stations of $V \setminus \{s\}$ as $\{1, \ldots, n-1\}$ such that the first $m = |\overline{\mathcal{P}}|$ stations are those in $\overline{\mathcal{P}}$, and they are ordered in clockwise order around the source station, starting with an arbitrary station in $\overline{\mathcal{P}}$. (The source station is still denoted by s.)

Definition 1. *Let the interval $[l, r]$ denote the set of stations i such that $l \leq i \ \leq r$.*

Definition 2. *For an interval I, let $A(I)$ be the minimum cost required to cover all the stations in I by stations from $\mathcal{P}$.*

According to the above definition, the value we are looking for in our procedure is $A([1, m]) + \texttt{cost}(s, f)$ where f is the station defining the source range. Consider an optimal covering of the interval $[l, r]$ expressed by the ranges of all stations in $\mathcal{P}$. Geometrically, this solution is represented as an arrangement $\mathcal{A}$ of disks, in which every disk represents the range of the station in its center. Denote by Δ the set of points contained in these disks. The disk of a station c in the arrangement is denoted by Δ_c.

An alternative representation of $\mathcal{A}$ is to assign to each station $p \in \overline{\mathcal{P}}$ the station in $\mathcal{P}$ that reaches p. In general, there may be many ways to define such an assignment. We do it by the λ function defined below.

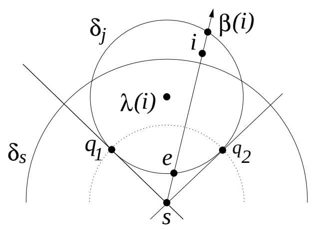

Fig. 1. Point i is always between e and $\beta(i)$

Definition 3. *For $i \in \overline{\mathcal{P}}$, let $\beta(i)$ be the last point in Δ on the ray $\bar{\rho}$ emerging from s in the direction of i. Define $\lambda(i)$ to be the station in $\mathcal{P}$ whose disk contains $\beta(i)$. If there is more than one, choose an arbitrary one. Let σ denote the segment $\overline{s\beta(i)}$.*

Note that $\beta(i)$ is on the border of some disk in the arrangement, and is not necessarily a station.

In order for λ to be a valid assignment, it is necessary to show the following.

Lemma 1. *In the arrangement $\mathcal{A}$, station i is contained in the range of station $\lambda(i)$.*

Proof. Clearly, $i \in \sigma$. If $s \in \Delta_j$, then the entire segment σ is contained in Δ_j and we are done. Otherwise, consider Fig. 1. Let e be the point in which the ray $\bar{\rho}$ enters Δ_j. We just have to prove that $e \in \Delta_s$. Let q_1 and q_2 be the intersection points of the tangents from s to Δ_j. The lemma follows since $\texttt{dist}(s, e) \leq \texttt{dist}(s, q_1) = \texttt{dist}(s, q_2) \leq \texttt{dist}(s, j)$ and $j \in \Delta_s$. This also proves that the entire segment σ is contained in Δ. □

If $j = \lambda(i)$, we say that i is λ-*covered* by j. By using the λ-assignment, we now establish the following lemma which states that the cost of $\mathcal{A}$ can be obtained by combining optimal solutions of smaller intervals, thus allowing the use of dynamic programming.

Lemma 2. *There exist stations $c \in \mathcal{P}$, $p \in \overline{\mathcal{P}}$ and subintervals $J_1, \ldots J_t$ of $[l, r]$ such that the cost of $\mathcal{A}$ can be expressed as*

$$\texttt{cost}(c, p) + \sum_{k=1}^{t} A(J_k).$$

Towards proving the lemma, we identify some properties of the arrangement. Let $D(c, p)$ be a disk of the arrangement with center c and radius $\mathtt{dist}(c, p)$, $c \neq p$. Let $[l, r]$ be the smallest interval that contains all stations that are λ-covered by c. Let I^- be the set of stations in $[l, r]$ that are not λ-covered by c. Let $J_1, \ldots, J_t$ be the partition of I^- into intervals such that i and j are in the same interval if and only if there is no station $q \in [i, j]$ which is λ-covered by c. For $1 \leq k \leq t$, define M_k to be the set of stations in $\mathcal{P}$ which λ-cover some station in J_k. The partitioning $J_1, \ldots, J_t$ has two key properties (whose proof is deferred to the full paper).

Proposition 1.
(P1) *For every* $1 \leq k \leq t$*, no station in* J_k *is* λ*-covered by station* c*.*
(P2) $M_i \cap M_j = \emptyset$ *for every* $1 \leq i < j \leq t$*.*

Proof of Lemma 2. We clearly have to pay the cost of the range of c, denoted by $\mathtt{cost}(c, p)$. For the second part, consider a set J_k. Let $\mathcal{A}_k$ be the optimal solution for covering the stations in J_k. Assume the cost of $\mathcal{A}_k$ was strictly smaller than the sum of the cost of the stations in M_k. Because of Property (P2), we could remove the ranges of the stations in M_k from $\mathcal{A}$ and add the ranges of $\mathcal{A}_k$. This new solution would be cheaper than the previous one, which is a contradiction to the optimality of $\mathcal{A}$. □

Let $S([l, r], c, p)$ be the set of stations in interval $[r, l]$ which are *not* covered by $D(c, p)$. In order to make use of Lemma 2, we have to solve the problem of finding the $J_1, \ldots, J_t$ for given $[l, r]$, c, and p. This is a kind of one-dimensional set covering problem: We have to find a set $\mathcal{N}$ of subintervals of $[l, r]$ such that any station in $S([l, r], c, p)$ is contained in at least one interval of $\mathcal{N}$ and $\sum_{J \in \mathcal{N}} A(J)$ is minimized.

Let $B([l, r], c, p)$ be the cost of the optimal covering of the stations in $S([l, r], c, p)$. Note that $A(J) \leq A(J')$ if $J \subseteq J'$. This implies that the sets in $\mathcal{N}$ can be chosen such that they do not intersect then $B([l, r], c, p) = B([l + 1, r], c, p)$ if $l \in D(c, p)$ and $B([l, r], c, p) = B([l, r - 1], c, p)$ if $r \in D(c, p)$. The general case comes from the fact that an optimal partitioning is composed of a first interval $[l, k]$ (which will have $k \in S([l, r], c, p)$) and an optimal partitioning of the remaining interval $[k+1, r]$ then

$$B([l,r],c,p) = \min_{k \in S([l,r],c,p)} \{A([l,k]) + B([k+1,r],c,p)\}.$$

Finally, $B([l,r],c,p) = 0$ for $l > r$ or $S([l,r],c,p) = \emptyset$.

Having $B([l,r],c,p)$ at hand, $A([l,r])$ can be computed considering the optimal partitioning for all pairs of c,p:

$$A([l,r]) = \min_{c \in \mathcal{P}, p \in \overline{\mathcal{P}} \,|\, l \in D(c,p)} B([l+1,r],c,p) + \mathtt{cost}(c,p).$$

The tables must be filled alternatingly starting with the smallest intervals. Since $B([l,r],c,p)$ might use $A([l,r])$, the latter should be computed first. The optimal value can be found in $A([l,r])$. The running time of the algorithm is $O(n^7)$.

3 A PTAS for any Constant h

The set of n stations is specified by the set of points $X = \{x_1, \ldots, x_n\}$ in the Euclidean plane, where $x_1 = s$. Without loss of generality assume that the points are ordered by their distance from s, and in particular, x_n is the point farthest from s, and let $R = \mathtt{dist}(s, x_n)$. Every range assignment induces the set of disks $\{D(x_1), \ldots, D(x_n)\}$, each with center x_i and radius $r(x_i)$. If $r(x_i) > 0$, we say that the disk $D(x_i)$ *belongs* to the assignment. For every two points $x_i, x_j \in X$, a *path* from x_i to x_j in the assignment is an ordered set of disks $\{D_1, \ldots, D_k\}$ belonging to the range assignment with centers $y_1, \ldots, y_k$ respectively, such that $y_1 = x_i$, $x_j \in D_k$ and $y_i \in D_{i-1}$ for each $i = 2, \ldots, k$. A *minimum hop path* from x_i to x_j is a path containing a minimum number of disks among all paths from x_i to x_j. In an h-broadcast assignment, the radii must be assigned such that for every $2 \le i \le n$, there exists a path containing at most h disks from s to x_i. Let S^* be an optimal solution to the problem, and denote its cost by C^*.

For every $\epsilon > 0$, let $k = \alpha 2^\alpha h^\alpha / \epsilon$. Define the sequence $\{k_i\}_{i=2}^{h}$ where $k_2 = k$ and $k_{i+1} = k \cdot k_i^\alpha$ for each $i = 2, \ldots, h-1$.

Notice that in any optimal solution, the disk around any point x_j is of radius $\mathtt{dist}(x_j, x_i)$ for some $1 \le i \le n$. For $2 \le i \le n$, let D_i be the disk of radius $\mathtt{dist}(s, x_i)$ centered at s. A range assignment is called a *principal h-Broadcast* if it consists of one such disk D_i around s, for some $2 \leq i \leq n$, plus up to k_h^α disks around other points, each of radius at least R/k_h.

Our algorithm operates as follows. For given fixed ϵ and h, if $\epsilon > h^{\alpha-1}$ then the algorithm returns a single disk of radius R centered at s. Otherwise, the algorithm examines all principal h-Broadcasts, and outputs the one attaining the minimal cost.

3.1 Analysis

We first observe that the algorithm is polynomial for fixed ϵ and h. For fixed $h \geq 1$ and $\epsilon > 0$, its time complexity is $O(n^\mu)$ where $\mu = O((\alpha 2^\alpha h^\alpha/\epsilon)^{\alpha^h})$.

Our approximation ratio analysis is based on the observation that the *single disk* solution, obtained by taking a single disk of radius R centered at s, yields a constant approximation to the optimal solution. (The proof is deferred to the full version of the paper.)

Lemma 3. *The single disk solution provides an approximation of ratio $h^{\alpha-1}$, namely, $R^\alpha \leq h^{\alpha-1} \cdot C^*$.*

We now prove that the cost of the solution produced by our algorithm is at most $(1+\epsilon)C^*$. Notice that if $\epsilon > h^{\alpha-1}$, then the single disk solution generated by the algorithm attains the desired bound trivially from Lemma 3. Hence hereafter we assume that $\epsilon \leq h^{\alpha-1}$.

Consider the disks that belong to the optimal solution S^*. For each such disk $D^*(x)$, define its *level* to be the number of disks in the minimum hop path from s to x. Define a disk $D^*(x)$ of level j, $2 \leq j \leq h$, to be *large* if $r^*(x) \geq R/k_j$, otherwise it is *small.* For uniformity, define also the disk $D^*(s)$ to be large. For each level $j \geq 1$, let m_j be the number of large disks of level j in the optimal solution S^*. (Always $m_1 = 1$.) Thus the large disks of level j contribute at least $m_j R^\alpha/k_j^\alpha$ to C^*, the cost of the optimal solution. As C^* cannot exceed the cost of the single disk solution, namely R^α, we have the following.

Proposition 2. *For each level $j \geq 2$, $m_j \leq k_j^\alpha$.*

Also, noting that every large disk of level $j \geq 2$ has radius at least R/k_h, we have the following.

Proposition 3. *S^* contains at most k_h^α large disks.*

Now, consider the range assignment $\hat{S}$ derived from S^* in the following way. For each large disk $D^*(x)$ in S^*, let $f(x)$ be the farthest point from x of higher level than x for which there is a minimum hop path from x to $f(x)$ that contains only small disks (other than $D^*(x)$). For each large disk $D^*(x)$ in S^*, let $\hat{r}(x) = \mathtt{dist}(x, f(x))$ and let $\varphi(D^*(x))$ be the disk of radius $\hat{r}(x)$ around x. We now take $\hat{S}$ to contain the disk $\varphi(D^*(x))$ for every large disk $D^*(x)$ in S^*.

Since the minimum hop path from x to $f(x)$ contains at most $h-1$ disks of increasing levels, which are all small to their level, $\mathtt{dist}(x, f(x)) \leq \sum_{\ell=1}^{h-1} \frac{R}{k_{j+\ell}}$, and as $R/k_j > h \cdot R/k_{j+1}$ by the choice of k and the assumption that $\epsilon \leq h^{\alpha-1}$, we have the following.

Proposition 4. *For every large disk $D^*(x)$ in S^* of level j:*

1. *If $2 \leq j \leq h-1$ then $\hat{r}(x) \leq r^*(x) + 2R/k_{j+1}$.*
2. *If $j = h$ then $\hat{r}(x) = r^*(x)$.*

Lemma 4. *The assignment $\hat{S}$ is a principal h-broadcast.*

Proof. By Proposition 3, $\hat{S}$ contains at most k_h^α disks, all of which (except maybe the one centered at s) have radius at least R/k_h. It remains to argue that every point $x \in X$ is reachable by a path of h or fewer hops. Consider such a point x and suppose a minimum hop path from s to x in S^* is established by the disks $D_1^*, \ldots, D_\ell^*$, where $\ell \leq h$. Note that the minimality of the path ensures that the level of each of the disks D_j^* is exactly j. Each small disk D_i^* in this list is now contained in the large disk $\varphi(D_j^*) \in \hat{S}$ such that D_j^* is the closest large disk in S^* preceding D_i^* in the list. Therefore the number of hops in the path to x in $\hat{S}$ is no greater than in S^*. □

We thus conclude that the assignment $\hat{S}$ was checked by our algorithm. We now bound the cost of this assignment, denoted $\hat{C}$. For each $j \geq 1$ and $i = 1, \ldots, m_j$, let $r_{i,j}$ be the radii of the large disks of level j. Then $\hat{C}$ satisfies

$$\hat{C} \;\leq\; \sum_{j=1}^{h-1} \sum_{i=1}^{m_j} \left(r_{i,j} + \frac{2R}{k_{j+1}} \right)^\alpha + \sum_{i=1}^{m_h} r_{i,h}^\alpha. \tag{2}$$

We rely on the following technical fact (which can be verified, say, by looking at the Taylor expansion of the function $(1+z)^\alpha$).

Fact 1. *For* $\alpha > 1$ *and* $0 < z \leq 1$, $(1+z)^\alpha \leq 1 + \alpha z(1+z)^{\alpha-1}$.

Proposition 5. $\left(r_{i,j} + \frac{2R}{k_{j+1}}\right)^\alpha \;\leq\; r_{i,j}^\alpha + \frac{\alpha 2^\alpha R^\alpha}{k_{j+1}}$.

Proof. Note that $r_{i,j} > 2R/k_{j+1}$ for every i and j, so $z = \frac{2R}{k_{j+1} r_{i,j}}$ satisfies $0 < z < 1$. This allows us to use Fact 1 and get

$$\left(r_{i,j} + \frac{2R}{k_{j+1}}\right)^\alpha \leq r_{i,j}^\alpha \left(1 + \frac{2R\alpha}{r_{i,j}k_{j+1}}\left(1 + \frac{2R}{r_{i,j}k_{j+1}}\right)^{\alpha-1}\right)$$
$$= r_{i,j}^\alpha + \frac{2R\alpha}{k_{j+1}}\left(r_{i,j} + \frac{2R}{k_{j+1}}\right)^{\alpha-1} \leq r_{i,j}^\alpha + \frac{2R\alpha(2r_{i,j})^{\alpha-1}}{k_{j+1}}.$$

The thesis follows from by observing that $R \geq r_{i,j}$. □

Combining Inequality (2) with Proposition 5 yields that

$$\hat{C} \;\leq\; \sum_{i,j} r_{i,j}^\alpha + \sum_{j=1}^{h-1}\sum_{i=1}^{m_j} \frac{\alpha 2^\alpha R^\alpha}{k_{j+1}} \; .$$

As $C^* \geq \sum_{i,j} r_{i,j}^\alpha$, and using Proposition 2 and the definition of k_j,

$$\hat{C} \;\leq\; C^* + \sum_{j=1}^{h-1}\left(k_j^\alpha \cdot \frac{\alpha 2^\alpha R^\alpha}{k_{j+1}}\right) \;<\; C^* + h \cdot \frac{\alpha 2^\alpha R^\alpha}{k} \;\leq\; C^* + \epsilon \cdot C^*,$$

where the last inequality is established by the choice of k and Lemma 3. We have thus established the following.

Lemma 5. *The algorithm yields a solution of cost at most* $(1+\epsilon)C^*$.

4 Conclusions and Open Problems

In this paper we investigated the problem of computing a minimal cost range assignment in ad-hoc wireless networks that guarantees the broadcast operation from a given source station in at most h hops. We provide a polynomial-time algorithm for the case $h = 2$ and a PTAS for any constant $h \geq 1$. Nothing is known about the hardness of the case $h > 2$. We conjecture that there exists some constant h for which the problem is NP-hard. This is the main problem left open by this paper. Finally, nothing is known when h is any function of n (but $h = n - 1$).

References

1. R. Bar-Yehuda, O. Goldreich, and A. Itai. On the Time Complexity of Broadcast Operations in Multi-Hop Radio Networks: An Exponential Gap Between Determinism and Randomization. *J. of Computer and Systems Science*, 45:104–126, 1992.
2. R. Bar-Yehuda, A. Israeli, and A. Itai. Multiple Communication in Multi-Hop Radio Networks. *SIAM J. on Computing*, 22:875–887, 1993.
3. G. Călinescu, X.Y. Li, O. Frieder, and P.J. Wan. Minimum-Energy Broadcast Routing in Static Ad Hoc Wireless Networks. In *Proc. 20th Joint Conf. of IEEE Computer and Communications Societies (INFOCOM)*, 1162–1171, 2001.
4. A.E.F. Clementi, P. Crescenzi, P. Penna, G. Rossi, and P. Vocca. A Worst-case Analysis of an MST-based Heuristic to Construct Energy-Efficient Broadcast Trees in Wireless Networks. In *Proc. 3th Workshop on Wireles, Mobile and Ad-Hoc Networks (WMAN-IPDPS)*, 2003.
5. A.E.F. Clementi, P. Crescenzi, P. Penna, G. Rossi, and P. Vocca. On the Complexity of Computing Minimum Energy Consumption Broadcast Subgraphs. In *Proc. 18th Symp. on Theoretical Aspects of Computer Science (STACS)*, 121–131, 2001.
6. A.E.F. Clementi, G. Huiban, P. Penna, G. Rossi, and Y.C. Verhoeven. Some Recent Theoretical Advances and Open Questions on Energy Consumption in Ad-Hoc Wireless Networks. In *Proc. 3rd Workshop on Approximation and Randomization Algorithms in Communication Networks (ARACNE)*, 23–38, 2002.
7. A.E.F. Clementi, M. Di Ianni, and R. Silvestri. The Minimum Broadcast Range Assignment Problem on Linear Multi-Hop Wireless Networks. *Theoretical Computer Science*, 299:751–761, 2003.
8. A.E.F. Clementi, P. Penna, and R. Silvestri. Hardness Results for the Power Range Assignment Problem in Packet Radio Networks. In *Proc. 2nd Int. Workshop on Approximation Algorithms for Combinatorial Optimization Problems (APPROX)*, 197–208, 1999.
9. A. Ephremides, G.D. Nguyen, and J.E. Wieselthier. On the Construction of Energy-Efficient Broadcast and Multicast Trees in Wireless Networks. In *Proc. 19th Joint Conf. of IEEE Computer and Communications Societies (INFOCOM)*, 585–594, 2000.
10. S. Guha and S. Khuller. Improved Methods for Approximating Node Weighted Steiner Trees and Connected Dominating Sets. *Information and Computation*, 150:57–74, 1999.
11. Z. Haas and S. Tabrizi. On Some Challenges and Design Choices in Ad-Hoc Communications. In *Proc. IEEE Military Communication Conf. (MILCOM)*, 1998.
12. L. M. Kirousis, E. Kranakis, D. Krizanc, and A. Pelc. Power Consumption in Packet Radio Networks. *Theoretical Computer Science*, 243:289–305, 2000.
13. G.S. Lauer. *Packet radio routing*, Chapt. 11 of *Routing in communication networks*, M. Streenstrup (ed.), 351–396. Prentice-Hall, 1995.
14. K. Pahlavan and A. Levesque. *Wireless information networks.* Wiley-Interscience, 1995.
15. R. Raz and S. Safra. A Sub-Constant Error-Probability Low-Degree Test, and a Sub-Constant Error-Probability PCP Characterization of NP. In *Proc. 29th ACM Symp. on Theory of Computing (STOC)*, 475–484, 1997.
16. M. Sharir and P.K. Agarwal. *Davenport-Schinzel Sequences and Their Geometric Applications.* Cambridge University Press, Cambridge-New York-Melbourne, 1995.