On the Approximation Ratio of the MST-based Heuristic for the Energy-Efficient Broadcast Problem in Static Ad-Hoc Radio Networks*

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Abstract

We present a technique to evaluate the approximation ratio on random instances of the Minimum Energy Broadcast Problem in Ad-Hoc Radio Networks which is known to be NP*-hard and approximable within 12. Our technique relies on polynomial-time computable lower bound on the optimal cost of any instance.*

The main result of this paper is that the approximation ratio has never *achieved a value greater than* 6.4*. Furthermore, the worst values of this ratio are achieved for small network sizes. We also provide a clear geometrical motivation of such good approximation results.*

1. Introduction

1.1. Motivations and preliminary definitions.

Wireless networking technology will play a key role in future communications and the choice of the network architecture model will strongly impact the effectiveness of the applications proposed for the mobile networks of the future. Broadly speaking, there are two major models for wireless networking: *single-hop* and *multi-hop*. The single-hop model [32], based on the cellular network model, provides one-hop wireless connectivity between mobile hosts and static nodes known as *base stations*. This type of networks relies on a fixed backbone infrastructure that interconnects all base stations by high-speed wired links. On the other hand, the multi-hop model [23] requires neither fixed, wired infrastructure nor predetermined interconnectivity. *Ad-hoc* networking [19] is the most popular type of multi-hop wireless networks because of its simplicity: Indeed, an *ad hoc* wireless network is constituted by a homogeneous system of stations connected by wireless links.

In ad-hoc networks, to every station is assigned a transmission range: The overall range assignment determines a transmission (directed) graph since one station s can transmit to another station t if and only if t is within the transmission range of s. The range transmission of a station depends, in turn, on the energy power supplied to the station: In particular, the power $p(s)$ required by a station s to correctly transmit data to another station t must satisfy the inequality

$$\frac{p(s)}{\mathtt{dist}(s,t)^{\alpha}} \geq 1 \qquad (1)$$

where $\mathtt{dist}(s,t)$ is the Euclidean distance between s and t, and $\alpha \geq 1$ is the *distance-power gradient*. The most studied case is $\alpha = 2$ [37, 8, 36] since this corresponds to the empty space and, moreover, it is a good approximation of the environment where wireless networks are located (see [27, 31]).

Energy conservation is a critical issue in an ad-hoc wireless network: It is important to minimize the energy consumption of the network provided that a connectivity property on the induced transmission graph is guaranteed (for a survey on this topic see [9]). Current transceivers and com-

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munication protocols are designed for a fixed transmission range (e.g. IEEE 802.11 standard [21]). However, a scenario in which the transmission range is not fixed is compatible with current technology. In particular, the transmission range can be varied dynamically in presence of mobility or when the physical node placement is unknown. Distributed topology control protocols, aimed at dynamically changing the transmission range assignment in order to guarantee a certain connectivity property of the network and minimize energy consumption, have recently presented in [25, 30, 33]

In this paper we address the case in which the connectivity property is the following: Given a source station s, the transmission graph induced by the range assignment must contain a directed spanning tree rooted at s. This is one of the crucial problems underlying ad-hoc wireless networks because any transmission graph satisfying the above property allows the source station to perform *broadcast* operations. Broadcast is a task initiated by the source station which transmits a message to all stations in the wireless networks: This task constitutes a major part of real life multi-hop radio networks [3, 4].

A trivial solution for the above problem consists in assigning to the source s a transmission power that suffices to directly communicate (within one hop) with all the other stations. However, this solution could be very expensive: In fact, due to Equation (1), the total power (i.e. the sum of the powers assigned to every stations) required by the network could be very large with respect to the optimal solution. This fact can be better explained by an example: Consider n nodes s_1, s_2, …, s_n on a line such that $d(s_i, s_{i+1}) = 1$ ($i = 1, \ldots, n-1$) and let s_1 be the source node. If $\alpha = 2$, an assignment that allows s_1 to directly communicate to all the other stations requires a total energy at $n2$ whereas the best assignment is $p(s_i) = 1$ ($i = 1, \ldots, n-1$), thus yielding total power $n - 1$.

Let S be a set of n nodes located on the Euclidean plane. A *range assignment* for S is a function $r : S \to \mathbb{R}^+$. The *transmission (directed) graph* $G_r = (S, E)$, induced by r, is defined as

$$E = \bigcup_{v \in S} \{(v, u) : u \in S \wedge \mathtt{dist}(v, u) \leq r(v)\}.$$

The MINIMUM ENERGY CONSUMPTION BROADCAST SUBGRAPH (in short, MECBS) problem is then defined as follows: Given a set of stations S on the Euclidean plane and a *source node* $s \in S$, find a range assignment r such that G_r contains a directed spanning tree rooted at s and the function

$$\mathtt{cost}(r) = \sum_{v \in S} r(v)^\alpha \qquad (2)$$

is minimized.

This problem was introduced in [37] where three greedy heuristics are proposed. Here, the performances of such heuristics, for the standard case $\alpha = 2$, have been compared (one to each other) on random instances, i.e., instances in which points are chosen independently and uniformly on a square region. The best heuristic appears to be the one based on the construction of an Euclidean *Minimum Spanning Tree* (MST) routed at the source node. This algorithm, denoted as MST-ALG, is sketched in Figure 1. The MST-ALG heuristic clearly runs in polynomial time and always returns a feasible solution. It achieves the best experimental results [37] and it is also easy to implement. Moreover, in network with dynamic power control (where stations are allowed to make small and/or slow movings), the range assigned to the stations can be modified at any time: The algorithm can thus take advantage of all known techniques to dynamically maintain MST's (see, for example, [13, 14, 26]).

begin
  T := DIR-MST(S, dist, s);
  forall $v \in S$ **do**
    $r^{\mathsf{mst}}(v)$:= $\max_{u:(v,u)\in T}\{\mathtt{dist}(v,u)\}$;
end

Figure 1. The MST-ALG for computing the MECST. The DIR-MST procedure returns the directed MST rooted at s (according to the input distance function dist).

Finally, MST-ALG is the only heuristic for which theoretical results are known: In fact, simultaneously and independently, in [8] and in [36], it is shown that the MST-ALG heuristic achieves a constant approximation ratio. More formally, given an instance $\langle S, s\rangle$, define

$$\mathtt{cost}(\langle S, s\rangle, r^{\mathsf{mst}}) = \sum_{v \in S} r^{\mathsf{mst}}(v)2$$

and $\mathtt{opt}(\langle S, s\rangle)$ as the cost of a minimum range assignment for this instance. Then, they prove that a constant $\rho > 0$ exists such that, for any instance $\langle S, s\rangle$, the *approximation ratio* is such that

$$\mathsf{R}^{\mathsf{mst}}(S, s) = \frac{\mathtt{cost}(\langle S, s\rangle, r^{\mathsf{mst}})}{\mathtt{opt}(\langle S, s\rangle)} \leq \rho. \qquad (3)$$

This constant is proved to be 40 in [8], it was then reduced to 20 by the same authors in [7], and it is shown to be 12 in [36]. On the other hand, [36] provides a "bad" instance (i.e., a star of 6 nodes, see Figure 2) in which MST-ALG returns a solution whose cost is almost 6 times the optimal.

We emphasize that the use of approximation algorithms is motivated by the fact that the MECBS problem is NP-hard even in the Euclidean plane (see [8, 7]). More recently, a simpler proof of the NP-hardness for a different version of

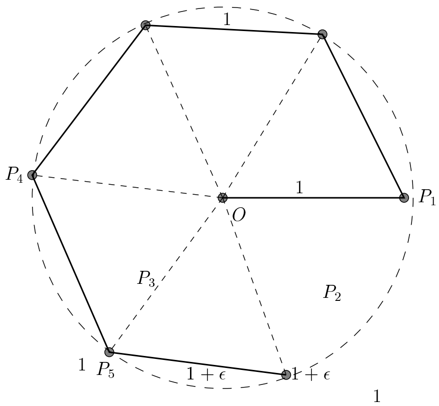


Figure 2. A bad instance for MST-ALG.

$\lvert S\rvert$	$R^{\texttt{mst}}(S,s)$	$\lvert S\rvert$	$R^{\texttt{mst}}(S,s)$	$\lvert S\rvert$	$R^{\texttt{mst}}(S,s)$
$5 \leq \cdot \leq 9$	6.4	60	2.1	375	1.4
10	4.4	65	2.0	500	1.3
15	3.3	70	2.0	1000	1.2
20	3.0	75	2.0	1500	1.2
25	2.7	80	1.9	2000	1.2
30	2.7	85	1.9	1250	1.2
35	2.5	90	1.9	1750	1.2
40	2.3	95	1.9	2250	1.1
45	2.4	100	1.8	5000	1.1
50	2.2	125	1.7	7000	1.1
55	2.2	250	1.5	9000	1.1

Table 1. The experimental results for the approximation ratio $R^{\texttt{mst}}(S,s)$ for several dimensions of the set S. We report the largest values from thousands of experiments.

the problem (in which the set of possible node transmission ranges is fixed and given as input) is presented in [16, 6]. It thus follows that an important open question is to determine the "real" quality of approximation achieved by the `MST-ALG` heuristic.

1.2. Our results.

We show that the large approximation ratio achieved in [36] is not tight for random instances. Actually, our intuition here is that it might be possible to almost match the lower bound 6 also in the worst case.

In order to support our intuition, we present and discuss the results of a new massive experimental analysis of the `MST-ALG` performances on random instances. According to most of the experimental analysis of computational problems on static ad-hoc radio networks (see for example the papers [37, 6, 5, 22]), we consider the *uniform random model*, in which nodes are chosen uniformly and independently at random from a square region of a given size and, then, the (complete) distance graph is considered. Besides having a *per se* theoretical interest, the use of the uniform random model is well motivated by theoretical and experimental results [18, 17, 24] showing that the topology of efficient static ad-hoc radio networks must be *sparse* and *well-spread* [10, 11]. We refer here to topologies arising from applications in emergencies, battlefield, monitoring remote geographical regions, etc. [15, 20, 28, 29, 34]. As in [37, 36, 6, 16], we address the case $\alpha = 2$.

The main novelty of our contribution consists in comparing the cost of the `MST-ALG` solution to a *lower bound* of the relative optimum. Indeed, from the theoretical analysis in [7], we first derive an easy-to-compute lower bound (which is not the direct lower bound yielded by the approximation ratio) on the optimal cost of any instance of the problem. We then exploit this lower bound in order to evaluate the approximation ratio over several thousands of random instances. The main result of this paper is that, for *all* the random instances, the approximation ratio has *never* achieved a value greater than 6.4. Notice that this value somewhat implies that the uniform random model "takes care" of "bad" instances like the one in Figure 2.

The above lower bound on the optima establishes a direct connection between the approximation ratio of the `MST`-based solution and the ratio $c(S)$ between the cost[1] of the `MST` of a set of nodes S on the plane and the minimal-area disk that contains S. It can in fact be proved that the approximation ratio of `MST-ALG` is not larger than $4 \cdot c(S)$. Thanks to this connection, we can evaluate the `MST-ALG` approximation ratio by performing experimental results on $c(S)$. We concentrate and report only the maximal value achieved by $c(S)$ (and, thus, by the approximation ratio) as function of the input parameters. Clearly, the average values are bounded by the relative maximal values.

Two input parameters are considered: the number n of nodes and the side length ℓ of the square region in which the n nodes are independently placed according to the uniform distribution. From these two parameters, we can define the *density* of the radio network as the ratio between the number of nodes and the size of the smallest region containing all the nodes. Number of nodes and region size characterize the network topology. For example, in radio networks implemented in buildings of few hundreds of square meters, the number of nodes can vary from few dozens to some hundreds, whereas wide area networks, spread over thousands of squared kilometers, may contain few thousands of nodes [12, 2]. However, we perform our experiments over larger ranges of the input parameters.

Our results are summarized in Table 1: the approximation ratio $R^{\texttt{mst}}(S,s)$ is shown for different sizes of the node set S. The choice of reporting $R^{\texttt{mst}}(S,s)$ as function of

[1] Notice that the cost of an edge (u,v) is $\texttt{dist}(u,v)^2$: see Section 2.

the (only) parameter $|S|$ is motivated by the fact that, from the experimental data, $R^{\text{mst}}(S,s)$ does not depends on the region size. In particular, the values of $R^{\text{mst}}(S,s)$ greater than 6 (as the value returned by the "bad" instance in Figure 2) are all obtained for $|S| \leq 9$: This might implies that this "bad" instance is one of the (absolute) worst instances. More importantly, the worst-case approximation ratio $R^{\text{mst}}(S,s)$ seems to be a *decreasing* function of $|S|$: It seems to tend to a constant slightly greater than one. This trend is consistent to that of the *asymptotical expected value* of $c(S)$ determined in [38] (this asymptotical average-case analysis gives no information about the "worst-case" instances of reasonable, small size - see Section 3).

¿From Table 1 and the above discussion, it thus turns out that the worst-case instances are likely to have small sizes. This well-motivates our massive simulation on random networks of relatively small sizes.

Finally, we can state that the quality of the approximation yielded by the `MST-ALG` heuristic is thus rather good on random instances, much better than that arising from the previous theoretical worst-case analysis in [8, 7, 36]. In Section 2, we will show some specific geometrical properties of the `MST-ALG` solutions that motivate the achieved quality.

1.3. Organization of the paper.

Section 2 shows a simple and efficient method to derive the lower bound on the optimal from the worst-case analysis in [7]. We also describe the main geometrical facts the worst-case analysis relies on, and we then conjecture a more likely worst-case geometrical scenario. In Section 3, we present our experimental results. Finally, in Section 4, we discuss the obtained results.

2. Fast-computable lower bound for the optima

Given any set of nodes S, $\mathcal{D}(S)$ denotes the smallest disk containing all the nodes S and its diameter is denoted as $\mathsf{diam}(S)$. Given the weighted complete graph $(G(S,E), \mathtt{dist}2)$, where the weight of every edge (u,v) is defined as $\mathtt{dist}(u,v)2$, the weight of a subgraph $G'(S,E')$ of G is defined as

$$w(G') = \sum_{(u,v)\in E'} \mathtt{dist}(u,v)2.$$

Now, let r^{opt} be an optimal range assignment for the instance $\langle S,s\rangle$ of `MECBS`. For any $v \in S$, let

$$K(v) = \{u \in S : \mathtt{dist}(v,u) \leq r^{\text{opt}}(v)\}$$

and let $\mathsf{MST}(v)$ be a minimum spanning tree of the subgraph of $G_{r^{\text{opt}}}$ induced by $K(v)$. For any $v \in S$, let

$$c(v) = \frac{w(\mathsf{MST}(v))}{\mathsf{diam}(K(v))2} \text{ and } c_{max} = \max\{c(v) \mid v \in S\}.$$

Then, it holds that

$$\begin{aligned}\mathtt{opt}(\langle S,s\rangle) = \sum_{v\in S} r^{\text{opt}}(v)2 &\geq \frac{1}{4}\cdot\sum_{v\in S}\frac{w(\mathsf{MST}(v))}{c(v)} \\ &\geq \frac{1}{4c_{max}}\cdot\sum_{v\in S} w(\mathsf{MST}(v))\end{aligned}$$

Since the graph $G' = (S,E')$ where

$$E' = \bigcup_{v\in S}\{e \in E : e \in \mathsf{MST}(v)\},$$

is a spanning subgraph for S, it follows that

$$\begin{aligned}\mathtt{opt}(\langle S,s\rangle) &\geq \frac{1}{4c_{max}}\cdot\sum_{v\in S} w(\mathsf{MST}(v)) \\ &\geq \frac{1}{4c_{max}}\cdot w(\mathsf{MST}(S)) \\ &\geq \frac{1}{4c_{max}}\cdot \mathtt{cost}(\mathtt{MST\text{-}ALG}(S,s)) \quad (4)\end{aligned}$$

From the above inequality, it should be clear that any upper bound for c_{max} determines a lower bound on the optimum of any instance of the MINIMUM ENERGY CONSUMPTION BROADCAST SUBGRAPH problem.

Notice that, given any set of points S on the plane, the ratio $w(\mathsf{MST}(S))/\mathsf{diam}(S)2$ can be easily computed in $O(|S|^2)$ time (as we will see later, this is the only computation made by our experimental tests!).

In [7], the following result is proved

Theorem 1 ([7]) *For any set S of points on the plane,*

$$c(S) = \frac{w(\mathsf{MST}(S))}{\mathsf{diam}(S)2} \leq 5. \quad (5)$$

By replacing $c_{max} \leq 5$ in Equation 4, [8] showed that `MST-ALG` is a 20-approximation algorithm for MINIMUM ENERGY CONSUMPTION BROADCAST SUBGRAPH. However, our opinion is that this upper bound is due to a rough and pessimistic theoretical analysis. In what follows, we argument this opinion.

2.1. A more realistic analysis.

In order to determine an upper bound for c_{max}, we need to compare the area of the disk $\mathcal{D}(S)$ and the weight of

$\mathcal{D}(S)$

Figure 3. **A minimum spanning tree (and the relative edge diameter disks) of a set S of 75 points randomly generated inside a disk of diameter 100.**

$\mathtt{MST}(S)$ for a generic set S of nodes on the plane (where the weight of every edge (u,v) is $w((u,v)) = \mathtt{dist}(u,v)2$).

Let $e = (u,v)$ be an edge of a Euclidean $\mathtt{MST}(S)$ and D_e be the *diametral* disk whose center is on the midpoint of e and whose diameter is $\mathtt{dist}(u,v)$. The contribution of e to the cost of MST can be "represented" as the area of D_e (up to the constant factor $\pi/4$); so, the cost of MST is thereabout the sum of the areas of the diametral disks associated to all the edge of the tree (see Figure 3). Then, roughly speaking, Theorem 1 is proved by showing that *no more than* 5 *of such disks can overlap over any point of* $\mathcal{D}(S)$.

In this analysis, it is thus assumed that, in the worst case, every point of $\mathcal{D}(S)$ is covered by 5 overlapping diametral disks! In other words, the worst-case scenario in which the MST solution "pays" 5 times the area of $\mathcal{D}(S)$ is considered. It is easy to convince the reader that this situation never happens. Moreover, as for random instances, the total area covered by the diametral disks appears very small with respect to the area of the disk $\mathcal{D}(S)$ (see Figure 3)! We even tried to draw 4 diametral disks of a minimum spanning tree so that they all cover a same region of positive area with no success. This really seems a geometrical property of minimum spanning trees for points of plane: unfortunately, until now, we were not able to prove it. We have run experiments devoted to the evaluation of the number of overlapping diametral disks that can occur (see the java applet in `http://mat.uniroma2.it/~verhoeve/`). From these simulations it turns out that never more than 3 disks overlap and the size of the region covered by more than one disk is almost negligible with respect to the area of $\mathcal{D}(S)$. Our opinion can thus be summarized into the following

	10×10	50×50	100×100
5%	1.448513	0.433417	0.333548
10%	0.978396	0.372507	0.30637
15%	0.807885	0.344493	0.295351
20%	0.738916	0.33153	0.293037
50%	0.543291	0.290886	0.270258
70%	0.507716	0.289506	0.268491
90%	0.457731	0.28003	0.263674

Table 2. The $c(S)$ values for some node densities and some region sizes.

Conjecture 1 *Let S be a set of points on the Euclidean plane and let* MST *be an Euclidean minimum spanning tree of S. Then, no more than 3 diametral disks of edges of T can overlap on a region of positive area. Furthermore, the area of the overall region which is covered by more than two diametral disks is almost negligible with respect to the area of $\mathcal{D}(S)$.*

3. Experimental Results

As mentioned in the previous section, our experimental task consists in computing the *worst* ratio $c(S)$ from several thousands of random node sets S's. In particular, the simulation is performed by varying the side length ℓ of the square region containing S and by varying the size $|S| = n$ from 5 to some thousands. The nodes are independently placed according to the uniform distribution. For each ℓ and $|S| \leq 1000$, we run $10,000$ experiments from which only the maximum value of $c(S)$ is considered. While, due to the high computational time and to the discovered trend of the experiments, few hundreds of experiments have been run for larger values of n. The experimental tests consider three region sizes (10×10, 50×50, 100×100). The results are summarized in Table 2. The table shows that $c(S)$ is a decreasing function of density. Observe that, fixing the density and increasing the region size corresponds to increasing the number of nodes! This might imply that, similarly to the *theoretical* asymptotical expected value (see Theorem 2), the maximum value of $c(S)$ only depends on n.

In order to support the above statement, we have performed experiments by varying the number of nodes and keeping the region size fixed. Table 3 shows the results for every $n \in \{5, \ldots, 100\}$ and for $\ell \in \{10, 50, 100, 1000\}$. The obtained data show that, for the same number of nodes, there is no relevant difference among the four considered regions. It seems thus confirmed our claim that $c(S)$ (and hence $R^{\mathtt{mst}}(S,s)$) only depends on the number of nodes and does not depend on the region size. Actually, this claim is also confirmed by a simple scaling operation.

$\|S\|$	10 $\times$ 10	50 $\times$ 50	100 $\times$ 100	1000 $\times$ 1000	max	$R^{\mathsf{mst}}(S,s)$
5	1.448	1.378	1.448	1.462	1.462	5.846
6	1.420	1.387	1.611	1.412	1.612	6.447
7	1.283	1.254	1.261	1.332	1.332	5.330
8	1.209	1.162	1.221	1.143	1.221	4.886
9	1.082	1.155	1.096	1.103	1.155	4.619
10	0.978	1.103	0.981	0.995	1.103	4.413
15	0.808	0.795	0.822	0.797	0.822	3.288
20	0.739	0.717	0.750	0.759	0.759	3.036
25	0.668	0.679	0.664	0.662	0.679	2.716
30	0.614	0.678	0.653	0.647	0.678	2.711
35	0.594	0.625	0.613	0.609	0.625	2.501
40	0.576	0.568	0.569	0.566	0.576	2.304
45	0.535	0.528	0.568	0.599	0.599	2.395
50	0.543	0.553	0.526	0.554	0.554	2.215
55	0.543	0.541	0.523	0.503	0.543	2.174
60	0.506	0.528	0.511	0.498	0.528	2.111
65	0.483	0.508	0.490	0.506	0.508	2.034
70	0.508	0.502	0.490	0.480	0.508	2.031
75	0.493	0.461	0.490	0.473	0.493	1.971
80	0.468	0.471	0.468	0.469	0.471	1.885
85	0.452	0.454	0.478	0.475	0.478	1.914
90	0.458	0.473	0.481	0.456	0.481	1.924
95	0.464	0.455	0.479	0.446	0.479	1.917
100	0.441	0.450	0.440	0.446	0.450	1.801

Table 3. The values of $c(S)$ and $R^{\mathsf{mst}}(S,s)$ for several sizes of S and several network sizes. For each region size, we have reported the worst value of $c(S)$ obtained from 10 thousands trials. The fourth column reports the worst value between the first three columns whereas the last column is the corresponding approximation ratio.

We emphasize that the maximal values of $c(S)$ returned by our experimental results seem to yield a decreasing function of n (see also Figure 4). This is fully compatible with the asymptotical behavior of the expected value of $c(S)$. Indeed, [38] proved the following theoretical result.

Theorem 2 ([38]) *Let S be a set of points chosen independently and uniformly at random from a square region of area A. Then, two positive constants k and 0 exist such that,* for any $n > 0$, *it holds that*

$$|w(\mathsf{MST}(S)) - k \cdot A| \leq \frac{p}{\sqrt{n}}.$$

For this reason, in order to find "bad" instances, we have considered instances S of size not too large ($|S| \leq 100$): The relative data are reported in Table 3.

We finally remark that determining the exact value of the constant k in Theorem 2 is still an open problem [35, 1]. However, on the ground of our experimental data, we may conjecture that this constant is widely smaller than 1.

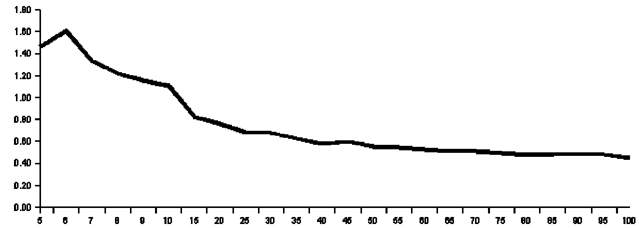


Figure 4. Trend for the worst values for $c(S)$ obtained by the fourth column of Table 3.

3.1. Notes on the Implementation

Our claim is that the performance ratio of the `MST-ALG` algorithm is 6 but the worst value found by our experiments is a little greater than this value. This discrepancy is due to our implementation choices. Since our experiments run over thousands of big instances, we have adopted the choice of computing the ratio

$$C'(S) = \frac{w(\mathsf{MST}(S))}{\max_{u,v \in S}\{\mathtt{dist}(u,v)^{\alpha}\}} \qquad (6)$$

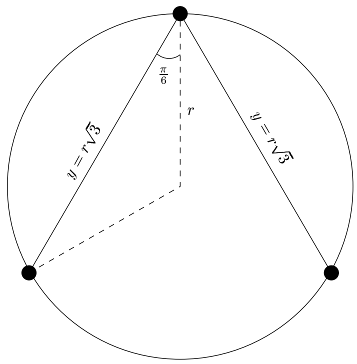
that can be computed faster than the real value of $c(S)$

$$C(S) = \frac{w(\mathsf{MST}(S))}{\mathsf{diam}^{\alpha}}.$$

Observe that $\max_{u,v \in S}\{\mathtt{dist}(u,v)\} \leq \mathsf{diam}$. Then, an upper bound for $C'(S)$ is also an upper bound for $C(S)$. However, this approximation can be too "rough" for small values of $|S|$. Indeed, let us consider three stations forming an equilateral triangle: by using Equation (6), we get $C(S) \leq 2$ and a performance ratio of 8. On the contrary, the real value of $C(S)$ is $3/2$ (see Figure 5) that implies a performance ratio of 6. We also observe that the worst instance leading the 6.4 approximation factor found by our experiments yield a shape similar to Figure 5. This instance is represented in Figure 6.

4. Conclusion and open questions

We have presented the first experimental results on the approximation ratio achieved by the `MST-ALG` heuristic for the Minimum Energy Consumption Broadcast Subgraph problem on 2-dimensional radio networks. Such experiments show that the achieved quality is good, much better than that derived from the best-known theoretical worst-case analysis. We strongly believe that this quality is due to a set of geometrical properties of the `MST-ALG` solutions which are not considered by such worst-case analysis: these properties seem to hold for *any* 2-dimensional instance of reasonable large size.



$$C(S) = \frac{w(\texttt{MST-ALG}(S))}{\mathsf{diam}^2} = \frac{6r^2}{4r^2} = \frac{3}{2}$$

$$C'(S) = \frac{w(\texttt{MST-ALG}(S))}{y^2} = \frac{2y^2}{y^2} = 2$$

Figure 5. The discrepancy between the "real" performance ratio of the MST-ALG algorithm and the performance ratio computed in our experiments.

The main theoretical open question is proving Conjecture 1, thus achieving a better theoretical worst-case approximation ratio for the `MST-ALG`.

Moreover, another important open problem is whether other algorithmic techniques can achieve better worst-case approximation for the MINIMUM ENERGY CONSUMPTION BROADCAST SUBGRAPH problem.

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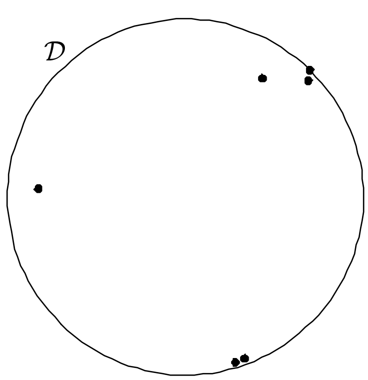

Figure 6. The worst instance returned by our experiments.

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