# The range assignment problem in *ad-hoc* wireless networks: a survey

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## 1 Introduction and Motivations

During the last decade, *wireless networks* face a tremendous development in the network area, mostly caused by the recent drop in equipment prices. Traditionally, wireless networks were developed LANs (Local Area Networks) [12], like an office environment where they bring flexibility for the users, to WANs (Wide Area Networks) where they are likely to open new possibility in networking [5]. Indeed, the major advantage of ad-hoc networks relies on the *needless of an infrastructure*: the network is simply a collection of *hosts*, that is, radio transmitter/receivers which can communicate with each other by sending/receiving radio signals. In general, message communication takes place via *multi-hop* transmission, that is, a message is sent to its destination through a set of intermediate hosts. This is due to the fact that, because of radio signal propagation [19], an host may not be able to directly communicate with another one. Indeed, the *transmission power* of the sender, its *distance* to the receiver, and other *environmental* *conditions* determine whether the received radio signal is strong enough in order to make possible the decoding of the message. The attenuation of a signal transmitted with power $P_s$ equals to

$$P_r = \frac{P_s}{\mathtt{dist}(s,t)^\alpha},$$

where $\mathtt{dist}(s,t)$ denotes the distance between the two hosts $s$ and $t$ ([21]), and $\alpha \geq 1$ is the *distance-power gradient*. In addition, a message can be correctly decoded whenever $P_r \geq \gamma$, where the constant $\gamma \geq 1$ is the *transmission-quality* parameter. Therefore, the so called *transmission range* of a station is determined by its transmission power: the (maximum) distance a station $s$ can transmit to is not bounded by $(P_s/\gamma)^{1/\alpha}$, where $P_s$ is the (maximum) power $s$ can transmit with.

Notice that, in some cases hosts are portable devices which benefits only of limited power resources. One of the main benefits of ad-hoc *power controlled* networks is the ability of the hosts to vary the power used in the transmission (and therefore the transmission range) in order to avoid interference problems and reduce the power consumption.

As we will see in the sequel, deciding the transmission power of the single hosts so to (i) guarantee a "good" communication between hosts, and (ii) minimize the overall power consumption of the network, gives rise to interesting algorithmic questions. In particular, these two aspects yield a a class of fundamental optimization problems, denoted as *range assignment* problems, that has been the subject of several works in the area of wireless network theory. The primary aim of this survey is to thus to describe the most important recent advances on the problems described above. Rather than completeness, the survey will try to provide results and techniques that seem to be the most promising to address the several important related problems which are still far to be solved. Discussing the related open problems are indeed the other main goal of this survey.

## 1.1 Range Assignment Problems

In what follows, we provide a formal definition of the range assignment problems. Stations are here considered as points of an Euclidean space.

Given a set of stations $S$, a *range assignment* for $S$ is a function $r : S \to \mathbb{R}^+$. The *cost* of a range assignment $r$ is the overall power consumption, that is

$$\mathtt{cost}(r) = \sum_{v \in S} [r(v)]^\alpha.$$

A range assignment $r$ for a set $S$ of stations yields a directed *communication graph* $G_r = (S, E)$ such that, for each $(u,v) \in S^2$, the directed edge $(u,v)$ belongs to $E$ if and only if $v$ is at distance at most $r(u)$ from $u$. Notice that we have here fixed $\gamma = 1$; However, all the results hold for any $\gamma \geq 1$.

An ad-hoc wireless network can then be viewed as a communication graph associated with a certain range assignment as shown in Figure 1.

Depending on the particular application, the communication graph is required to satisfy a given property $\Pi$. By varying property $\Pi$, we can obtain a class of range assignment problem.

**Definition 1** *Given a graph property* $\Pi$*, we define the problem* MIN-RANGE($\Pi$) *such that,*

Input: *A set of points* $S$*.*

Output: *A range assignment* $r$ *for* $S$ *such that* $G_r$ *satisfies* $\Pi$ *and* $\mathtt{cost}(r)$ *is minimized.*

In the following, we will denote $\mathtt{opt}_\Pi(I)$ as the cost of the solution of MIN-RANGE($\Pi$) for instance $I$. The range assignment problems that have attracted the attention of researchers are mainly those in which the graph property enable to implement a network primitive. Such properties are listed below.

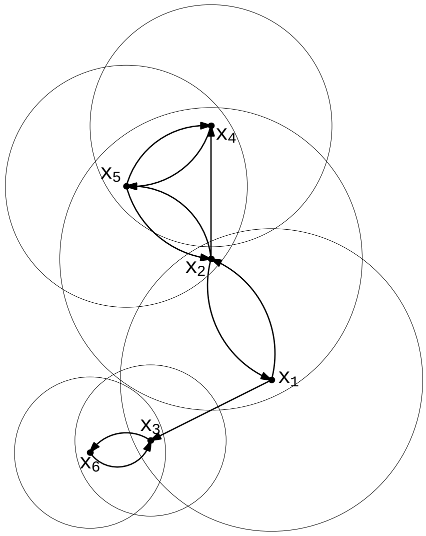


Figure 1: Example of a communication graph

- *strong connectivity* (SC); $G_r$ must be strongly connected.
- *h-strong connectivity* ($h$SC); From every stations $s$ to any other $t$, $G_r$ must contain a directed path of length at most $h$;
- *broadcast* (B), given a particular node $s$ (called the *source*), $G_r$ must contain a directed source spanning tree rooted at $s$;
- *h-broadcast* ($h$B), as in the previous case but with the further property that the source tree must have depth at most $h$.

In the next sections, we will see that the computational complexity of the above range assignment problems varies dramatically depending on the property $\Pi$ and on the other parameters of the wireless networks, such us the distance-power gradient $\alpha$ and the dimension of the Euclidean space the network is located on. As mentioned before, this survey will review the main complexity results and open theoretical questions related to the above problems

**Organization of the paper.**

# 2 The Strong Connectivity

The Min-Range(SC) problem was the first studied problem in this area [16]. The importance of this problem is due to the fact that, in a wireless, network it can be usefull the possibility that every station can communicate with all the other ones. All the complexity results regarding this problem are summarized in Table 2. In this section we briefly describe the polynomial time algorithm for the one dimensional case, after this, the approximation properties of the np-hard instances are investigated.

| | $d = 1$ | $d = 2$ | $d > 2$ |
|---|---|---|---|
| $\alpha = 1$ | $\in$ P ([16]) | $2-$ APX ([16]) | NP-hard ([16])<br>$2-$ APX ([16]) |
| $\alpha \geq 2$ | $\in$ P ([16]) | NP-hard ([10])<br>$2-$ APX ([16]) | APX-hard ([10])<br>$2-$ APX ([16]) |

## 2.1 Strong Connectivity on the Line

When the stations are located along a line the network is said *linear*. Linear radio networks have been the subject of several recent studies [6, **?**, 20]. As pointed out in [20], rather than a simplification, this version of the problem results in a more accurate analysis of the situation arising, for instance, in vehicular technology applications. It is common opinion to consider one-dimensional frameworks as in fact the most suitable ones in studying road traffic information systems [4, 13, 18, 20]. Vehicles follow roads, and messages are to be broadcasted along lanes. Typically, the curvature of roads is small in comparison to the transmission range (half a mile up to some few miles).

Kirousis *et al* [16] showed that the Min-Range(SC) problem restricted to linear instances can be solved in polynomial time.

**Theorem 2** *There exists an algorithm that finds an optimal solution for* Min-Range(SC) *in* $O(n^4)$ *time when its input is a set* $S$ *of* $n$ *points in a 1-dimensional Euclidean space,*

The algorithm makes a rather involved use of dynamic programming and it is thus omitted here (all the details of the proof can be found in [16]). However, as we will see in the next sections, the use of dynamic programming characterizes all the efficient algorithms for the range assignment problems when the input network is linear.

## 2.2 Strong Connectivity in Higher Dimensions

**NP-Hardness and APX-Hardness** When stations are spread on a multidimensional space, finding an optimal solution for the Min-Range(SC) problem is computationally hard.

**Theorem 3 (Hardness results)** *For any distance-power gradient* $\alpha > 1$*, the* Min-Range(SC) *problem is*

- *[11]* NP*-hard for* $d = 2$*;*
- *[16]* NP*-hard for* $d \geq 3$*;*
- *[11]* APX*-hard for* $d \geq 3$*, so it does not admit a PTAS unless* P = NP.

The above hardness results are all proved by using reductions from variants of the Min Vertex Cover problem. The reductions make use of suitable *local gadgets*. Even though the gadgets are rather difficult to construct (The interested reader is invited to consult the complete proofs in [16] and [11].) and their shape depend on the dimension and the kind of reduction, their role is always clear. For this reason, the idea of the reduction is rather simple and it is described below. For the sake of simplicity, we will consider the 2-dimensional case.

The Min Vertex Cover problem is to find a subset $K$ of the set $V$ of vertices of a graph $G(V, E)$ such that $K$ contains at least one endpoint of any edge in $E$ and $|K|$ is as small as possible. Min Vertex Cover is known to be NP-hard even when restricted to planar cubic graphs [14].

Given a graph $G$ a *planar orthogonal grid drawing* is a drawing of $G$ such that

1. Each vertex is represented as a point in the plane with integer coordinates;

2. Edges are represented as chains of horizontal and vertical segments (i.e. *polyline*) connecting the two endpoints and whose bends have integer coordinates;

3. Every polyline (representing an edge) crosses neither other polylines nor points representing vertices.

A drawing is said to be *straight-line* if all the edges are represented by one segment connecting the endpoints.

We will show a polynomial-time reduction from Min Vertex Cover restricted to planar, cubic graphs to Min-Range(SC).

We first outline which step have to be performed in order to derive an instance $S(G)$ of Min-Range(SC) corresponding to a planar at most cubic graph $G$. To this aim, we will make use of an intermediate representation of $G$, by means of a planar orthogonal grid drawing $D(G)$ of it. This intermediate step will make the construction of $S(G)$ simpler. The whole construction will basically take these steps:

1. Construct a planar orthogonal grid drawing of $G$;

2. Add two new vertices for each bend of the drawing so to obtain a straight-line drawing $D(G)$;

3. Replace each straight-line (edge) in $D(G)$ with a suitable set of stations (gadget).

Notice that, in the second step, the reduction preserves the optimality of the vertex cover solutions between $G$ and the new graph represented by $D(G)$. Indeed, if $2h$ is the number of vertices added by this operation, then $G$ has a vertex cover of size $k$ if and only if $D(G)$ has a vertex cover[1] of size $k+h$. Finally, in the third step, further vertices are added in $D(G)$ still preserving the above relationship between the vertex covers of $G$ and those of $D(G)$.

In what follows, we provide the key properties of these gadgets and the reduction to Min-Range(SC) that relies on such properties.

The type of gadget used to replace one edge of $D(G)$ depends on the local "situation" that occurs in the drawing (for example it depends on the degree of its endpoints). However, the following properties characterize any of these gadgets (see Figure 2).

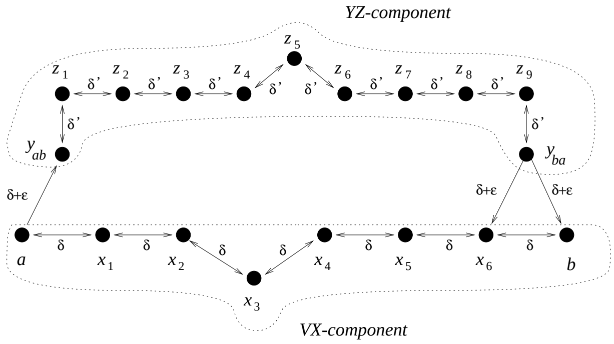


Figure 2: An example of a 2-dimensional gadget and a canonical assignment for it.

**Definition 4 (Gadget Properties)** *Let $\delta, \delta', \epsilon \geq 0$ such that $\delta + \epsilon > \delta'$ and $\alpha > 1$. For any edge $(a, b)$ the corresponding gadget $g_{ab}$ contains the sets of points $X_{ab} = \{x_1, \ldots, x_{l_1}\}$, $Y_{ab} = \{y_{ab}, y_{ba}\}$, $Z_{ab} = \{z_1, \ldots, z_{l_2}\}$ and $V_{ab} = \{a, b\}$, where $l_1$ and $l_2$ depend on the length of the drawing of $(a, b)$. These sets of points are drawn in $\mathbb{R}^2$ so that the following properties hold:*

[1] In what follows, we will improperly use $D(G)$ to denote both the drawing and the graph it represents.

*1.* $\texttt{dist}(a, y_{ab}) = \texttt{dist}(b, y_{ba}) = \delta + \epsilon$.

*2.* $X_{ab}$ *is a chain of points drawn so that*

$$\texttt{dist}(a, x_1) = \texttt{dist}(x_1, x_2) = \cdots = \texttt{dist}(x_{l_1-1}, x_{l_1}) = \texttt{dist}(x_{l_1}, b) = \delta$$

*and, for any* $i \neq j$, $\texttt{dist}(x_i, x_j) \geq \delta$.

*3.* $Z_{ab}$ *is a chain of points drawn so that*

$$\texttt{dist}(y_{ab}, z_1) = \texttt{dist}(z_1, z_2) = \cdots = \texttt{dist}(z_{l_2-1}, z_{l_2}) = \texttt{dist}(z_{l_1}, y_{ba}) = \delta'$$

*and, for any* $i \neq j$, $\texttt{dist}(z_i, z_j) \geq \delta'$.

*4. For any* $x_i \in X_{ab}$ *and* $z_j \in Z_{ab}$, $\texttt{dist}(x_i, z_j) > \delta + \epsilon$. *Furthermore, for any* $i = 1, \ldots, l_1$, $\texttt{dist}(x_i, y_{ab}) \geq \delta + \epsilon$ *and* $\texttt{dist}(x_i, y_{ba}) \geq \delta + \epsilon$.

*5. Given any two different gadgets* $g_{ab}$ *and* $g_{cd}$, *for any* $v \in g_{ab} \setminus g_{cd}$ *and* $w \in g_{cd} \setminus g_{ab}$, *we have that* $\texttt{dist}(v, w) \geq \delta$ *and if* $v \notin V_{ab} \cup X_{ab}$ *or* $w \notin V_{cd} \cup X_{cd}$ *then* $\texttt{dist}(v, w) \geq \alpha\delta$.

From the above definition, it turns out that the gadgets consist of two components whose relative distance is $\delta + \epsilon$: the $VX$*-component* consisting of the "chain" of points in $X_{ab} \cup V_{ab}$, and the $YZ$*-component* consisting of the chain of points in $Y_{ab} \cup Z_{ab}$.

Let $S(G)$ be the set of points obtained by replacing each edge of $D(G)$ by one gadget having the properties described above.

**Note 5** *Let* $r^{min}$ *be the range assignment of* $S(G)$ *in which every point in* $VX$ *and in* $YZ$ *have range* $\delta$ *and* $\delta'$, *respectively (notice that this assignment is not feasible). The corresponding communication graph consists of* $m + 1$ *strongly connected components, where* $m$ *is the number of edges: the* $YZ$*-components of the* $m$ *gadgets and the union* $\mathcal{U}$ *of all the* $VX$*-components of the gadgets. It thus follows that, in order to achieve a feasible assignment, we must define the "bridge-point" between* $\mathcal{U}$ *and every* $YZ$*-component.*

The above note leads us to define the following *canonical* (feasible) solutions for $S(G)$.

**Definition 6 (Canonical Solutions for $S(G)$)** *A range assignment* $r$ *for* $S(G)$ *is* canonical *if, for every gadget* $g_{ab}$ *of* $S(G)$, *the following properties hold.*

*1. Either* $r(y_{ab}) = \delta + \epsilon$ *and* $r(y_{ba}) = \delta'$ *(so,* $y_{ab}$ *is a radio "bridge" from the* $YZ$*-component to the* $VX$ *one) or vice versa.*

*2. For every* $v \in \{a, b\}$, *either* $r(v) = \delta$ *or* $r(v) = \delta + \epsilon$. *Furthermore, there exists* $v \in \{a, b\}$ *such that* $r(v) = \delta + \epsilon$ *(so,* $v$ *is a radio "bridge" from the* $VX$*-component to the* $YZ$ *one).*

*3. For every* $x \in X_{ab}$, $r(x) = \delta$.

*4. For every* $z \in Z_{ab}$, $r(z) = \delta'$.

The reduction is then based on the following ideas:

1. If we minimize the number of "bridge" stations in the $V$-components then we minimize the overall cost of any canonical solution (observe that the cost of all the $X$- and $YZ$-components is fixed);
2. The graph $D(G)$ has a vertex cover of size $k$ if and only if there exists a canonical solution for $S(G)$ with $k$ "bridge" stations of type $V$;
3. Any non-canonical feasible solution can be transformed in polynomial time into a canonical one without paying any extra cost (notice that any canonical assignment is feasible). This fact is guaranteed by Property 5 of the gadget definition, for a suitable choice of the parameters *delta*, $\delta'$, $\epsilon$, and $\alpha$.

**begin**
  $T$ := $\texttt{mst}(S, \texttt{dist})$;
  **forall** $v \in S$ **do**
    $r^{\texttt{mst}}(v)$ := $\max_{u:(v,u)\in T}\{\texttt{dist}(v,u)\}$;
**end**

Figure 3: The MstAlg.

**Approximability Results** Theorem 3 shows that the Min-Range(SC) problem is hard to solve exactly and, for $d \geq 3$, it does not admit a PTAS unless P = NP. On the other hand, Kirousis *et al* [16] derived a simple and efficient algorithm that, given any instance of the problem, returns a solution whose cost is at most twice the optimum: so, they provide a 2-approximation algorithm for Min-Range(SC). The algorithm MstAlg is based on the computation of a minimum spanning tree mst.

Given a set $S = \{x_1, \ldots, x_n\}$ of points in a $d$-dimensional space, consider the complete weighted graph $G = (S, E)$ where the weight of the edge $\{x_i, x_j\}$ is set to $\texttt{dist}(x_i, x_j)$. The algorithm is described in Figure 3.

Clearly this algorithm runs in $O(n^2)$ time and returns a feasible range assignment. In order to analyse the approximation degree achieved by the MstAlg we need the following definitions:

For any subgraph $G'(V, E')$ of the weighted graph $G$, define the cost of $G'$ as

$$\texttt{cost}(G') = \sum_{e \in E'} \texttt{cost}(e)^2$$

and let $\texttt{opt}_{sc}(S)$ be the optimal cost for the instance $S$ of the Min-Range(SC).

**Theorem 7 ([16])** *For any instance $S$, the* MstAlg *returns a range assignment $r^{\texttt{mst}}(S)$ such that*

$$\texttt{cost}(r^{\texttt{mst}}(S)) \leq 2 \cdot \texttt{opt}_{sc}(S).$$

Proof. The proof of the theorem is based on the following two inequalities:

$$\texttt{opt}_{sc}(S) \geq \texttt{cost}(T) \tag{1}$$

$$\texttt{cost}(r^{\texttt{mst}}(S)) \leq 2 \cdot \texttt{cost}(T) \tag{2}$$

Where $T$ is the minimum spanning tree computed by the MstAlg. To prove the Equation 1, we just notice that the undirected communication graph of any feasible solution must contain a spanning tree.

Equation 2 is proved observing that

$$\texttt{cost}(r^{\texttt{mst}}(S)) = \sum_{i=1}^{n} \max_{j:(x_i,x_j)\in T}\{\texttt{dist}(x_i, x_j)^2\} \; < \sum_{i=1}^{n} \sum_{\{j \mid \{x_i,x_j\}\in T\}} \texttt{dist}(x_i, x_j)^2 = 2\texttt{cost}(T).$$

□

The algorithm presented here works also when the input space is not Euclidean since the quality of the solution relies only on the properties of minimum spanning trees.

## 2.3 Open Problems

It is unknown whether Min-Range(SC) in the 2-dimensional Euclidean space is APX-hard or admits a PTAS. A more sophisticated gadget construction in the reduction shown in Section 2.2 might yield again an APX-hardness result. Another challenging goal is to improve the approximation factor achieved by the MST algorithm at least in some interesting restrictions of the problem. About this issue, we observe that the MST algorithm does not exploit at all the Euclidean properties of the instance, so we believe that a better algorithm should instead rely on some geometrical properties. A seemingly simpler case is that in which $\alpha = 1$. Since the communication graph satisfies the triangular inequality, we conjecture that this case is efficiently solvable or, at least, approximable within a factor smaller than 2. As for linear network, a relevant future work is that of designing a more efficient polynomial-time algorithm.

# 3 The $h$-Strong Connectivity

Recall that the MIN-RANGE($h$SC) problem is similar to the MIN-RANGE(SC). The difference consists in the fact that an additional request on the maximum number of hops $h$ for each communication between every pair of stations can be given. This means that the diameter of the communication graph $G_r$ is at most $h$.

The known results concerning the worst-case complexity of the MIN-RANGE($h$SC) problem is still unknown. Clearly, the problem is $NP$-hard for multi-dimensional instances since the MIN-RANGE(SC) problem is the special case of MIN-RANGE($h$SC) in which $h = n-1$. However, nothing is known for other (say, asymptotically smaller) values of the parameter $h$.

In what follows, we review and discuss some recent results concerning special cases of this problem.

## 3.1 $h$-Strong Connectivity on the Line

In [16], the following bounds on the optimal cost have been proved for a strong restriction of MIN-RANGE($h$SC) in the line.

**Theorem 8 (The Uniform Chain Case [16])** *Let $S$ be a set of $n$ points equally spaced at distance $\delta > 0$ on the same line; let $\mathtt{opt}_{h\mathtt{SC}}(S)$ be the cost of an optimal solution for* MIN-RANGE($h$SC) *on input $h$ and $S$. Then, it holds that*

1. $\mathtt{opt}_{h\mathtt{SC}}(S) = \Theta\left(\delta^2 n^{\frac{2^{h+1}-1}{2^h-1}}\right)$, *for any fixed positive integer $h$;*
2. $\mathtt{opt}_{h\mathtt{SC}}(S) = \Theta\left(\delta^2 \frac{n^2}{h}\right)$, *for any $h = \Omega(\log n)$.*

*Furthermore, the two above (implicit) upper bounds can be efficiently constructed.*

The approximation ratio guaranteed by the first result of Theorem 8 *increases with $h$.*

In [6], the following efficient algorithm is given

**Theorem 9 ([6])** *There exists an algorithm that, given any linear wireless network and any $h > 0$, guarantees a 2-approximation ratio and runs in $O(hn^3)$ time.*

The same work also provide a better approximation algorithm that works on any family of *well spaced* instances and for any constant $h$; in such instances, the ratio between the maximum and the minimum distance among adjacent stations is bounded by a polylogarithmic function of $n$. More precisely, it is shown that, for any well spaced instance and for any constant $h$, it is possible to compute in $O(hn^3)$ time a solution whose cost is at most $(1 + \epsilon(n))$ times the optimum, where $\epsilon(n) = o(1)$.

The above approximability results are obtained by exploiting exact solutions for two natural variants of the MIN-RANGE($h$SC) problem that may be of independent interest:

MIN ALL-TO-ONE ASSIGNMENT Given a set $S$ of stations on the line, a *sink* station $t \in S$, and an integer $h > 0$; find a minimum cost range assignment for $S$ ensuring that any station is able to reach $t$ in at most $h$ hops.

Min Assignment with Bases Given a set $S$ of stations on the line, and an integer $h > 0$; find a minimum cost range assignment for $S$ such that, any station in $S$ is either a *base* (a station is a base if it directly reaches any other station in $S$) or it reaches a base in at most $h-1$ hops.

For each of the two above problems, provide an algorithm, based on dynamic programming, that returns an optimal solution in $O(hn^3)$ time.

Finally, it is also proved that, for $h = 2$, the Min-Range($h$SC) problem can be solved in $O(n^3)$ time. This result is obtained by combining the algorithm for the Min Assignment with Bases problem with a simple characterization of the structure of any optimal 2-hops range assignment.

## 3.2 $h$-Strong Connectivity on Higher Dimensions

In [11, 10], the authors provide a lower bound and an upper bound that hold for the solution cost of any 2-dimensional instance, when $h$ is an arbitrary constant. The results are given for $\alpha = 2$ but they hold for any $\alpha \geq 1$.

Given a set of stations $S$, let us define

$$D(S) = \max\{\texttt{dist}(s, s') \mid s, s' \in S\}; \quad \delta(S) = \min\{\texttt{dist}(s, s') \mid s, s' \in S,\ s \neq s'\}$$

and let $\texttt{opt}_h(S)$ be the cost of an optimal solution for the set $S$ and for $h$ hops.

**Theorem 10** *[11, 10] For any constant $h > 0$, and for any set $S$ of stations on the plane, it holds that*

$$\texttt{opt}_h(S) = \Omega(\delta(S)^2 |S|^{1+1/h}),$$

The same papers provide an efficient solution for any instance, for constant values of $h$.

**Theorem 11** *[11, 10] For any set of stations $S$ on the plane, it is possible to construct in time[2] $O(|S|)$ a feasible range assignment $r_h(S)$ such that*

$$\mathsf{cost}(r_h(S)) = O(D(S)^2 |S|^{1/h}),$$

*for any constant $h > 0$.*

Let us now consider the planar configuration $G_n$ where $n$ stations are placed on a square grid of side $\sqrt{n}$ and the distance between adjacent pairs of stations is 1 (notice that this is the 2-dimensional version of the unit chain case studied in [16] - see Theorem 8). Then, by combining Theorem 10 and Theorem 11, we easily obtain the following optimal bound

$$\texttt{opt}_h(G_n) = \Theta\left(n^{1+1/h}\right). \tag{3}$$

The square grid configuration is the most regular case of *well-spread* instances. In general, we say that a family $\mathcal{S}$ of planar instances is *well-spread* if, for any $S \in \mathcal{S}$, $\delta(S) \geq cD(S)/\sqrt{|S|}$ (for some positive constant $c > 0$). Notice that the above property is rather natural: informally speaking, in a well-spread instance, any two stations must be not "too close". Because of interference problems, this is the typical situation in radio networks adopted in practice [20, 21]. It turns out that the optimal bound in Equation 3 holds for any family of well-spread instances. The following corollary is thus an easy consequence of Theorems 10 and 11.

[2] The constant hidden by the $O$ notation is linear in $h$.

**Corollary 12** *Let $\mathcal{S}$ be a family of well-spread instances. For any $S \in \mathcal{S}$, it holds that*

$$\texttt{opt}_h(S) = \Theta\left(\delta(S)^2 |S|^{1+1/h}\right),$$

*for any positive constant $h$.*

Beside being interesting in itself, the well-spread concept turns out to be useful to analyse another important family of instances: the *random instances*. It is not hard to show that a family $\mathcal{S}^R$ of uniformly distributed random instances, with high probability, does not satisfy the well-spread property. However, in [10], it is shown that, given a family $\mathcal{S}^R$ of random instances, it is possible to construct a family $\mathcal{S}^W$ of well-spread instances having the following property. For any $S^r \in \mathcal{S}^R$, there is an $S^w \in \mathcal{S}^W$ such that $|S^w| = \Theta(|S^r|)$ and, with high probability, $\texttt{opt}_h(S^r) = \Theta(\texttt{opt}_h(S^w))$. This equivalence yields the following result.

**Theorem 13** *[10] Let $\ell$ be any positive real. Let $S^r$ be a set of $n$ stations chosen uniformly and independently at random on a square of side $\ell$. Then, with high probability, it holds that*

$$\texttt{opt}_h(S^r) = \Theta\left(\ell^2 n^{1/h}\right),$$

*for any constant $h$.*

The lower bound obtained in Theorem 10 holds for any instance, so the constructive (and efficient) method of Theorem 11 and the equivalence yielding Theorem 13 easily imply the following result. Let Av-APX be the class of optimization problems (together with a probability function on the instance set) that admit a polynomial time algorithm that, with high probability, returns a feasible solution having performance ratio bounded by a fixed constant [1].

**Corollary 14** *[10]*

- *Let $\mathcal{S}$ be any family of well-spread instances. Then, the* MIN-RANGE($h$SC) *problem restricted to $\mathcal{S}$ admits a polynomial-time approximation algorithm with constant performance ratio (i.e. the restriction is in* APX*), for any contant $h > 0$.*
- *The* MIN-RANGE($h$SC) *problem (with uniform instance probability) is in* Av-APX*, for any constant $h > 0$.*

### 3.3 Open Problems

Several questions related to MIN-RANGE($h$SC) are still open. We here discuss only our favorite ones. As for the multi-dimensional case, we conjecture that the problem remains NP-hard even for constant values of $h$ and for any $\alpha \geq 1$. We believe that a good starting point, in order to prove this conjecture, might be that of considering suitable versions of Facility Location Problems, optimization problems which are known to be NP-hard. As for the linear case, it is not known whether the problem is NP-hard for some range of $h$. We here conjecture the existence of a dynamic-programming algorithm that solves the linear case in $O(n^{O(h)})$ time, for any $h > 0$. Notice that both cases are still unsolved for general instances even for $\alpha = 1$.

## 4 The Broadcast

Broadcast is one of the fundamental tasks that constitutes a major part of real life multi-hop radio network [2, 3]. In particular, it consists in a transmission of a message from a source station $s$ to *all* stations in the wireless network.

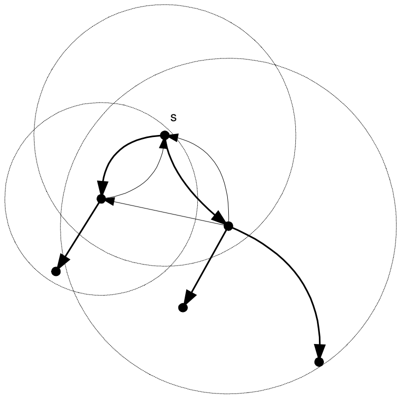


Figure 4: The spanning tree (bold arrows) from the source $s$ of the transmission graph induced by the range assignment represented by the dashed circles

| | $d = 1$ | $d = 2$ | $d > 2$ |
|---|---|---|---|
| $\alpha = 1$ | $\in$ P (*folklore*) | $\in$ P (*folklore*) | $\in$ P (*folklore*) |
| $\alpha \geq 2$ | $\in$ P ([9]) | NP-hard ([8])<br>$3^{\alpha/2}2^{\alpha}$ − APX ([22]) | NP-hard ([8]) |
| $2 < \alpha < d$ | $\times$ | $\times$ | NP-hard ([8]) |
| $\alpha \geq d$ | $\times$ | $\times$ | NP-hard ([8])<br>$2^{O(d)}$-APX ([8]) |

Table 1: Complexity of the Min-Range(B) problem based on the dimension $d$ and the distance power gradient $\alpha$.

The broadcast operation can be possible if the transmission graph induced by the range assignment contains a directed spanning tree rooted at the source station $s$ (see Figure 4) because there exist paths from $s$ towards all the nodes in the network. With this observation in mind, have a sense the study of the Min-Range(B) problem that, given a set of station in a space and a source station $s$, consists in finding a range assignment $r$ such that its induced subgraph contains a spanning tree rooted at $s$.

This problem differs from the preceding in the sense that only one station must be able to communicate with all the other ones and in the computation of the cost, we only need to consider unidirectional links.

The Min-Range(B) problem was introduced in [23] for the bi-dimensional case and when $\alpha = 2$. In this work, the performances of three heuristics have been experimentally compared (one to each other) on the random uniform model without providing theoretical results.

A theoretical analysis of this problem was independently took up in the papers [8, 7, 9, 22] where both positive and negative results are given. In Table 4 are summarized these results.

The Min-Range(B) problem is NP-hard when the stations lay in a $d$-dimensional space for all $d \geq 2$ and for all $\alpha > 1$, this result is given in [8] where is proved the following theorem:

**Theorem 15 ([8])** *In a Euclidean d-dimensional space the problem* Min-Range(B) *is* NP*-hard for all* $\alpha > 1$ *and* $d \geq 2$.

However, when $\alpha = 1$ the problem is trivially in P because it suffices to assign to the source station the minimum power that permits the communication with all the other stations with a single hop.

In [8], this result is just stated because its proof is very similar to the proof of Theorem 3. A complete proof is available in the technical report [7].


Figure 5: A right bridge.

## 4.1 Broadcast in the Line

The Min-Range(B) problem in the line is in P for any $\alpha$. In the paper [9] a dynamic programming algorithm for the case $\alpha \geq 2$ is proposed (the case $\alpha = 1$ is trivial). The case $\alpha \geq 2$ is not easily computable starting from smaller instances because the introduction of a new station could affect the previously computed ranges. An example can clarify the ideas: Consider the Figure 5, let $\alpha = 2$ and $s$ be the source node, the optimal range assignment of Figure 5a has cost 3. If we add a station $c$ on the right extreme of the line at distance 4 from the nearest station $r$ (see Figure 5b), we have to assign a range 4 to $r$. But now, from $r$ we can reach also the stations $b$ and $\ell$ so the old ranges of $a$ and $b$ are no longer necessary. The cost of this optimal solution is 17.

If in the optimal solution there exist a situation similar to the one showed in Figure 5b, that is, there exist a link from a station $r$ in the right side of $s$ towards the station $\ell$ in the left side of $s$ (or vice-versa), we say that this optimal solution contains a *right-bridge* (or *left-bridge* if the vice-versa follows).

The algorithm in [9] is based on the following result.

**Theorem 16** *All the optimal solutions for the* Min-Range(B) *problem in the line contains at most one right-bridge or left-bridge.*

Let us call *bridge* both the left-bridge and the right-bridge. Then, the optimal solutions for Min-Range(B) are bridge-free or contains one bridge. The dynamic programming algorithm is divided in two parts: First an optimal bridge free solution is computed, then a optimal solution with a single bridge is computed. Finally is chosen the solution with lower cost that is also the optimal one.

The algorithm works even if the maximum number of hops $h$ to reach all the stations is specified. Its complexity is $O(hn^2)$ where $n$ is the number of stations.

## 4.2 Broadcast in the Plane

Theorem 15 affirms that when the dimension of the space is 2 or more and when $\alpha > 1$, Min-Range(B) is NP-hard.

Regarding the positive result for the case $\alpha \geq 1$, as formerly mentioned, in [23] three heuristic are compared via simulation on random instances. The best heuristic appears to be the one based on the construction of an Euclidean *Minimum Spanning Tree* (`mst`) routed at the source node. In particular, given a set of stations $S$ in the Euclidean plane and a source node $s$, a `mst` $T$ routed at $s$ of the euclidean graph induced by $S$ and by the `dist` function is computed. Then, the range $r^{\mathtt{mst}}(u)$ of a station $u \in S$ is defined as the weight of the maximum outgoing edge from $u$ in $T$. This algorithm, denoted as MstAlg, is the same that guarantees a 2 approximation factor for the Min-Range(SC) in Section 2.2. The only difference consists the orientation of the tree starting from the root $s$. This new version of the MstAlg is sketched in Figure 6.

**begin**
  $T$ := $\mathtt{mst}(S, \mathtt{dist})$;
  Root $T$ at $s$;
  **forall** $v \in S$ **do**
    $r^{\mathtt{mst}}(v)$ := $\max_{u:(v,u)\in T}\{\mathtt{dist}(v,u)\}$;
**end**

Figure 6: The MstAlg.

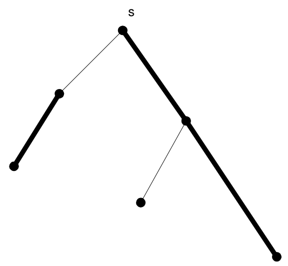


Figure 7: The `mst` of the graph in Figure 4 in which the maximum outgoing edge is emphasized.

Clearly, the cost of the MstAlg with input $S$ and $s$ is defined as:

$$\mathtt{cost}(\text{MstAlg}(S, s)) = \sum_{v \in S} r^{\mathtt{mst}}(s)^{\alpha}$$

In Figure 7 is showed the `mst` of the graph in Figure 4. Here, the maximum outgoing edges are denoted with wider lines.

Theoretical upper bounds on the approximation performances of the MstAlg are proposed for the first time in [8] and, simultaneously, in [22]. In the Euclidean plane, the analysis of the first paper proves an approximation factor of $10^{\alpha/2}2^{\alpha}$ whereas the approximation factor proved in the second paper is much more smaller: $3^{\alpha/2}2^{\alpha}$. However, the same authors of [8] improve their analysis in the technical report [7] leading the performance ratio to $5^{\alpha/2}2^{\alpha}$.

The proofs of the three previously mentioned results are obtained showing that, the cost of the minimum spanning tree built by the MstAlg is $O(\mathsf{diam}^{\alpha})$ where $\mathsf{diam}$ is the diameter of the smaller disk containing the set $S$. In particular the following result follows:

**Theorem 17 ([8, 7, 22])** *Let $T$ the minimum Euclidean spanning tree of any set of points $S$ induced by the* `dist` *function on these points. There exist a constant $c$ depending on $\alpha$ such that*

$$\mathtt{cost}(\mathtt{mst}(S)) = c \cdot \mathsf{diam}^{\alpha}.$$

In [8, 7, 22] three upper bounds for the constant $c$ are given and in particular:

**Theorem 18**

- $c \leq 10^{\alpha/2}$ *[8];*
- $c \leq 5^{\alpha/2}$ *[7];*

- $c \leq 3^{\alpha/2}$ *[22].*

In the following we will show how from Theorem 17 can be obtained an approximation result.

**The Approximation Result.** Given any set of stations $S$, $\mathcal{D}(S)$ denotes the smallest disk containing all the stations $S$ and its diameter is denoted as $\mathsf{diam}(S)$. Given the weighted complete graph $(G(S,E), \mathtt{dist}^{\alpha})$, where the weight of every edge $(u,v)$ is defined as $\mathtt{dist}(u,v)^{\alpha}$, the weight of a subgraph $G'(S,E')$ of $G$ is defined as

$$\mathtt{cost}(G') = \sum_{(u,v)\in E'} \mathtt{dist}(u,v)^{\alpha}.$$

Now, let $r^{\text{opt}}$ be an optimal range assignment for the instance $\langle S, s\rangle$ of MIN-RANGE(B) . For any $v \in S$, let

$$K(v) = \{u \in S : \mathtt{dist}(v,u) \leq r^{\text{opt}}(v)\}$$

and let $\mathtt{mst}(v)$ be a minimum spanning tree of the subgraph of $G_{r^{\text{opt}}}$ induced by $K(v)$. For any $v \in S$, let

$$c(v) = \frac{\mathtt{cost}(\mathtt{mst}(v))}{\mathsf{diam}(K(v))^{\alpha}} \quad \text{and} \quad c = \max\{c(v) \mid v \in S\}.$$

Then, it holds that

$$\begin{aligned} \mathtt{opt}(\langle S,s\rangle) = \sum_{v\in S} r^{\text{opt}}(v)^{\alpha} \geq \frac{1}{2^{\alpha}} \cdot \sum_{v\in S} \frac{\mathtt{cost}(\mathtt{mst}(v))}{c(v)} & \\ \geq \frac{1}{2^{\alpha}\cdot c} \cdot \sum_{v\in S} \mathtt{cost}(\mathtt{mst}(v)) & \quad (4) \end{aligned}$$

Since the graph $G' = (S,E')$ where

$$E' = \bigcup_{v\in S} \{e \in E : e \in \mathtt{mst}(v)\},$$

contains a minimum spanning tree for $S$, it follows that

$$\begin{aligned} \mathtt{opt}(\langle S,s\rangle) \geq \frac{1}{2^{\alpha}\cdot c} \cdot \sum_{v\in S} \mathtt{cost}(\mathtt{mst}(v)) & \\ \geq \frac{1}{2^{\alpha}\cdot c} \cdot \mathtt{cost}(\mathtt{mst}(S)) & \\ \geq \frac{1}{2^{\alpha}\cdot c} \cdot \mathtt{cost}(\text{MSTALG}(S,s)) & \quad (5) \end{aligned}$$

From the above inequality, it should be clear that any upper bound for $c$ determines a lower bound on the optimum of any instance of the MIN-RANGE(B) problem. So, the following result is proved.

**Theorem 19** *The* MIN-RANGE(B) *problem in the Euclidean plane is approximable within* $c \cdot 2^{\alpha}$ *for every* $\alpha \geq 2$.

Notice that, given any set of points $S$ on the plane, the ratio $w(\mathtt{mst}(S))/\mathsf{diam}(S)^\alpha$ can be easily computed in $O(|S|^2)$ time.

The next paragraphs of this section explain how the constant $c$ is obtained in the three cited papers. The three approaches are similar infact, they are based on computing the areas of some geometrical figures built on the edges of the minimum spanning tree. Let $e = (u, v)$ be an edge of the minimum spanning tree $T$ and $F(e)$ be the figure built on $e$. If we are able to prove that no more than $q$ of such figure can overlap then

$$\sum_{(u,v)\in T} \mathtt{area}(F(u,v)) \le q \cdot \mathtt{area}\begin{pmatrix}\text{Minimum} \\ \text{disk con-} \\ \text{taining all} \\ \text{the } F(u,v)\end{pmatrix} \tag{6}$$

In the three papers, $F(u, v)$ is chosen such that its area is equal to $\rho \cdot \mathtt{dist}(u, v)^2$ and the area of the smaller disk containing all the $F(u, v)$ is $\sigma \cdot \mathsf{diam}^2$ for some constants $\rho$ and $\sigma$. From this, Equation (6) becomes

$$\rho \sum_{(u,v)\in T} \mathtt{dist}(u,v)^2 \le q\ \sigma\ \mathsf{diam}^2.$$

Notice that, the previous equation bounds the cost of the minimum spanning tree solution for the Min-Range(B) when $\alpha = 2$. This result can be extended also in the case $\alpha > 2$ infact by simple calculations, we get

$$\sum_{(u,v)\in T} \mathtt{dist}(u,v)^\alpha = \sum_{(u,v)\in T} \left(\mathtt{dist}(u,v)^2\right)^{\alpha/2}$$
$$\le \left(\sum_{(u,v)\in T} \mathtt{dist}(u,v)^2\right)^{\alpha/2} \le \left(\frac{q\ \sigma}{\rho}\right)^{\alpha/2} \mathsf{diam}^\alpha. \tag{7}$$

We have shown the general paradigm for the construction of the constant $c$ of Theorems 17 and 18. Before starting the survey of the single constructions, note that, till the end of this section we always assume that: $T$ is a minimum Euclidean spanning tree with edges $e_1, \ldots, e_{n-1}$; $u_i$ and $v_i$ are the vertices of the edge $e_i$ and $c_i$ is its center. Moreover, the smaller disks $\mathcal{D}$ containing $T$ has diameter $\mathsf{diam}$.

**The Overlapping Diametral Disks ([8]).** The figures considered in [8] are the diametral disks $D_i$ of the edges of the minimum spanning tree $T$, that is, the open disks whose diameters are the edges of $T$. The core of their analysis is the following lemma.

**Lemma 20** *For any set of points $S$ and for any point $p$ (eventually not in $S$) in the Euclidean plane, there exist at most five diametral disks of the minimum spanning tree $T$ that contain $p$.*

The proof of the previous lemma is very easy but needs the following important property of the minimum Euclidean spanning tree.

**Lemma 21 ([8])** *For any $i, j \in [n-1]$ with $i \ne j$, $c_i$ is not contained in $D_j$.*

Proof ofLemma 20 Suppose by contradiction that there exist a point $p$ covered by (at least) six diametral circles. Then, there must exist two edge $e_i = (u_i, v_i)$ and $e_j = (u_j, v_j)$ such that their respective centers $c_i$ and $c_j$ form with $p$ an angle $\beta \le \pi/3$ (see Figure 8(a)). Since $\beta \le \pi/3$, we have that

$$\mathtt{dist}(c_i, c_j) \le \max\{\mathtt{dist}(c_i, p), \mathtt{dist}(c_j, p)\} < \max\{\mathtt{dist}(u_i, v_i), \mathtt{dist}(u_j, v_j)\}$$

where the strict inequality is due to the fact that $p \in D_i \cap D_j$ and that both $D_i$ and $D_j$ are open disks. Hence, either $c_i \in D_j$ or $c_j \in D_i$, thus contradicting Lemma 21. $\square$

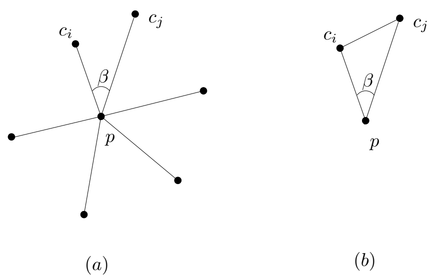


Figure 8: The proof of Lemma 20

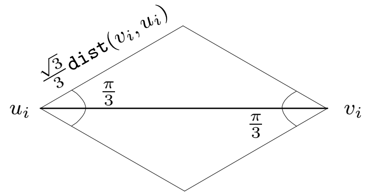


Figure 9: The diamond $\Delta(e_i)$

The value of $c$ of Theorem 18 is then obtained showing that all the diametral disks of the edges of a minimum spanning tree are contained in a disk of diameter $\sqrt{2}\mathsf{diam}$. The proof of this result is very simple and the interested reader can find it in [8].

Finally, we have shown that $q = 5$, $\rho = \pi/4$ and $\sigma = \pi/2$ then, putting this values in Equation (7) we obtain the upper bound $10^{\alpha/2}$ for $c$.

**"Half" Disks Never Overlap ([7]).** In order to improve the bound for $c$, in [7] the authors consider smaller disks respect to the previous work. They prove the following lemma.

**Lemma 22** *The disks $D'(e_i)$ whose centers are in $c_i$ and whose diameters are $\mathtt{dist}(u_i, v_i)/2$ never overlap.*

Furthermore, is simple to prove that the smaller disk containing the disks $D'(e_i)$ for all edges $e_i$ of $T$ has diameter $(\sqrt{5}/2)\mathsf{diam}$. These facts immediately yield the upper bound $5^{\alpha/2}$ for $c$. Infact, this implies that the parameters of Equation (7) are $q = 1$, $\rho = \pi/16$ and $\sigma = 5\ (\pi/16)$.

We conclude this treatment with the proof of Lemma 22.

Proof ofLemma 22 Let $c_i$ and $c_j$ be the centers of $D'_i$ and $D'_j$, respectively. Also, for $\ell \in \{i, j\}$, let $r_\ell$ the radius of $D'_\ell$. From Lemma 21 and from the definition of $D'_i$ and of $D'_j$, it follows that

$$\frac{\mathtt{dist}(c_i, c_j)}{2} \geq \frac{\max\{2r_i, 2r_j\}}{2} = \max\{r_i, r_j\}.$$

This implies that $D'_i$ does not intersect $D'_j$. □

**Diamonds are Better ([22]).** The upper bound for $c$ can be notably lowered if, instead of considering disks, we consider *diamonds*. The diamond $\Delta(e_i)$ of an edge $e_i$ of $T$ is a open rhombus with sides of length $\frac{\sqrt{3}}{3}\mathtt{dist}(v_i, u_i)$ and with major diagonal of length $\mathtt{dist}(v_i, u_i)$ (see Figure 9). Observe that the interior angles in $v_i$ and $v_j$ are equal to $\pi/3$.

As in the case of the "half disks", is it possible to prove that the diamonds never overlap. A complete proof of this result require some pages than we remand the interested reader to the original paper.

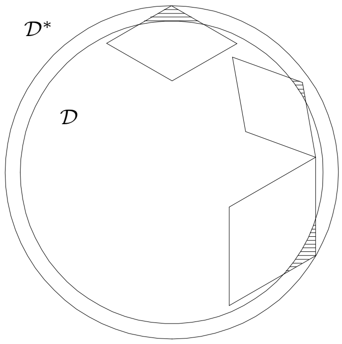


Figure 10: The areas of diamonds never cover the area of $\mathcal{D}^*$ outside $\mathcal{D}$

An interesting feature of the analysis of [22] is a more accurate computation of the total area of the diamonds outside the disk $\mathcal{D}$. They don't stop to the computation of the size of the minimum disk $\mathcal{D}^*$ containing all the diamonds. Infact, the total area of the pieces of diamonds outside $\mathcal{D}$ cannot completely cover the portion of $\mathcal{D}^*$ outside $\mathcal{D}$ (see Figure 10). The interesting result is the following.

**Lemma 23** *The total area of the pieces of diamonds outside $\mathcal{D}$ is no more than $(\frac{2\sqrt{3}-\pi}{4})\mathsf{diam}^2$.*

The bound for $c$ follows observing that the area of the diamond $\Delta(e_i)$ is $\rho = \sqrt{3}/6$ and the total area containing all the diamonds is $(\frac{2\sqrt{3}-\pi}{4})\mathsf{diam}^2 + \frac{\pi}{4}\mathsf{diam}^2$, that is, $\sigma = \sqrt{3}/2$. Putting these values in Equation (7) we obtain that $c \leq 3^{\alpha/2}$.

### 4.3 Broadcast in Higher Dimensions

In [8] is shown that the algorithm MstAlg achieves a constant (i.e. independent of the graph size) approximation ratio even on higher dimensions. That is the following theorem can be proved.

**Theorem 24** *There exists a function $f : \mathbb{N} \times \mathbb{R} \to \mathbb{R}$ such that, for any $d \geq 2$ and for any $\alpha \geq d$,* Min-Range($\mathtt{B}$) *problem in a $d$-dimensional Euclidean space is approximable within factor $f(d, \alpha)$.*

The proof uses the same geometric arguments of those used for the two-dimensional case

On the other hand, the following theorem shows that the function $f$ in the statement of the previous theorem grows exponentially with respect to $d$.

**Theorem 25** *There exists a positive constant $\gamma$ such that, for any $d$ and for any $k$, an instance $x_{k,d}$ of* Min-Range($\mathtt{B}$) *in the $d$-dimensional Euclidean space exists such that $\mathtt{opt}(x_{k,d}) = k^d$ while the cost of the range assignment computed by* MstAlg *is at least $k^d \cdot 2^{\gamma d}$.*

In order to prove the above theorem, we first describe the idea with an example. In the two-dimensional Euclidean space, we can construct a worst-case instance for MstAlg by considering the star of center $c$ and six adjacent points $c_1, \ldots, c_6$ such that $\mathtt{dist}(c, c_i) = \mathtt{dist}(c_i, c_{(i+1) \bmod 6}) = 1$, for $i = 1, \ldots, 6$. Also let $c$ be the source node. This configuration can be viewed as an arrangement of six disks of unit diameter such that they all touch (kiss) a central disk (see Figure 11).

Notice that there are (at least) two $\mathtt{msts}$ for this point set: the star with root $c$, and the path $c, c_1, c_2, \ldots, c_6$. The corresponding solutions have however quite different costs: 1 against 6. Now we modify the instance in such a way that MstAlg is forced to "pay" all the edges in the star. Indeed, for each $i$ with $1 \leq i \leq 6$, we place a point $c'_i$ on the segment joining $c$ and $c_i$ and at distance $\epsilon > 0$ from $c$.

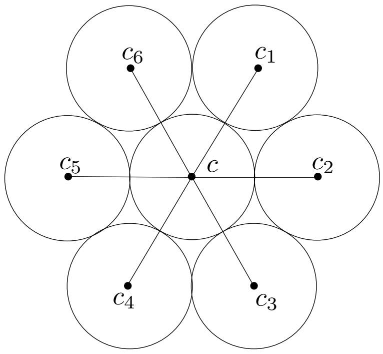


Figure 11: A worst-case instance of MstAlg on the plane and the corresponding kissing disk arrangement.

Now there is only one `mst` and the corresponding cost is $\epsilon^2 + 6(1-\epsilon)^2$. This implies that, on the plane, MstAlg cannot achieve an approximation ratio better than 6.

In higher dimensions, this idea extends to *spheres* of unit diameter. What is the maximum number $n_d$ of spheres that can simultaneously touch another sphere? The following theorem gives the answer.

**Theorem 26 ([15, 24])** *For any positive integer $d$,*

$$n_d = \Omega(2^d).$$

Moreover, it can be proven, in a similar way of the two-dimensional case, that there exists an instance of Min-Range(B) whose corresponding minimum spanning tree has cost $\epsilon + n_d(1-\epsilon)^d$, while a solution of cost 1 exists.

This proves that MstAlg has approximation ratio at least $\Omega(n_d)$.

### 4.4 Open Problems

## 5 The $h$-Broadcast

Another important range assignment problem is that in which, a source station $s$ is given, and the resulting communication graph must contain a source spanning tree of depth at most $h$. This problem is denoted as Min-Range($h$B). Since Min-Range(B) can be seen as the case of Min-Range($h$B) where $h = n-1$, the latter is NP-hard in any multi-dimensional space (see Theorem 15). However, it is still unknown whether, for some (non trivial) ranges of $h$, the problem is efficiently solvable. The only positive result for this problem concerns the linear case. Infact, as previously observed in Section 4.1, the dynamic algorithm proposed in [9] works for the $h$-broadcast as well.

It is also interesting to mention that [6] provide a dynamic programming algorithm that optimally solves the *all-to-one* version of the min-range assignment problem, i.e., the case in which $\Pi$ consists in requiring that, for each station $v$, the range assignment must guarantee the existence of a path of length at most $h$ from $v$ to a *sink* $s$ given in input. Their algorithm works in $O(hn^3)$ time. Informally speaking, this is the opposite version of the Min-Range($h$B) problem. Even though, at a first look, the problems seem to be mutually related, this is not the case: in particular, the latter problem cannot be (at least easily) reduced to the all-to-one version. Among the others, we emphasize two key-differences. *i*). In the all-to-one version, any feasible solution must assign a positive range to *every* station: this does not clearly hold for the broadcast version. *ii*). On the other hand, bridges are not useful for the all-to-one version while, as discussed above, it is a crucial issue in proving.

### 5.1 Open Problems

The real complexity of all versions of the MIN-RANGE($h$B) problem is still unknown but the linear case. Among the others, we find very interesting the following two cases:

- The 2-dimensional case in which $\alpha = 1$ and/or $h$ is set to some small constant. We believe that, in this case, the use a suitable combination of geometrical arguments and dynamic programming could yield efficient optimal algorithms.
- The multi-dimensional case in which $\alpha > 1$ and $h$ is bounded by some small constant. The intuition here is that the problem remains hard to solve and only approximating solutions can be found in polynomial time.

## References


[1] G. Ausiello, P. Crescenzi, G. Gambosi, V. Kann, A. Marchetti-Spaccamela, and M. Protasi. *Complexity and Approximation - Combinatorial Optimization Problems and their Approximability Properties.* Springer-Verlag, 1999.

[2] R. Bar-Yehuda, O. Goldreich, and A. Itai. On the Time Complexity of Broadcast Operations in Multi-Hop Radio Networks: An Exponential Gap Between Determinism and Randomization. *Journal of Computer and Systems Science*, 45:104–126, 1992.

[3] R. Bar-Yehuda, A. Israeli, and A. Itai. Multiple Communication in Multi-Hop Radio Networks. *SIAM Journal on Computing*, 22:875–887, 1993.

[4] M.A. Bassiouni and C. Fang. Dynamic Channel Allocation for Linear Macrocellular Topology. In *Proceedings of the 13th ACM Symposium on Applied Computing (SAC)*, pages 382–388, 1998.

[5] L. Blazevic, L. Buttyan, S. Capkun, S. Giordano, J.P. Hubaux, and J.Y. Le Boudec. Self-Organization in Mobile Ad-Hoc Networks: the Approach of Terminodes. *IEEE Communications Magazine*, 2001.

[6] A. Clementi, A. Ferreira, P. Penna, S. Perennes, and R. Silvestri. The minimum range assignment problem on linear radio networks. In *Proc. ESA'00, 8th Annual European Symposium on Algorithms*, volume 1879 of *Lecture Notes in Computer Science*, pages 143–154. Springer-Verlag, 2000.

[7] A.E.F. Clementi, P. Crescenzi, P. Penna, G. Rossi, and P. Vocca. A Worst-case Analysis of an MST-based Heuristic to Construct Energy-Efficient Broadcast Trees in Wireless Networks. Technical Report 010, University of Rome "Tor Vergata", Math Department, 2001. Available at http://www.mat.uniroma2.it/∼penna/papers/stacs01-TR.ps.gz (Extended version of [8]).

[8] A.E.F. Clementi, P. Crescenzi, P. Penna, G. Rossi, and P. Vocca. On the Complexity of Computing Minimum Energy Consumption Broadcast Subgraphs. In *Proceedings of the 18th Annual Symposium on Theoretical Aspects of Computer Science (STACS)*, volume 2010 of *LNCS*, pages 121–131. Springer-Verlag, 2001.

[9] A.E.F. Clementi, M. Di Ianni, and R. Silvestri. The Minimum Broadcast Range Assignment Problem on Linear Multi-Hop Wireless Networks. *Theoretical Computer Science*, to appear.

[10] A.E.F. Clementi, P. Penna, and R. Silvestri. On the Power Assignment Problem in Radio Networks. Technical Report TR00-054, Electronic Colloquium on Computational Complexity (ECCC), 2000.

[11] A.E.F. Clementi, P. Penna, and R. Silvestri. The Power Range Assignment Problem in Radio Networks on the Plane. In *Proceedings of the 17th Annual Symposium on Theoretical Aspects of Computer Science (STACS)*, pages 651–660, 2000. Full version in [10].

[12] IEEE Computer Society LAN MAN Standards Committee. *IEEE Standard for Wireless LAN Medium Access Control and Physical Layer specifications*, 1997. Project IEEE 802.11.

[13] K. Diks, E. Kranakis, D. Krizanc, and A. Pelc. The Impact of Knowledge on Broadcasting Time in Radio Networks. In *Proceedings of the 7th Annual European Symposium on Algorithms (ESA)*, volume 1643 of *Lecture Notes in Computer Science*, pages 41–52. Springer-Verlag, 1999.

[14] M.R. Garey and D.S. Johnson. *Computers and Intractability - A Guide to the Theory of NP-Completness*. Freeman and Co., New York, 1979.

[15] G.A. Kabatiansky and V.I. Levenshtein. Bounds for Packings on a Sphere and in Space (in Russian). *Problemy Peredachi Informatsii*, 14(1):3–25, 1978. English translation: *Problems of Information Theory*, 14(1):1–17, 1978.

[16] L. M. Kirousis, E. Kranakis, D. Krizanc, and A. Pelc. Power Consumption in Packet Radio Networks. In *Proceedings of the 14th Annual Symposium on Theoretical Aspects of Computer Science (STACS)*, volume 1200 of *Lecture Notes in Computer Science*, pages 363 – 374. Springer-Verlag, 1997. Full version in [17].

[17] L. M. Kirousis, E. Kranakis, D. Krizanc, and A. Pelc. Power Consumption in Packet Radio Networks. *Theoretical Computer Science*, 243:289–305, 2000.

[18] E. Kranakis, D. Krizanc, and A. Pelc. Fault-Tolerant Broadcasting in Radio Networks. *Proceedings of the 6th Annual European Symposium on Algorithms (ESA)*, Lecture Notes in Computer Science(1461):283–294, 1998.

[19] G.S. Lauer. *Packet radio routing*, chapter 11 of *Routing in communication networks*, M. Streenstrup (ed.), pages 351–396. Prentice-Hall, 1995.

[20] R. Mathar and J. Mattfeldt. Optimal Transmission Ranges for Mobile Communication in Linear Multihop packet Radio Netwoks. *Wireless Networks*, 2:329–342, 1996.

[21] K. Pahlavan and A. Levesque. *Wireless information networks*. Wiley-Interscience, 1995.

[22] P.-J. Wan, G. Călinescu, X.-Y. Li, and O. Frieder. Minimum-Energy Broadcast Routing in Static Ad Hoc Wireless Networks. In *Proceedings of the 20th Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM)*, pages 1162–1171, 2001.

[23] J. E. Wieselthier, G. D. Nguyen, and A. Ephremides. On the Construction of Energy-Efficient Broadcast and Multicast Trees in Wireless Networks. In *Proceedings of the 19th Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM)*, pages 585–594, 2000.

[24] A.D. Wyner. Capabilities of Bounded Discrepancy Decoding. *Bell System Technical Journal*, 44:1061–1122, 1965.