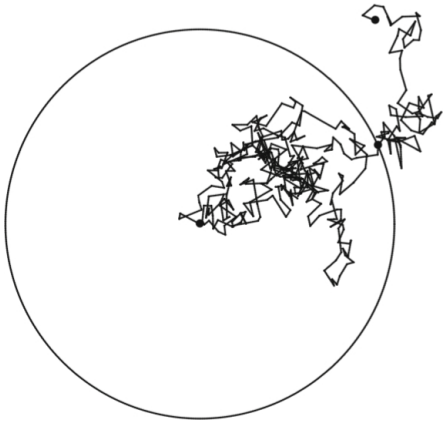
Recent Advances in Random Processes

A CONFERENCE IN HONOR OF
PAOLO BALDI's 70TH BIRTHDAY

10-11 September 2018, Rome

Program
Titles & Abstracts



Program

September 10, Monday

09:15-09:45 - *Registration*
09:45-10:30 - **Maurizio Pratelli**
10:30-11:15 - **Marta Sanz-Solè**
11:15-11:45 - *Coffee break*
11:45-12:30 - **Eugenio Regazzini**
12:30-14:30 - *Lunch break*
14:30-15:15 - **Franco Flandoli**
15:15-16:00 - **Annie Millet**
16:00-16:30 - *Coffee break*
16:30-17:15 - **Vlad Bally**
17:15-18:00 - *Memories break*
20:00- - *Social dinner*

September 11, Tuesday

09:45-10:30 - **Wolfgang J. Runggaldier**
10:30-11:15 - **Mauro Piccioni**
11:15-11:45 - *Coffee break*
11:45-12:30 - **Dario Trevisan**
12:30-14:30 - *Lunch break*
14:30-15:15 - **Maurizia Rossi**
15:15-16:00 - **Domenico Marinucci**
16:00-17:00 - *Coffee break*

Titles & Abstracts

Malliavin Calculus and Invariance Principles

Vlad Bally
Université Paris-Est Marne-la-Vallèe

We consider a sequence of independent random variables $X_n, n \in \mathbb{N}$ with law locally lower bounded by the Lebesgue measure, that is $\mathbb{P}(X_n \in A) \geq \varepsilon\lambda(A)$ for every measurable set A included in a bounded domain D, λ denoting the Lebesgue measure. Using Nummelin's splitting method one may write such a random variable as $X_n \overset{\text{law}}{=} \xi_n Y_n + (1-\xi_n)Z_n$ with ξ_n a Bernoulli random variable, $Y_n \sim \phi_n(x)dx, \phi_n \in C^\infty(\mathbb{R}^d)$ and ξ_n, Y_n, Z_n independent. We establish a differential calculus (inspired from Malliavin calculus) based on $\phi_n, n \in \mathbb{N}$ which produces integration by parts formulas for functionals of $X_n, n \in \mathbb{N}$.
This abstract Malliavin calculus is then used in order to obtain convergence in distribution norms in the CLT associated to $X_n, n \in \mathbb{N}$. Moreover, as an application of this result and of the Kac-Rice formula, we give invariance principles for the mean and the variance of the number of zeros of trigonometric polynomials.

References

[1] Bally, V., Caramellino, L., Poly, G. Convergence in distribution norms in the CLT for non identical distributed random variables. *Electronic Journal of Probability*, to appear.
[2] Bally, V., Caramellino, L., Poly, G. On the non universality for the variance of the number of real roots of random trigonometric polynomials. *Preprint,* 2017, arXiv:1711.0336v1.

The Paradigm of Probabilistic Selection in the Case of Non-Uniqueness

Franco Flandoli
Scuola Normale Superiore, Pisa

Roberto Bafico and Paolo Baldi, around 1981, understood the limit in law of solutions to stochastic differential equations, when the noise goes to zero, in the case when the limit ordinary differential equation presents a Peano phenomenon. This is probably the most remarkable result of probabilistic selection in the case of non-uniqueness of an underlying deterministic problem. In the talk some ideas of that original work will be recalled along with

more recent contributions and open problems and other examples of probabilistic selection will also be mentioned; with remarks also on the related problem of intrinsic randomness, namely the construction of probabilistic solutions to deterministic dynamics.

The Geometry of Random Eigenfunctions

Domenico Marinucci
Università di Roma "Tor Vergata"

We shall review some recent results concerning the asymptotic behaviour (in the high-frequency regime) of geometric functionals on the excursion sets of random spherical eigenfunctions. We shall show in particular how the Lipschitz-Killing curvatures (equivalently, the Minkowski functionals) evaluated on these excursion sets are dominated by a single component, corresponding to their projection on the so-called Wiener chaos of order 2. This component disappears for the excursion set corresponding to a zero threshold, where the asymptotic behaviour is hence different (the so-called Berry cancellation phenomenon). A similar behaviour can also be established for random eigenfunctions on the torus (arithmetic random waves). The talk is based on some joint works with Valentina Cammarota, Giovanni Peccati, Maurizia Rossi and Igor Wigman.

On Stochastic Brinkman-Forchheimer Anisotropic 3D Navier-Stokes Equations

Annie Millet
Université Paris 1 Panthéon Sorbonne

Navier-Stokes equations in the whole 3D space subject to an anisotropic viscosity and a random perturbation of multiplicative type are described. By adding a term of Brinkman-Forchheimer type to the model, existence and uniqueness of global weak solutions in the PDE sense are proved. These are strong solutions in the probability sense. The convective term given in terms of the Brinkman-Forchheirmer provides some extra regularity. This implies that the nonlinear term has better properties which allow global well posedeness. The proof of existence is performed through a control method. A Large Deviations Principle is also obtained using the "weak convergence approach".
Joint work with H. Bessaih.

Vingt Ans Après: the Level 2.5 Large Deviation Principle

Mauro Piccioni
Università di Roma "La Sapienza"

Almost twenty years ago Paolo and I published a small paper [1] on the level 2 LDP governing the exponential convergence of the empirical measure to stationary one, for continuous time finite state Markov chains. Following a note in [2] on the binary case, we established that the rate function of the above LDP, as derived in the pioneering paper [3], could be obtained by "contracting" a functional defined on pairs of states, formally corresponding to an intermediate LDP between level 3 (the empirical process) and level 2. Quite recently, this formal LDP principle has been rigorously fixed in [4,5] in the more general context of chains with countable states. We will review some of the features of the solution to the problem, providing an illustration of some new techniques available in this area of research.

References

[1] Baldi, P. and Piccioni, M. A representation formula for the large deviation rate function for the empirical law of a continuous time Markov chain. *Statist. Probab. Lett.* 41(2), 1999, 107–115.
[2] Kesidis, G. and Walrand, J. Relative entropy between Markov transition rate matrices. *IEEE Trans. Inform. Theory* 39(3), 1993, 1056–1057.
[3] Donsker, M. D. and Varadhan, S. R. S. Asymptotic evaluation of certain Markov process expectations for large time. I *Comm. Pure Appl. Math.* 28(1), 1975, 1–47.
[4] Bertini, L., Faggionato, A., and Gabrielli, D. From level 2.5 to level 2 large deviations for continuous time Markov chains. *Markov Process. Related Fields* 20(3), 2014, 545–562.
[5] Bertini, L., Faggionato, A. and Gabrielli, D. Large deviations of the empirical flow for continuous time Markov chains. *Ann. Inst. H. Poincaré* Probab. Stat. 51(3), 2015, 867–900.

BV functions on the Wiener Space and some analogies with Geometric Measure Theory.

Maurizio Pratelli
Università di Pisa

Classical Bounded Variation functions are functions whose (distributional) derivative is a measure, and BV functions on the Wiener space are in some sense functions whose 'Malliavin derivative' is a measure. The talk will concentrate on some applications of BV functions on the Wiener space (e.g. an extension of the Clark-Ocone formula) and on analogies with some concepts of Geometric Measure Theory. (Based on joint work with Dario Trevisan).

References
[**1**] Pratelli M., Trevisan D.: Functions of Bounded Variation on the classical Wiener space and an extended Ocone-Karatzas formula. *Stochastic Proc. and Appl.*, vol. 122 (6) **2012**.
[**2**] Trevisan D.: BV regularity for the Maximum of the Wiener Process. *Electronic Communications in Probability*, vol. 18 (29) **2013**.

Probabilistic and Statistical Issues in the Interplay between Frequentism and Bayesanism

Eugenio Regazzini
Università di Pavia

The connection between frequentism and Bayesanism shows itself, for example, in implementation of empirical Bayes methods (eBm's, for short), whose most common versions are characterized by frequentist estimation of initial distributions. On the contrary, the eBm discussed in this talk assumes the estimation of a predictive distribution, according to a procedure devised, and applied to a particular case, by Corrado Gini (1911). The talk especially focuses on the proposal of adequate estimators of a predictive distribution, and the study in "Wasserstein spaces" of their errors in approximating.

Spin Random Fields on the Sphere

Maurizia Rossi
Université Paris 5 Paris Descartes

In this talk we focus on random fields on algebraic structures, in particular spherical spin random fields, i.e. random sections of the spin line bundles of the two-dimensional sphere. The latter are currently used e.g. in Cosmology to model Cosmic Microwave Background data. We refer to a joint work with Paolo Baldi, and present a technique for the construction of these random fields (which is a representation formula for complex Gaussian isotropic spin random fields) that extends P. Lévy's original idea for his spherical Brownian Motion, and on the way find a new approach for the study of random sections of homogeneous vector bundles. If time permits, we present some other recent results on the geometry of spin random fields.

Monte Carlo Variance Reduction by Conditioning in the Pricing of Complex Derivatives with Underlying a Continuous-time Markov Chain

Wolfgang J. Runggaldier
Università di Padova

We consider pricing of complex/path dependent derivatives when the underlying evolves as a continuous time Markov chain. We present a semi-analytic approach: i) simulate the number of transitions of the underlying up to a given horizon; ii) compute via an explicit analytic formula the price for each simulated sequence of transitions; iii) approximate the actual price by an empirical average over the values computed in ii). With respect to a full Monte Carlo, this can be considered as a Monte Carlo approach with variance reduction by conditioning thus leading to more precise values.
Joint work with J.M. Montes and V. Prezioso.

Polarity of Sets for Random Field Solutions of SPDEs

Marta Sanz-Solè
Universidad de Barcelona

In probability theory, the notion of polarity is about sojourn of stochastic processes in deterministic sets. In this talk, we will study this notion for processes which are solutions of SPDEs. For linear equations, the results are derived from the analysis of this question in the general setting of anisotropic Gaussian processes. For nonlinear equations, existence and properties of univariate and bivariate densities play a crucial role. This suggests a study based on Malliavin calculus. We will give a survey of methods and discuss results proved in the last ten years.

On Empirical L^p-Medians of I.I.D. Random Variables

Dario Trevisan
Università di Pisa

In this talk we present some results on the asymptotic behavior of empirical L^p-medians of multidimensional i.i.d. random variables, generalizing the case of sample means, i.e., $p = 2$, to the non linear case of $p > 1$, with some results valid also for Euclidean L^1-medians (also known as spatial medians). Under natural assumptions on the common distribution we prove strong laws of large numbers, large deviation principles and central limit theorems.
Joint work with R. Giuliano and C. Macci.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