REGULARITY RESULTS FOR ANISOTROPIC ELLIPTIC EQUATIONS

GIUSEPPE RIEY

For $n \geq 2$, let $\Omega \subset \mathbb{R}^n$ be a smooth bounded domain. We consider the functional $I(u) = \int_\Omega [B(H(\nabla u)) - F(u)]\, dx$, whose Euler-Lagrange equation is given by

$$-\operatorname{div}\left(B'(H(\nabla u))\nabla H(\nabla u)\right) = f(u), \tag{1}$$

where H is a Finsler norm, f is a positive continuous function on $[0,\infty)$, locally Lipschitz continuous on $(0,\infty)$ and B satisfies

(i) $B \in C^{3,\beta}_{loc}((0,+\infty)) \cap C^1([0,+\infty))$, with $\beta \in (0,1)$

(ii) $B(0) = B'(0) = 0, \quad B(t), B'(t), B''(t) > 0 \;\forall t \in (0,+\infty)$

(iii) $\exists\, p > 1, k \in [0,1], \gamma > 0, \Gamma > 0$:
$\gamma(k+t)^{p-2}t \leq B'(t) \leq \Gamma(k+t)^{p-2}t\,, \quad \gamma(k+t)^{p-2} \leq B''(t) \leq \Gamma(k+t)^{p-2}\,.$

Taking $H(\xi) = |\xi|$ and $B(t) = \frac{t^p}{p}$, the operator at left-hand side of (1) becomes the usual p-Laplace operator. If $H(\xi) = |\xi|$ and $B(t) = \sqrt{1+t^2}$, $I(u)$ is the euclidean area functional.
We first prove local regularity estimates for positive weak solutions of (1): a weighted integral hessian estimate and the integrability of the inverse of the gradient.
Moreover, adding a suitable hypothesis on f, we also prove a Hopf type Lemma and, thanks to this result, the local regularity estimates are then extended to the whole Ω.