

# MONOTONICITY AND SYMMETRY OF SOLUTIONS TO A NON-COOPERATIVE SYSTEM OF GROSS-PITAEVSKII-TYPE

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ABSTRACT. The talk is devoted to the study of the qualitative properties of solutions to the elliptic system

$$(P) \quad \begin{cases} -\Delta u = u - u^3 - \Lambda uv^2 & \text{in } \mathbb{R}^N \\ -\Delta v = v - v^3 - \Lambda u^2 v & \text{in } \mathbb{R}^N \\ u, v \geq 0 & \text{in } \mathbb{R}^N, \end{cases}$$

where  $N \geq 1$  and  $\Lambda > 1$ .

This problem arises in the study of two-components Bose-Einstein condensates in the segregation regime (= coupling parameter  $\Lambda > 1$ ).

In particular, we show a (sharp) universal  $L^\infty$  estimate for any solution of (P), as well as the monotonicity and the one-dimensional symmetry of any solution of (P) satisfying the asymptotic conditions

$$(h_\infty) \quad \begin{array}{llll} u(x', x_N) \rightarrow 1 & v(x', x_N) \rightarrow 0 & \text{as } x_N \rightarrow +\infty, \\ u(x', x_N) \rightarrow 0 & v(x', x_N) \rightarrow 1 & \text{as } x_N \rightarrow -\infty. \end{array}$$

The talk is based on a joint work with Berardino Sciunzi and Nicola Soave.