WORKSHOP ON

"THE INTERPLAY OF REPRESENTATION THEORY, POISSON GEOMETRY AND QUANTIZATION"

ROMA, 28-29/04/2004

Università di Roma "Tor Vergata", Dipartimento di Matematica - Room 1101
Via della ricerca scientifica 1, I-00133 Roma - ITALY

Organizing & Scientific Committee: *G. Carnovale, N. Ciccoli, F. Gavarini*

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WEDNESDAY 28-04-2004

[1] h. 9:30 **Alessandro D'ANDREA** (Università di Roma "La Sapienza")
"An introduction to Lie algebra representations in characteristic p"

h. 11:00 = Coffee Break =

[2] h. 11:30 **Alberto S. CATTANEO** (Universitat Zurich)
"Deformation quantization of coisotropic submanifolds (1)"

h. 13:00 α Lunch Time α

[3] h. 14:30 **Davide FATTORI** (Università di Padova)
"An introduction to conformal (super)algebras"

h. 16:00 = Coffee Break =

[4] h. 16:30 **Andrea MAFFEI** (Università di Roma "La Sapienza")
"Modular representations of Lie algebras and geometry of Springer fibres"

h. 21:00 ααα Social Dinner ααα

THURSDAY 29-04-2004

[5] h. 9:00 **Alberto S. CATTANEO** (Universitat Zurich)
"Deformation quantization of coisotropic submanifolds (2)"

h. 10:30 = Coffee Break =

[6] h. 11:00 **Alessandro D'ANDREA** (Università di Roma "La Sapienza")
"Conformal algebras and pseudoalgebras"

h. 12:30 α Lunch Time α

[7] h. 14:00 **Gilles HALBOUT** (Université de Strasbourg)
"Tamarkin's formality and globalization"

h. 15:30 = Coffee Break =

[8] h. 16:00 **Paolo LORENZONI** (Università di Milano "Bicocca")
"Deformations of bihamiltonian structures of hydrodynamic type"

h. 17:30 **o d t-u** Free Discussion and Departure **w-x ^ m**

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ABSTRACTS & REFERENCES

WARNING: there are logical dependence relations among the talks, summarized by the following scheme (where [A] ===> [B] means: "talk [B] logically follows talk [A]"):

[1] ===> [4] , [2] ===> [5] ===> [7] , [3] ===> [6] ===> [8]

[1]: **A. D'ANDREA**, "An introduction to Lie algebra representations in characteristic p"

ABSTRACT: The aim of this talk is to provide an introduction to the subsequent one by Andrea Maffei. I shall describe some introductory results of the representation theory in characteristic p, stressing the main differences with the case of characteristic 0.

REFERENCES:

J. C. Jantzen, *"Representations of Lie algebras in prime characteristic"*, Notes by Iain Gordon. NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., **514**, "Representation theories and algebraic geometry" (Montreal, PQ, 1997), 185-235, Kluwer Acad. Publ., Dordrecht, 1998;

N. Jacobson, *"Lie algebras"*, Interscience Tracts in Pure and Applied Mathematics, New York, 1996, Interscience/Wiley;

J. C. Jantzen, *"Representations of Algebraic Groups"*, Pure and Applied
Mathematics, Orlando, 1987, Academic Press.

[4]: **A. MAFFEI**, "Modular representations of Lie algebras and geometry of Springer fibres"

ABSTRACT: The goal of this lecture is to provide an introduction to the ideas in the paper by Bezrukavnikov, Mirkovic and Rumynin about representations of Lie algebras of semisimple Lie groups in characteristic p.

Their methods work splits into two subsequent steps. The first one is an analogue, in characteristic p, of the theorem of Beilinson and Bernstein on the localization of g-modules, and "reduces" the study of the representation theory of Lie algebras (with Harish-Chandra character 0 and Frobenius character \chi) to the study of the D-modules on the flag variety supported onto a formal neighbourhood of the Springer fibre defined by \chi.

The second step instead reduces the study of D-modules to that of coherent sheaves on the Springer fibre.

As an application of these methods one proves Lusztig's conjecture on the number of irreducible representations with fixed character, and one gives a new proof of Kac-Weisfeiler's conjecture on the dimension of irreducible representations.

REFERENCES:

R. Bezrukavnikov, I. Mirkovic, D. Rumynin,*"Localization of modules for a semisimple Lie algebra in prime characteristic"*, arXiv math.RT/0205144;

A. Beilinson, J. Bernstein, *"Localisation de g-modules"*, C. R. Acad. Sci. Paris Series I Math. **292** (1981), no. 1, 15-18.

[2]+[5]: A. S. CATTANEO, "Deformation quantization of coisotropic submanifolds (1)+(2)"

ABSTRACT: Coisotropic submanifolds play a fundamental role in symplectic and Poisson geometry as they describe systems with symmetries ("first-class constraints") and provide a method to generate new symplectic or Poisson spaces ("symplectic or Poisson reduction").

Though often singular, these reduced spaces possess anyway a Poisson algebra of functions that is defined as a quotient from the Poisson algebra of the original Poisson manifold. Then an interesting question is whether it is possible to quantize these Poisson algebras (i.e., to deform the underlying commutative algebras in the direction of the Poisson bracket) even when the spaces are singular.

In the first talk, I will review some basic concepts in Poisson geometry, coisotropic submanifold, Poisson reduction, deformation quantization and its relation with the Poisson sigma model (a topological field theory).

In the second talk, I will first discuss how coisotropic submanifolds are related to the possible boundary conditions for the Poisson sigma model and how to use it to get a generalization of Kontsevich's formula in the presence of coisotropic submanifolds. Then I will present the relevant formality theorem and from it I will derive the possible obstructions to the deformation quantization of a reduced Poisson manifold. An interpretation in terms of supermanifolds and the relation with the Batalin-Fradkin formalism may also be given.

Time permitting, I will discuss how to associate to two intersecting coisotropic submanifolds a bimodule for the deformed algebras of the two reduced spaces. This suggests that deformation quantization should be regarded as a (partially defined?) functor.

REFERENCES:

A. Cattaneo, G. Felder*"A path integral approach to the Kontsevich quantization formula"*, Commun. Math. Phys. **212** (2000) , 591-611;

A. Cattaneo, G. Felder,*"Coisotropic submanifolds in Poisson geometry and branes in the Poisson sigma model"*, arXiv math.QA/0309180.

For more information on coisotropic submanifolds:

A. Weinstein, *"Coisotropic calculus and Poisson groupoids"*, J. Math. Soc. Japan **40** (1988), 705-727.

For more information on deformation quantization: any review on it, e.g.:

A. Cattaneo, D. Indelicato, *"Formality and Star Products"*, arXiv math.QA/0403135; *(and references therein)*
M. Kontsevich, *"Deformation quantization of Poisson manifolds, I"*, arXiv q-alg/9709040.

[7]: **G. HALBOUT**, "Tamarkin's formality and globalization"

ABSTRACT: I will give the general framework of D. Tamarkin's formality theorem. To do so, I will recall the definition of G_\infty structures, recall Etingof-Kazhdan's quantization-dequantization theorem for Lie bialgebras (and introduce recent works of Enriquez and Enriquez-Etingof), and I will speak about globalization procedure. I will end with a few questions, recent works and works in progress related to that topic:
- globalization of Tamarkin's (and Tamarkin-Tsygan's) formality theorem,
- comparison of the construction of M. Kontsevich and D. Tamarkin,
- application to the problem of star-representations on coisotropic submanifolds,
- quantization of quasi-Poisson manifolds.

REFERENCES:

D. Tamarkin, *"Another proof of M. Kontsevich's formality theorem"*, arXiv math.QA/9803025;
V. Hinich, *"Tamarkin's proof of Kontsevich's formality theorem"*, arXiv math.QA/9803025;
G. Ginot, G. Halbout, *"A formality theorem for Poisson manifold"*, Preprint Institut de Recherche Mathematique Avancee n° **42** (2001), Strasbourg;
M. Bordemann, G. Ginot, G. Halbout, H.-C. Herbig, S. Waldmann, *"Star-Representations sur des sous-varietés co-isotropes"*, arXiv math.QA/0309321.

[3]: **D. FATTORI**, "An introduction to conformal (super)algebras"

ABSTRACT: In this talk we will provide an overview based on examples of the theory of finite Lie conformal (super)algebras. Formal distributions will be introduced as a motivation. The notion of locality will then lead to the notion of a formal distribution Lie (super)algebra. Next, we will define Lie conformal (super)algebras and illustrate some of their features on examples. Also, we will discuss central extensions, physical Virasoro pairs and their connection to superconformal algebras.

REFERENCES:

V. G. Kac, *"Vertex algebras for beginners"*. Second edition. University Lecture Series **10**; American Mathematical Society, Providence, RI, 1998;

A. D'Andrea, V. G. Kac, *"Structure theory of finite conformal algebras"*, Selecta Math. (N.S.) **4** (1998), no. 3, 377-418;

D. Fattori, V. G. Kac, "Classification of finite simple Lie conformal superalgebras", Special issue in celebration of Claudio Procesi's 60th birthday. J. Algebra **258** (2002), no. 1, 23-59;

V. G Kac, *"Classification of supersymmetries"*, arXiv math-ph/0302016.

[6]: A. D'ANDREA, "Conformal algebras and pseudoalgebras"

ABSTRACT: I will provide a well-motivated introduction to the study of conformal algebras and of pseudoalgebras. In particular, I will describe their relationship with vertex algebras, linearly compact Lie algebras, representation theory, Poisson algebras of hydrodynamical type.

REFERENCES:

A. D'Andrea, V. G. Kac, *"Structure theory of finite conformal algebras"*, Selecta Math. (N.S.) **4** (1998), no. 3, 377-418;

B. Bakalov, A. D'Andrea, V. G. Kac, *"Theory of finite pseudoalgebras"*, Advances in Mathematics **162** (2001), no. 1, 1-140.

[8]: P. LORENZONI, "Deformations of bihamiltonian structures of hydrodynamic type"

ABSTRACT: We discuss a perturbative approach to the classification problem of a certain class of bihamiltonian hierachies of PDEs depending on a small parameter and its applications to the theory of dispersive waves. The r.h.s of the equations of these hierarchies usually are formal series in the dispersion parameter. Truncating these series one obtains an "approximately integrable systems" since the vector fields of the truncated hierarchy commute up to a certain order in the deformation parameter. This fact suggest that these approximately integrable systems could have, at least for small times, multi-soliton solutions. The numerical experiments we have performed confirm this hypothesis.

REFERENCES:

P. Lorenzoni, *"Deformation of bihamiltonian structures of hydrodynamic type"*, J. Geom. Phys. **44** (2002) 331-375.

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