OCTOGON MATHEMATICAL MAGAZINE
Vol. 26 No.1, April 2018, pp 296-310
Print: ISSN 1222-5657, Online: ISSN 2248-1893
http://www.uni-miskolc.hu/~matsefi/Octogon/

József Wildt International Mathematical Competition

The Edition XXVIIIth , 2018 [27]

The solution of the problems W.1 - W.60 must be mailed before 26. October 2018, to Mihály Bencze, str. Hărmanului 6, 505600 Săcele - Négyfalu, Jud. Braşov, Romania, E-mail: benczemihaly@gmail.com; benczemihaly@yahoo.com

W1. We consider a prime number $n \geq 3$, two matrices $A, B \in M_{n-1}(Q)$, $AB = BA$, and

$$\varepsilon = \cos \frac{2(n-1)\pi}{n} + i \sin \frac{2(n-1)\pi}{n}$$

such that

a). $\det(A - \varepsilon B) = 0$
b).

$$\sum_{k=0}^{n-1} \varepsilon^{n-ki} \det(1 - \varepsilon^{-k}) \det(\varepsilon^k B - A) \geq 0$$

for all $i \in \{0, 1, ..., n-2\}$
c).

$$\det B > 0$$

Show that whatever $m \geq 1$ is the natural number and whatever the rational numbers $b_0, b_1, ..., b_m$, white $\sum\limits_{k=0}^{m} b_k A^k B^{m-k} \neq O_n$ then

[27]Received 15.03.2018
2000 *Mathematics Subject Classification.* 11-06.
Key words and phrases. Contest.

$$\det\left(\sum_{k=0}^{m} b_k A^k B^{m-k}\right) \neq 0$$

Florin Stănescu

W2. Determine the biggest real number α, with the propoerty that for any function $f : [0,1] \to [0,\infty)$ which meets the requirements:
i). f it's convex and $f(0) = 0$
ii). exist $\varepsilon \in (0,1)$ such that f it's differentiable on $[0,\varepsilon)$ and $f'(0) \neq 0$;
inequality holds

$$\int_0^1 \left[\frac{x^2}{\int_0^x f(t)\,dt}\right] dx \geq \int_0^1 \left[(x+\alpha)\cdot\frac{f(x)}{\left(\int_0^1 f(t)\,dt\right)^2}\right] dx$$

Florin Stănescu

W3. Consider the complex numbers a, b, c, d, white module one, which have the following properties:
a).

$$\arg a < \arg b < \arg c < \arg d$$

b).

$$2\sqrt{\left|(b+ai)(c+bi)(d+i)(a+di)\right|} - \sqrt{\frac{\left[(a-c)(b-d)\right]^2}{abcd}} = 4$$

Show that:

$$\max\left\{\left|1-\frac{i+4a^2}{i+8a(b-d)}\right|;\left|\frac{i+4b^2}{i+4b(a-c)}\right|\right\} \geq \frac{4}{\sqrt{17}}$$

Note. If $a = r(\cos t + i\sin t)$, $t \in [0, 2\pi)$, then $t = \arg z$.

Florin Stănescu

W4. Let $(F_n)_{n\geq 0}$ and $(L_n)_{n\geq 0}$ be the Fibonacci and the Lucas sequence, respectively. Compute the following limits:
a).

$$\lim_{n\to\infty}\left(\sqrt[2n+2]{(2n+1)!!F_{n+1}}-\sqrt[2n]{(2n-1)!!F_n}\right)\sqrt{n}$$

b).

$$\lim_{n\to\infty}\left(\sqrt[2n+2]{(2n+1)!!L_{n+1}}-\sqrt[2n]{(2n-1)!!L_n}\right)\sqrt{n}$$

D.M. Bătineţu-Giurgiu and Neculai Stanciu

W5. Show that in any triangle ABC (with usual notations) holds the following inequalites:
a).

$$\left(s^2+r^2+4Rr\right)^2\geq 8r\left(4R+r\right)\left(s^2-r^2-4Rr\right)$$

b).

$$\left(s^2+r^2-8Rr\right)^2\geq 8\left(8R^2+r^2-s^2\right)\left(s^2-4R\left(R+r\right)\right)$$

D.M. Bătineţu-Giurgiu and Neculai Stanciu

W6. For $n\in N^*-\{1\}, x_k\in R_+^*$, $(\forall)\, k=\overline{1,n}$, $X_n\left(t\right)=\sum\limits_{k=1}^{n}x_k^t$, $t\in R$, $X_n\left(1\right)=\sum\limits_{k=1}^{n}x_k=X_n$, $m\in[1,\infty)$ show that

$$\sum_{k=1}^{n}x_k\left(X_n\left(-m\right)-x_k^{-m}\right)\geq\frac{\left(n-1\right)n^m}{X_n^{m-1}}$$

D.M. Bătineţu-Giurgiu and Neculai Stanciu

W7. Find the sum

$$\sum_{n=0}^{+\infty}\frac{n!}{1\times 3\times ...\times(2n+1)}$$

Moubinool Omarjee

W8. Consider the real sequence $a_0 = 0, a_1 = 1, a_{2n} = a_n, a_{2n+1} = a_n + a_{n+1}$.

Prove that $\forall n \in N$, $2^{a_n} \le \varphi^n$ where $\varphi = \frac{1+\sqrt{5}}{2}$.

Moubinool Omarjee

W9. Find an example of field K, an integer $d \ge 1$, a infinite subgroup G of $GL_d\,(K)$ such that there exist $N \ge 1$, that verify $\forall g \in G$, $g^N = I_d$.

Moubinool Omarjee

W10. Let $a > 0$ be a real number. Find the value of the sum

$$\sum_{n\ge 1} \frac{n^3 a^n}{(n-1)!}$$

(Here, $0! = 1! = 1$).

José Luis Díaz-Barrero

W11. Let $a \ge 1$ be an integer. Calculate

$$\lim_{n\to\infty} \left(\sqrt[n+1]{\Gamma\left(\frac{(n+1)a+1}{a}\right)} - \sqrt[n]{\Gamma\left(\frac{na+1}{a}\right)} \right),$$

where Γ is the Euler gamma function.

José Luis Díaz-Barrero

W12. Let $A_1, A_2, \cdots, A_n \in M_2(\mathbb{C}), (n \ge 2)$ be the solutions of the equation

$$X^n = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}.$$

Prove that $\displaystyle\sum_{k=1}^{n} Tr(A_k) = 0$.

José Luis Díaz-Barrero

W13. Find in closed form the value of

$$\sum_{n=3}^{\infty}\frac{(-1)^n}{2n-3}\sum_{k=n+1}^{\infty}\frac{(-1)^k}{2k-3}+\sum_{n=0}^{\infty}\sum_{k=1}^{\infty}\frac{(-1)^n}{(2k-1)(2n+2k+1)}$$

Paolo Perfetti

W14. Let $a,b,c\geq 0$ and $a+b+c=1$. Prove that

$$\sqrt[3]{4+17a^2b}+\sqrt[3]{4+17b^2c}+\sqrt[3]{4+17c^2a}+10\left(\frac{1}{27}-abc\right)\geq 5$$

Paolo Perfetti

W15. Evaluate

$$\sum_{k=0}^{\infty}(-1)^k\frac{\Gamma(\frac{k}{2}+1)}{\Gamma(\frac{k+3}{2})}$$

Paolo Perfetti

W16. Let $n\in N, n\geq 2$ and the numbers $a_i, b_i\in R, b_i>0$ where $i\in\{1,2,...,n\}$. Prove that

$$(b_1+b_2+...+b_n)^2\left(\frac{a_1^2}{b_1^2}+\frac{a_2^2}{b_2^2}+...+\frac{a_n^2}{b_n^2}\right)\geq(a_1+a_2+...+a_n)^2+$$

$$+n^2\left(n-1\right)\sqrt[n]{\left(a_1a_2...a_n\right)^2}$$

Ovidiu T. Pop

W17. Prove that

$$\left|\frac{z}{z}+a\cdot\frac{\overline{z}}{z}\right|\leq 2,\ \forall z\in C^*$$

where $a\in\{-1,1\}$.

Ovidiu T. Pop

W18. Let a, b, c be nonnegative real numbers such that $ab + bc + ca = 3\alpha^2$, $\alpha \geq 0$. Find the minimum of

$$(1 + a^2)(1 + b^2)(1 + c^2)$$

and the values (a, b, c) where the minimum is attained.

Paolo Perfetti

W19. 1). Prove that the function $f : (0, +\infty) \to R$, where

$$f(x) = e^x + \ln x$$

is injective
2). Solve the equation

$$e^{2x} + e^x \ln \frac{2x}{1 - 2x} = \sqrt{e}$$

Ionel Tudor

W20. Let $f : [0, 1] \to [0, \infty)$ be a convex and integrable function, with $f(0) = 0$. Show that
a).

$$\int_0^1 x^{2n} f(x)dx \geq \frac{1}{n+1} \int_0^1 f(x)dx$$

b).

$$\int_0^1 x^{2018} \ln(1 + x^2)dx > \frac{1}{4040}$$

Mihaela Berindeanu

W21. Let $G(n, m)$ be the 1-graphs with n vertices and m edges. Draw these graphs and describe them about their planarity and convexity, if they have the next characteristic polynomial:

$$P(\lambda) = \lambda^4 - 3\lambda^2 + 1.$$

Find $Spec\,(G)$ and study if these graphs are cospectral.

Laurenţiu Modan

W22. Let $(x_n)_n$ be the recurrent sequence:

$$x_{n+1} = 1 + \frac{x_n}{2},\ n \geq 0, x_0 = \frac{1}{2}$$

If we consider the recurent sequece $y_n = 2 - x_n, n \geq n_0$, study the convergence of the series:

$$\sum_{n\geq 0} y_n$$

and in the convergence case, compute its sum.

Laurenţiu Modan

W23. Find the smallest natural number $n \geq 2$, so that $ord\left(\widehat{3}\right) = 4$ in $(Z_n, \cdot)$ and $ord\left(\widehat{3}\right) = 5$ in $(Z_n, +)$.
Let $P \in Z_n\,[x]$ be the polynomial:

$$P\,(x) = \widehat{4}x^4 - x^3 + \widehat{6}x^2 - \widehat{9}$$

Find the decompsition of P in irreductible factors.

Laurenţiu Modan

W24. Prove that the inequality

$$\frac{L_1^6}{(L_1^4 + L_1^2L_2^2 + L_2^4)(\sqrt{2}L_1 + L_2)} + \frac{L_2^6}{(L_2^4 + L_2^2L_3^2 + L_3^4)(\sqrt{2}L_2 + L_3)} + \cdots$$
$$+ \frac{L_{n-1}^6}{(L_{n-1}^4 + L_{n-1}^2L_n^2 + L_n^4)(\sqrt{2}L_{n-1} + L_n)} + \frac{L_n^6}{(L_n^4 + L_n^2L_1^2 + L_1^4)(\sqrt{2}L_n + L_1)}$$
$$\geq \frac{\sqrt{2}-1}{3}\left(L_{n+2} - 1\right).$$

Ángel Plaza

W25. Let a, b, c be non negative integer numbers such that $a+b=c$. Prove that

$$\left(\frac{a}{c}\right)^n+\left(\frac{b}{c}\right)^n=\sum_{i=0}^{\left[\frac{n}{2}\right]}\left(\binom{n-i}{i}+\binom{n-i-1}{i-1}\right)\left(-\frac{ab}{c^2}\right)^i.$$

Ángel Plaza

W26. Let x, y, z be positive real numbers, and n and m integers. Find the maximal value of the expression

$$\frac{x+ny}{(m+1)x+(n+m)y+mz}+\frac{y+nz}{(m+1)y+(n+m)z+mx}+$$
$$+\frac{z+nx}{(m+1)z+(n+m)x+my}.$$

Ángel Plaza

W27. If $a_1, a_2, ..., a_n \geq e$, with the notations:

$$A_n := \frac{1}{n}\sum_{k=1}^{n} a_k,\ G_n := \sqrt[n]{\prod_{k=1}^{n} a_k}, H_n := \frac{n}{\sum\limits_{k=1}^{n}\frac{1}{a_k}}$$

prove that:

$$A_n^{G_n H_n} \leq G_n^{A_n H_n} \leq H_n^{A_n G_n}$$

Dorin Mărghidanu

W28. If $a_1, a_2, ..., a_n > 0$, with the notations:

$$A_n\left[a_1, a_2, ..., a_n\right] := \frac{1}{n}\sum_{k=1}^{n} a_k,\ G_n\left[a_1, a_2, ..., a_n\right] :=$$
$$= \sqrt[n]{\prod_{k=1}^{n} a_k}, H_n\left[a_1, a_2, ..., a_n\right] := \frac{n}{\sum\limits_{k=1}^{n}\frac{1}{a_k}}$$

prove that:

a).

$$\left(A_n\left[a_1,a_2,...,a_n\right]\right)^{A_n[a_1,a_2,...,a_n]} \leq G_n\left[a_1^{a_1},a_2^{a_2},...,a_n^{a_n}\right];$$

b).

$$G_n\left[a_1^{1/a_1},a_2^{1/a_2},...,a_n^{1/a_n}\right] \leq \left(H_n\left[a_1,a_2,...,a_n\right]\right)^{\frac{1}{H[a_1,a_2,...,a_n]}}$$

Dorin Mărghidanu

W29. Let $u:R\to R$ be a continuos function and $f:(0,\infty)\to(0,\infty)$ be a solution of differential equation $y'(x)-y(x)-u(x)=0$ for any $x\in(0,\infty)$. Find

$$\int\frac{e^x u(x)}{\left(e^x+f(x)\right)^2}dx.$$

D.M. Bătineţu-Giurgiu and Neculai Stanciu

W30. If $x_k>0$, $k=1,2,...,n$, then

$$\left(\sum_{k=1}^{n}x_k^2\right)\left(\sum_{k=1}^{n}x_k^4\right)\leq\left(\sum_{k=1}^{n}x_k^3\right)\sqrt{n\sum_{k=1}^{n}x_k^6}$$

Li Yin

W31. For all $n\in N$, then

$$W_n\sim\left(1-\frac{\ln n}{2n}\right)^n$$

where $W_n:=\frac{(2n-1)!!}{(2n)!!}$ is Wallis product.

Li Yin

W32. If $x_0=1$ and

$$x_{n+1}^3+1=(x_n+1)^3$$

for all $n\geq 0$, then $[x_n]=n$ for all $n\geq 1$, when $[\cdot]$ denote the integer part.

Tibor Jakab

W33. Prove that if $m, n \in N^*$ then:

$$\left(\int_0^{\frac{\pi}{2}} \sin^n x dx\right)\left(\int_0^{\frac{\pi}{2}} \cos^m x dx\right) \geq \left(\frac{2}{\pi}\right)^{n+m}$$

Daniel Sitaru

W34. Let be

$$\varepsilon_k = \cos\frac{2k\pi}{2n+1} + i\sin\frac{2k\pi}{2n+1}; k \in \overline{1,2n}; n \in N^*.$$

Prove that if $|x| < 1; |y| < 1$ then:

$$\left|\prod_{i=1}^{2n}(x-\varepsilon_i)(y-\varepsilon_i)\right| < 4n(n+1)$$

Daniel Sitaru

W35. Compute

$$L = \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\int_0^1 \frac{x\sin\pi x}{x+(1-x)k^{1-2x}}dx$$

Daniel Sitaru

W36. Prove that if $a, b, c \in R; a^2+b^2+c^2 \neq 0$ then:

$$\frac{(ab+bc+ca)^2(b-a)^2(c-a)^2(c-b)^2}{a^6+b^6+c^6} \leq 2\left(a^4+b^4+c^4-a^2b^2-b^2c^2-a^2c^2\right)$$

Daniel Sitaru

W37. If ABCD its an inscriptible quadrilateral: $AB = a$, $BC = b$, $CD = c$, $DA = d$, then

$$\frac{a+b+c+d}{\sqrt{abcd}} > 2\left(\frac{1}{\sqrt{ab+cd}} + \frac{1}{\sqrt{ad+bc}}\right)$$

Daniel Sitaru

W38. Let $x, y, z > 0$, $k > 0$ such that

$$x^2 + y^2 + z^2 + xyz = 4$$

Then

$$\frac{1}{(x+2)^{\frac{k}{2}}} + \frac{1}{(y+2)^{\frac{k}{2}}} + \frac{1}{(z+2)^{\frac{k}{2}}} < 2^{-k} + 2^{1-\frac{k}{2}}$$

Marius Drăgan

W39. Compute

$$\sum_{k=0}^{n-1} \cos^3 \left(x + \frac{2k\pi}{n} \right), n \in N^*$$

Liviu Bordianu

W40. Let $x_1, x_2, y_1, y_2, z_1, z_2 > 0$. Then

$$\frac{(x_1 + x_2)^2}{(y_1 + y_2)(z_1 + z_2)} \leq \max \left\{ \frac{x_1^2}{y_1 z_1}, \frac{x_2^2}{y_2 z_2} \right\}$$

Marius Drăgan and Sorin Rădulescu

W41. Let A, B be two square matrices from C, a, b, c, d integers such that $a < b, q$ the quotient of dividing b by a, such that $cq \neq d$ and $A^a B^c = A^b B^d = I_n$. Then there is a strictly integer number û such that $B^u = I_n$.

Mihály Bencze and Marius Drăgan

W42. Let be $n \geq 2, n \in N$ and $x_1, x_2, ..., x_n > 0$. Then

$$\frac{1}{x_1 x_2 ... x_n} \left(\frac{x_1 + x_2 + ... + x_n}{n} \right)^n \geq$$

$$\geq \max_{1 \leq i_1 < i_2 < ... < i_k \leq n} \frac{1}{x_{i_1} x_{i_2} ... x_{i_k}} \left(\frac{x_{i_1} + x_{i_2} + ... x_{i_k}}{k} \right)$$

Mihály Bencze and Marius Drăgan

W43. Let p_1, p_2, p_3 be positive real numbers and the functions $f, g : [0, +\infty) \to R$, $f(x) = [\sqrt{x}]$, $g(x) = \left[\sqrt{x - [x]^2}\right]$. Then is true the inequality:

$$g(p_1) \cdot f(p_2) f(p_3) + f(p_1) g(p_2) f(p_3) + f(p_2) f(p_1) g(p_3) \leq$$

$$\leq f(p_1 p_2 p_3) + g(p_1) g(p_2) g(p_3)$$

Mihály Bencze and Marius Drăgan

W44. Let $x, y, z > 0$ and $\alpha > 9$ such that

$$(x + y + z)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = \alpha$$

Then

$$\left(x^5 + y^5 + z^5\right)\left(\frac{1}{x^5} + \frac{1}{y^5} + \frac{1}{z^5}\right) \geq$$

$$\geq \frac{1}{16}\left(\alpha^5 - 25\alpha^4 + 230\alpha^3 - 950\alpha^2 + 1705\alpha - 945\right)$$

Marian Cucoaneş and Marius Drăgan

W45. Let O an interior point of ABC triangle and $\{M\} = AO \cap BC$, $\{N\} = BO \cap AC$, $\{P\} = CO \cap AB$. We denote $S_1 = \sigma[BOM]$, $S_2 = \sigma[MOC]$, $S_3 = \sigma[NOC]$, $S_4 = \sigma[AON]$, $S_5 = \sigma[AOP]$, $S_6 = \sigma[BOP]$ such that $S_5 \leq S_3$. Then

$$2S_1 + 2S_5 \leq 3|S_1 - S_5| + S_2 + S_3 + S_4 + S_6$$

Marian Cucoaneş and Marius Drăgan

W46. Compute

$$\cot^n\left(\frac{\pi}{7}\right) + \cot^n\left(\frac{2\pi}{7}\right) + \cot^n\left(\frac{4\pi}{7}\right), \ n \in N$$

Marian Cucoaneş and Marius Drăgan

W47. Let ABC be an acute triangle, denote A_1, B_1, C_1 the midpoints of sides BC, CA, AB. The lines OA_1, OB_1, OC_1 intersect the circumcircle in points A_2, B_2, C_2. Denote $x = A_1A_2, y = B_1B_2, z = C_1C_2$. prove that
1).

$$x + y + z \geq 3r$$

2).

$$(R - x)(R - y)(R - z) = R^3 \cos A \cos B \cos C$$

Nicuşor Minculete

W48. Let be $f : [0, 1] \to (0, +\infty)$ a continuously function. Prove that if exist $\alpha > 0$ for which

$$\int_0^1 x^\alpha f^n(x)dx \geq \frac{1}{(n+1)\alpha + 1} \geq \int_0^1 f^{n+1}(x)dx$$

where $n \in N$ then α is unique.

Mihály Bencze

W49. If $x_k > 0$ $(k = 1, 2, ..., n)$, then

$$\sum_{k=1}^{n} x_k - n \sqrt[n]{\prod_{k=1}^{n} x_k} \leq \sum_{1 \leq i < j \leq n} (\sqrt{x_i} - \sqrt{x_j})^2$$

Mihály Bencze

W50. Compute

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(x^3 \sin x + \lambda \cos x)dx}{\sqrt{5 + x^3 \cos x + \sqrt{5 + x^6 \cos^2 x}}}$$

Mihály Bencze

W51. Prove that

$$\left[\sum_{k=1}^{n} \frac{1}{\sqrt{k^4 + 2k^3 + 2k^2 + k + 1} - \sqrt{k^4 + 2k^3 - k + 1}}\right] = n$$

where [·] denote the integer part.

Mihály Bencze

W52. Let $A_1, A_2, ..., A_n$ be a convex polygon. Prove that

$$\sum_{k=1}^{n} \frac{1}{\sin A_k} \geq \sum_{k=1}^{n} \frac{1}{\sin \frac{\pi + A_k}{n-1}}$$

Mihály Bencze

W53. Let $a, b \in \{2, 3, 4, 5, 6, 7, 8, 9\}$ and $m \in N$. Prove that exist $n \in N$ such that the first digit of a^n are b^m.

Mihály Bencze

W54. For all invertable matrices $A, B \in M_2(C)$ holds
1).

$$2Tr(AB) + Tr(A^{-1}B)detA + Tr(B^{-1}A)detB = 2Tr(A)Tr(B)$$

2).

$$Tr(ABA) + Tr(BAB) + Tr(A)detB + Tr(B)detA = (Tr(A) + Tr(B))Tr(AB)$$

Mihály Bencze

W55. If $a \in N^*$, $(a, p) = 1$ where p is a prime, then

$$\prod_{k=1}^{n} \left(a^{p^{k-1}(p-1)} - 1 \right)$$

is divisible by $p^{\frac{n(n+1)}{2}}$.

Mihály Bencze

W56. In all tetrahedron $ABCD$ holds:
1).

$$\frac{1}{h_a h_b} + \frac{2}{h_a h_c} + \frac{1}{h_a h_d} + \frac{1}{h_b h_c} + \frac{2}{h_b h_d} + \frac{1}{h_c h_d} \leq \frac{1}{2r^2}$$

2).

$$\frac{1}{r_a r_b}+\frac{2}{r_a r_c}+\frac{1}{r_a r_d}+\frac{1}{r_b r_c}+\frac{2}{r_b r_d}+\frac{1}{r_c r_d}\le\frac{2}{r^2}$$

Mihály Bencze

W57. Prove that

$$\int_0^{\frac{\pi}{2}}\prod_{k=1}^{n}\left(\sin^{2k+1}x+\cos^{2k+1}x\right)dx\ge\frac{\pi}{2\left(\sqrt{2}\right)n^2};n\in N^*$$

Daniel Sitaru

W58. Let f_n be $n-th$ Fibonacci number defined by recurrence

$$f_{n+1}-f_n-f_{n-1}=0,n\in N$$

and initial conditions $f_0=0,f_1=1$.
Prove that $\left(5n^2+3n-2\right)f_n-6nf_{n+1}$ is divisible by 50 for any $n\in\mathbb{N}$

Arkady Alt

W59. Let E be a Inner Product Space with dot product $-\cdot-$ and F be proper nonzero subspace. Let $P:\mathbf{E}\longrightarrow\mathbf{E}$ be orthogonal projection $\mathbf{E}$ on $\mathbf{F}$.
a). Prove that for any $\mathbf{x},\mathbf{y}\in\mathbf{E}$, holds inequality

$$|\mathbf{x}\cdot\mathbf{y}-\mathbf{x}\cdot P(\mathbf{y})-\mathbf{y}\cdot P(\mathbf{x})|\le\|\mathbf{x}\|\,\|\mathbf{y}\|$$

b). Determine all cases when equality occurs.

Arkady Alt

W60. Let x,a,h be arbitrary real numbers such that
$x>0,a\ge-1$, $h>0$ and let sequence (x_n)
defined recursively by $x_1=x,\ x_{n+1}=\dfrac{n+a}{n+a+h}x_n,\ n\in N\cup\{0\}$.
Explore for which h the infinite sum $\sum\limits_{n=1}^{\infty}x_n$ converges and find it in the case of convergence.

Arkady Alt