SOLUTION

1| The cartesian equation of a plane parallel to π is:

$x+y-z=d$ with $d \in \mathbb{R}$.

Let us ~~denote~~ denote π_d such a plane.

The distance from $\underline{O}$ to π_d is:

$$D = \frac{|(\underline{O}-\underline{P})\cdot N|}{\|N\|}$$

where $N = (1,1,-1)$ is a normal vector and $\underline{P}$ is any point of π_d. We know that $\underline{P}\cdot N = d$. Therefore

$$D = \frac{|0-d|}{\sqrt{3}} = \frac{|-d|}{\sqrt{3}} = \frac{|d|}{\sqrt{3}}.$$

We want $D=3$ therefore $|d| = 3\sqrt{3}$, that is $d = \pm\sqrt{3}$

In conclusion the planes are two:

$x+y-z=\sqrt{3}$ and $x+y-z=-\sqrt{3}$.

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2| The conic section $\mathcal{C}$ is the ellipse whose points are the $X \in V_2$ such that $d(X,\underline{O}) = \|X\| = \frac{1}{2}\, d(X,L)$.

Let $N = \frac{(3,4)}{5}$ N is a unit normal vector to L and $\underline{O}$ is in the negative half-plane defined by N.

Therefore we have the polar equation for $\mathcal{C}$:

$$\rho = \frac{ed}{e\cos\vartheta + 1}.$$

where: $\rho = \|X\|$ ϑ = angle ~~be~~ of ~~X~~ and N

$d = d(O,L)$. By the known methods $d(O,L) = \frac{10}{5} = 2$