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~~[illegible]~~

1) Let $\pi = \{(1,1,0) + s(1,-1,0) + t(1,0,1) \mid s,t \in \mathbb{R}\}$
Find the cartesian equations of all planes parallel to π whose distance from the origin is 3

2) Let $\mathcal{C}$ be the conic section with a focus in $(0,0)$, eccentricity $e = 1/2$ and directrix L : $3x+4y=10$.
Find the point of $\mathcal{C}$ which is closest to L and the point of $\mathcal{C}$ ~~[illegible]~~ farthest from L. (these are the vertices of the conic section).

3) Let us consider a plane motion $\underline{r}(t) = (x(t), y(t))$ such that the acceleration $\underline{a}(t)$ is directed toward the origin and its norm is 9 times the norm of $\underline{r}(t)$. At time $t=0$ we have $\underline{r}(0) = (5,0)$ and $\underline{v}(0) = (0,6)$.
(a) Write down explicitly the functions $x(t)$ ~~[illegible]~~ and $y(t)$
(b) Write down the cartesian equation of the path
(c) How much time is required for the particle to go once around the path?

4) Let $\underline{v} = (1,2,1)$ $\underline{w} = (1,-1,-1)$.
(a) Are there matrices $A \in M_{3,3}(\mathbb{R})$ such that
$A\underline{v} = 2\underline{v}$, $A\underline{w} = -\underline{w}$ ~~[illegible]~~, $\det A = 6$
and $\underline{u}$ is eigenvector of A, where $\underline{u} = (-2,1,1)$?
If the answer is yes, compute all of them.
(b) Same question with $\underline{u} = (2,1,0)$

5) Let $Q(x,y,z) = 2x^2 + 4xy + 4xz + 5y^2 + 8yz + 5z^2$.
(a) Reduce the quadratic form Q to canonical form
(NOTE: this means to find an orthonormal basis B of V_3 such that $Q(x,y,z) = \lambda_1(x')^2 + \lambda_2(y')^2 + \lambda_3(z')^2$, where $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$ and (x',y',z') are the components of (x,y,z) with respect to the basis B).
(b) Is there a $(x,y,z) \in V_3$ with $(x,y,z) \neq (0,0,0)$ such that $Q(x,y,z) = 0$?