LINEAR ALGEBRA AND GEOMETRY

SUPPLEMENTARY EXERCISES, 2 (Straight lines)

1. (a) Let us consider the lines of $\mathcal{V}_2$: $L : 2x - y + 1 = 0$ and $R = \{(3,1) + t(4,3)\}$. Describe and find cartesian equations for the following subset of $\mathcal{V}_2$:

$$\{X \in \mathcal{V}_2 \mid d(X,L) = d(X,R)\}.$$

(b) Let $L = \{(1,2) + t(1,3)\}$ and $R = \{(-2,5) + t(2,6)\}$. Describe and find cartesian equations for the following subset of $\mathcal{V}_2$:

$$\{X \in \mathcal{V}_2 \mid d(X,L) = d(X,R)\}.$$

(c) Let L and R be two non-parallel lines in $\mathcal{V}_2$. Describe the subset of $\mathcal{V}_2$ whose elements are points $X \in \mathcal{V}_2$ such that $d(X,L) = d(X,R)$. What happens if L and R are parallel?
(d) For L and R as in (a), describe and find cartesian equations for the following subset of $\mathcal{V}_2$:

$$\{X \in \mathcal{V}_2 \mid d(X,L) = 2d(X,R)\}.$$

2. (a) Find the intersection of the lines $\{(1,1) + t(1,2)\}$ and $\{(1,1) + t(2,1)\}$. (b) Same question with the two lines $\{(2,-1,2) + t(1,1,-1)\}$ and $\{(2,0,0) + t(1,0,1)\}$.

3. Let us consider the lines of $\mathcal{V}_2$: $L = (1,-1) + t(1,1)$ and $R : x + 2y = 1$. How many are the points $X \in \mathcal{V}_2$ such that $\begin{cases} d(X,L) = \sqrt{2} \\ d(X,R) = \sqrt{3} \end{cases}$? Find one of them.

4. Let $A = (3,4)$. Describe and find parametric equations of the locus of all vectors $X \in \mathcal{V}_2$ such that the angle between A and X is $\pi/4$. (Suggestion: use a orthonormal basis of $\mathcal{V}_2$ such that one of its vectors is parallel to A.)

5. Let $A = (1,-2)$ and $P = (1,-1)$. Find both parametric and cartesian equations of the lines passing through P which are parallel to a vector B whose angle with A is $\theta = \pi/4$.

6. Let $L = \{(1,-1,1) + t(1,1,1)\}$ and $Q = (1,1,-1)$. Compute $d(Q,L)$ and the point of L which is nearest to Q.