

**LINEAR ALGEBRA AND GEOMETRY**  
**SUPPLEMENTARY EXERCISES ABOUT CHAPTERS 12 AND 13**

- 1.** Let  $A = (1, 1, -1)$ ,  $B = (1, 2, 0)$ ,  $C = (1, 1, 1)$ . (a) Is  $\{A, B, C\}$  a basis of  $\mathcal{V}_3$ ? (b) If the answer to (a) is in the affirmative, compute the components of the vector  $(0, 1, 1)$  with respect to the basis  $\{A, B, C\}$ .
- 2.** Let  $A = (1, 1, -1)$ ,  $B = (1, 2, 0)$ ,  $C = (-2, -5, -1)$ . (a) Is  $\{A, B, C\}$  a basis of  $\mathcal{V}_3$ ? (b) If the answer to (a) is in the negative, express a  $A, B$  or  $C$  as a linear combination of the remaining two vectors.
- 3.** Is there an orthogonal basis  $\{A, B, C\}$  of  $\mathcal{V}_3$  such that  $\|A - 2B + C\| = 12$  and  $\|A + B + C\| = 6$ ?
- 4.** Let  $\{A, B, C\}$  be a orthogonal basis such that  $\|A\| = \|B\| = 2$  and  $\|C\| = 1$ . Moreover let  $V = A + B + C$  and  $W = -A + B - C$ . Find the cosine of the angle between  $V$  and  $W$ .
- 5.** For  $t$  varying in  $\mathbf{R}$ , let  $A_t = (t + 1, 1, 1)$ ,  $B_t = (1, t + 2, 1)$ ,  $C_t = (1, 1, t + 2)$ .  
 (a) Find all  $t \in \mathbf{R}$  such that  $A_t, B_t, C_t$  form an independent set.  
 (b) Compute, for each  $h \in \mathbf{R}$ , the solutions of the system (depending on  $h$ ): 
$$\begin{cases} hx + y + z = 0 \\ x + hy + z = 0. \\ x + y + hz = 0 \end{cases}$$
- 6.** For  $t$  varying in  $\mathbf{R}$ , let  $A(t) = (t + 1, t + 2, 2t)$ ,  $B(t) = (1, t^2 + 1, 1)$ ,  $C(t) = (1, 2, 1)$ . (a) Find all  $t$  such that  $A(t)$ ,  $B(t)$  and  $C(t)$  do not form a basis of  $\mathcal{V}_3$ . (b) For all  $t, s \in \mathbf{R}$  let us consider the system

$$xA(t) + yB(t) + zC(t) = (s, 2s, 1).$$

For which values of  $t$  and  $s$  there is no solution? For which values of  $t$  and  $s$  there is more than one solution?

- 7.** For  $t, a$  varying in  $\mathbf{R}$ , let us consider the system 
$$\begin{cases} x + y + z = 1 \\ 2x + ty + z = -1. \\ 6x + 7y + 3z = a \end{cases}$$
 Find for which values of  $t$  and  $a$  there is a unique solution, no solution, infinitely many solutions.
- 8.** Let  $A = (3, 4)$ . Describe the set of all vectors  $X \in \mathcal{V}_2$  such that the angle between  $A$  and  $X$  is  $\pi/4$ .