First Name: Last Name:

Linear Algebra and Geometry, written test, 09.26.2011

NOTE: In the solution of a given exercise you must (briefly) explain the line of your argument and show the main points of your calculations. Solutions without adequate explanations will not be evaluated.

1. Let $L = \{(1,2)+t(1,3)\}$ and $R = \{(-2,5)+t(2,6)\}$. Describe and find cartesian equations for the locus of points $X \in \mathcal{V}_{\in}$ such that $d(X,L) = d(X,R)$.

2. Let L be the intersection of the two planes $M_1 : x - y + z = 1$ and $M_2 : y - 2z = -1$. Let moreover Π be the plane $\{(1,2,1) + t(1,-1,-1) + s(-1,0,2)\}$. Find (if any) a plane of V_3 containing L and perpendicular to Π. Is such plane unique?

3. Let W be the linear subspace of V_4 defined by the equation $x + y - z + t = 0$ (that is, the elements of W are the solutions of that equation). Find the element Q of W nearest to the point $P = (1,2,-1,4)$. Compute also the distance $\| P - Q \|$.

4. (a) Find all symmetric matrices $A \in \mathcal{M}_3$ such that:
(i) the eigenvalues of A are 2 and -3, and (ii) the eigenspace $E(2) = L((1,2,0))$.
(b) For such matrices A compute A^{100}.
In the answers to both questions you can express the resulting matrices as products of (explicitly determined) matrices.

5. Let us consider the real quadratic form $Q(x,y,z) = 3x^2 + 8xy + 4xz + 3y^2 + 4yz$.
(a) Reduce Q to diagonal form by means of an orthonormal basis (that is: find an orthonormal basis $\mathcal{B}$ of V_3 and real numbers λ_i, $i = 1,2,3$ such that $Q(x,y,z) = \sum_{i=1}^{3} \lambda_i {x'}^2$, where the x_i''s are the components of (x,y,z) with respect to the basis $\mathcal{B}$).
(b) Find the maximum and minimum value of Q on the unit sphere and describe the points of maximum and of minimum.