Representations of the stabilizer subgroup at the point of infinity in $\mathrm{Diff}(S^1)$

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Introduction

Conformal symmetry

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A traditional set of mathematical objects is

- Wightman field (operator valued distribution on Minkowski space)
- Unitary representation of conformal symmetry group with spectrum condition
- the vacuum vector

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Diff(S^1) symmetry

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4. the commutation relations are same as the Lie algebra of Diff(S^1).
5. the component of stress energy tensor is Diff(S^1) covariant.

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We need to include representations without vacuum, since they appear in charged sectors.
Moreover, if we drop the existence of vacuum, the extension of stress energy tensor to S^1 (Lüscher-Mack theorem) is no longer true in general.

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And our main result is

Theorem

There are representations of B_0 which does not extend to $\mathrm{Diff}(S^1)$.

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Diff(S^1) includes important one-parameter subgroups:

Möbius group

$$\begin{aligned} \text{rotation } \rho_s(z) &= e^{is}z, \text{ for } z \in S^1 \subset \mathbb{C} \\ \text{translation } \tau_s(x) &= x + s, \text{ for } x \in \mathbb{R} \\ \text{dilation } \delta_s(x) &= e^s x, \text{ for } x \in \mathbb{R} \end{aligned}$$

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Spectrum condition
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$\iff$ the spectrum of rotation is positive
The rotation group is compact $\Longrightarrow$ the rotation group has the lowest eigenvector with eigenvalue h.

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fact

the second cohomology of Diff(S^1) is one dimensional.

$c(g,h)$ is determined by a real number c.

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projective unitary, positive energy, irreducible representations of Diff(S^1) are completely classified by c and h. There exists such a representation if and only if there exist natural numbers m, r, s such that

$$\begin{aligned} c &= 1 - \frac{6}{(m+2)(m+3)}, 0 \leq m \\ h &= \frac{\{(m+3)r - (m+2)s\}^2 - 1}{4(m+2)(m+3)}, 1 \leq s \leq r \leq m+1, \end{aligned}$$

or $c \geq 1$ and $h \geq 0$.

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projective unitary, positive energy, irreducible representations of $\mathcal{P}$ is unique. Any restriction of irreducible representations of Möb is irreducible.

restrictions of representations of Diff(S^1) to B_0

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Any restriction of representations $\pi_h^c|_{B_0}$ of Diff(S^1) is irreducible.

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Two restricted representations $\pi_h^c|_{B_0}, \pi_{h'}^{c'}|_{B_0}$ are equivalent only if $c = c'$. For $c \leq 1$, all possible $\{\pi_h^1|_{B_0}\}$ are inequivalent. For $c > 2$, some examples $\{\pi_{h_1}^c|_{B_0}, \pi_{h_2}^c|_{B_0}\}$ of equivalent restrictions are exhibited.

What about nonextendable representations of B_0?

Lie algebra of $\mathrm{Diff}(S^1)$

group/algebra	elements	operation
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The Virasoro algebra has the following commutation relations.

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{Cn(n^2-1)}{12}\delta_{m,-n}$$

Lowest weight representations of Vir

fact

For any $c, h \in \mathbb{R}$ there is a representation of Vir with a contravariant sesquilinear form $\langle \cdot, \cdot \rangle$ and a lowest weight vector v such that

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The ideal structure of $\mathcal{K}_0$ is determined as an infinite sequence of ideals

$$\mathcal{K}_0 \supset \mathcal{K}_1 \supset \mathcal{K}_2 \supset \cdots$$

and an exceptional ideal $\mathcal{K}_{1,3} \supset \mathcal{K}_3$ and it holds that $[\mathcal{K}_n, \mathcal{K}_n] = \mathcal{K}_{2n+1}$. In particular, $\mathcal{K}_1 = [\mathcal{K}_0, \mathcal{K}_0]$ has codimension one in $\mathcal{K}_0$ and $\mathcal{K}_0$ has one dimensional representation.

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The second cohomology of $\mathcal{K}_0$ on $\mathbb{C}$ is one dimensional ($\mathcal{K}_0$ has the unique central extension $\mathcal{K}$).

several representations of B_0

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π has a big kernel.

Theorem (T. in preparation)

For $c \leq 1$, $\pi \otimes \pi_h^c$ is irreducible.

These representations are not associated with stress energy tensor.

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These modules are expected to integrate to representations of B_0 but not to $\mathrm{Diff}(S^1)$.

representations of $\mathrm{Diff}(\mathbb{R})_c$

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Is there any representation of $\mathrm{Diff}(\mathbb{R})_c$ other than restrictions of π_h^c?

representations of $\mathrm{Diff}(\mathbb{R})_c$

Consider U(1)-current $J(z)$ on S^1 with $[J(z_1), J(z_2)] = i\delta'(z_1 - z_2)$ in vacuum representation.

representations of $\mathrm{Diff}(\mathbb{R})_c$

Consider U(1)-current $J(z)$ on S^1 with $[J(z_1), J(z_2)] = i\delta'(z_1 - z_2)$ in vacuum representation.
The Fourier components of $T(z) = \frac{1}{2} : J^2 : (z)$ satisfy the Virasoro commutation relations with $c = 1, h = 0$.

representations of $\mathrm{Diff}(\mathbb{R})_c$

Buchholz-Mack-Todorov automorphisms '90

For a smooth real function $\rho(z)$, $J(z) \mapsto J(z) + \rho(z)$ is an automorphism.
If $q := \int \frac{dz}{2\pi i z}\rho(z) \neq 0$, $T(z)$ is mapped to the representation $\pi^c_{q^2}$.

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Theorem (T. in preparation)

If ρ is a function on S^1 smooth except the point of infinity, divergent sufficently strongly at infinity, then the transformed $T(z)$ integrates to a representation of $\mathrm{Diff}(\mathbb{R})_c$, but not to $\mathrm{Diff}(S^1)$.

summary and outlook

- several representations of B_0 and $\mathrm{Diff}(\mathbb{R})_c$ exist.

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- what about vacuum representation?

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- what about vacuum representation?
- integrability and inequivalence of Verma modules? good invariant?

summary and outlook

- several representations of B_0 and $\mathrm{Diff}(\mathbb{R})_c$ exist.
- some are not associated with stress energy tensor.

- what about vacuum representation?
- integrability and inequivalence of Verma modules? good invariant?
- characterization of extendability to $\mathrm{Diff}(S^1)$?