Spectral Triples and JLO Cocycles for Conformal Field Theory

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joint work in progress with Sebastiano Carpi, Yasuyuki Kawahigashi, and Roberto Longo

Outline

- representations and states of conformal nets are algebraic objects in noncommutative (=quantum) field theory
- noncommutative geometry described by spectral triples and cyclic homology
- overall question: possible to encode representation and states in these NCG objects? e.g., as (co-)homology classes, or K(K)-classes?

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Entire Cyclic Cohomology and the JLO-Cocycle

First and Main Example: The Supersymmetric Free Field – *nice toy*

Second Example: The $(N = 1)$ Super-Virasoro Net – *universal*

Further Models

Conclusions... and then?

Entire cyclic homology

given a $\mathbb{Z}/2$-graded Banach algebra A.

- chains $C_*(A) := \sum_{n=0}^{\infty} A \otimes (A/\mathbb{C})^{\otimes n}$ so-called universal differential algebra of A
- consider $(a_0, a_1, ...) \in C_*(A)$ such that

$$z \in \mathbb{C} \mapsto ||(a_1, a_2, ...)||_z := \sum_n \frac{1}{\sqrt{n!}} ||a_n|| z^n$$

  is entire
- entire cochains defined dually
- usual (cyclic) boundary operator $d = b + B$

Entire cyclic homology (skip)

boundary operator $d = b + B$ where ...

$$b(a_0,\dots,a_n)_n = \sum_{k=0}^{n}(-1)^k(a_0,\dots,a_k a_{k+1},\dots,a_n)_{n-1} + (-1)^n(a_n a_0,\dots,a_{n-1})_{n-1}$$

$$B(a_0,\dots,a_n)_n = \sum_{i=0}^{n}(-1)^{kn}(1,a_k,\dots,a_n,a_0,\dots,a_{k-1})_{n+1}$$

The JLO cocycle

- here consider: θ-summable spectral triples $(A, \mathcal{H}, D)$, meaning e^{-tD^2} is trace-class for all $t > 0$, and $D^2 = L_0$ up to a constant
- various cohomologous expressions for the EC cocycle associated to $(A, \mathcal{H}, D)$, here JLO-cocycle:

$$\tau_n(a_1 \dots a_n) := \int_{\Delta_n} \mathrm{tr}\left(\Gamma a_0 \alpha_{\mathrm{i}s_1}(\delta(\gamma^1(a_1))) \dots \alpha_{\mathrm{i}s_n}(\delta(\gamma^n(a_n)))\mathrm{e}^{-D^2}\right) \mathrm{d}s$$

  with $(\alpha_t)_{t\in\mathbb{R}}$ the one-parameter automorphism group with infinitesimal generator ad L_0, Γ the grading in the $\mathbb{Z}/2$-graded case
- for simplicity, identify A with $\pi_R(A) \subset B(\mathcal{H})$, corresponding to some fixed (Ramond) representation π_R of the net
- homotopy invariance yields pairing of cocycle with K-theory:

$$K^*(A) \ni \tau : K_*(A) \to K_0(\mathbb{C}) = \mathbb{Z}$$

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Cocycles in the sectors

what happens in a sector $\pi_1 \not\simeq \pi_R$? how to define cocycle? two answers:

1 consider different triple $(A, \mathcal{H}_{\pi_1}, D_{\pi_1})$ such that $D_{\pi_1}^2 = L_0^{\pi_1}$ and associated JLO-cocycle $\tau^{(\pi_1)}$

2 DHR-reps are described by localised endomorphisms of the net $\mathcal{A}$, namely $\pi_1 \simeq \pi_0 \circ \rho_{\pi_1}$, so

$$(\rho_{\pi_1}^* \tau_n)(a) := \tau_n(\rho_{\pi_1 *}(a))$$

well-defined for $a \in C_n(A)$

- the two descriptions expected to be cohomologous; at least, for certain models
- locally, on $\mathcal{A}(I)$, the second one cohomologous to τ, hence work on (global) universal C^*-algebra

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Now the questions:

- in models, are the cocycles nontrivial classes? when? under what assumptions?
- can we distiguish the representations, i.e., $[\rho_1^*\tau] \neq [\rho_2^*\tau]$ for $\rho_1 \not\simeq \rho_2$?
- if not, can we distiguish the induced metrics?
- abstract general arguments?
- explicit arguments: find suitable projections and unitaries in $\mathcal{M}_n(A)$ corresponding to classes $K_0(A)$ and $K_1(A)$, respectively, and explicit expressions for the localised endomorphisms in the respective model

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Definition

- consider (unbounded) bosonic and (bounded) fermionic fields: $J(f)$ and $\psi(f)$, $f \in C^\infty(\mathbb{S})$, on $\mathcal{H}$, the product of bosonic and fermionic Fock spaces on the circle
- field algebra $\mathcal{A}(I) = \{\mathrm{e}^{\mathrm{i}J(f)} : \ f \in C^\infty(I)\}'' \otimes \mathrm{Cliff}(C^\infty(I))''$ for $I \in \mathcal{I}$
- conformal Hamiltonian L_0 and supercharge $D = \sum_k J_k \psi_{-k}$ with $D^2 = L_0$, densely defined, essentially self-adjoint, domain containing the set of finite energy vectors $\subset \mathcal{H}$

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Some facts

mostly owing to Buchholz/Mack/Todorov:

- irreducible unitary positive energy Ramond representations of the net $\mathcal{A}$ described by $q \in \mathbb{R}_+$ such that $(L_0 - 1/16)\Omega = q^2/2\Omega$
- the (localisable) DHR-automorphism explicitly known
- $\mathcal{D}(\delta) \cap \mathcal{A}(I)$ is strongly dense in $\mathcal{A}(I)$

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Spectral triples and the Cocycle

Theorem

Let $\mathcal{A}$ be the net associated to the supersymmetric free field, represented on the ($\mathbb{Z}/2$-graded) Ramond Fock space $\mathcal{H}$. Let L_0 be the associated conformal Hamiltonian on $\mathcal{H}$, and D a self-adjoint (odd) square-root of L_0, as above.

- $(A, \mathcal{H}, D)$, *with* $A := \mathcal{D}(\delta) \cap \mathcal{A}(I)$ *for some fixed interval* I, *or* $A := \mathcal{D}(\delta)$ *are* θ*-summable spectral triples.*
- *For the corresponding even JLO-cocycle we have* $\tau_e = 0$; *doubling procedure gives nontrivial* τ_e.
- *For the corresponding odd JLO-cocycle and a DHR-representation* π_1 *on* $\mathcal{H}_1 \simeq \mathcal{H}$, *with DHR-endomorphism* ρ *such that* $\pi_1 \simeq \pi_R \circ \rho$, *we have:*

$$\rho_* \tau_o \simeq \tau_o^{\pi_1}$$

Search for unitaries

are the odd cocycles non-trivial? do they distinguish the sectors?

1. natural physically meaningful candidates for $[u] \in K_1(A)$ are: Weyl unitaries (the generators of the $\mathcal{A}(I)$), shifts on the spectrum of D, charge transporters, braiding operators,...
2. some necessary conditions for non-triviality $\tau(u) \neq 0$ are: $[D, u] \neq 0$ and $(\exists n_0 \in \mathbb{N})(\forall n > n_0) u^{1/n} \notin \mathcal{D}(\delta)$

- hence $\tau(u) = 0$ for all these u
- open question: do we even have $K_1(A) = 0$?

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SuSy rational extensions of the net

- idea: consider the commutative algebra $A = C^\infty(\mathbb{S})$ represented on $\mathcal{H} = L^2(\mathbb{S})$ (with basis $(e_k : \mathrm{e}^{\mathrm{i}t} \mapsto \mathrm{e}^{\mathrm{i}tk})_{k\in\mathbb{Z}}$) by multiplication, and $D = \mathrm{d}/\mathrm{d}t$; the "spectral shift unitary" $u = e_1$ acts as bilateral shift on the eigenvectors and gives $\tau(u) = 1$
- back to our setting: $u \notin A$, but u is in the maximal extensions $\mathcal{A}_N$ of the net $\mathcal{A}$
  (i.e., those super-conformal nets containing $\mathcal{A}$ as a subnet covariant w.r.t. the same representation of $\mathrm{SL}(2,\mathbb{R})$ as $\mathcal{A}$ itself; in fact,

$$\mathcal{A}_N = \mathcal{A}_{LU(1)_{2N}}, \quad \mathcal{A} = \mathcal{A}_{LU(1)_{conn-comp}})$$

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The "shift unitary"

Theorem

The rational extensions $\mathcal{A}_N$ of $\mathcal{A}$ are supersymmetric and give rise to spectral triples $(A := \mathcal{D}(\delta) \cap \mathcal{A}_N, \mathcal{H}, D)$ with the same representation space as before. δ extends to $\mathcal{A}_N$. The even cocycle vanishes. Concerning the odd cocycle, we have $\pi(u) \in \mathcal{D}(\delta)$, and $\delta(\pi(u)) = \pi(u)b_0$, and $\tau(u) = 1$. Moreover

$$\tau(\rho_\pi(u)) = \tau(u) = \tau^\pi(u) = 1$$

for all representations π of the N-th extension.

Rough definition

consider a unitary positive-energy representation of the $(N = 1)$ Ramond Super-Virasoro algebra for fixed central charge c and with finite-dimensional lowest-energy eigenspace; denote the generators in this representation by $(G_n, L_n)_{n\in\mathbb{Z}}$ and define the smoothed out fields $G(f), L(f)$ for $f \in C_c^\infty(S^1)$; set

$$\mathcal{A}(I) := \{e^{iL(f)}, e^{iG(f)} : f \in C_c^\infty(S^1) \text{ real}, \operatorname{supp} f \subset I\}'', \ I \in \mathcal{I}.$$

Spectal triples from the Ramond algebra (1)

Theorem

- *Given a unitary positive energy Ramond representation* π_R *of* $\mathcal{A}$*, we have a net of graded,* θ*-summable spectral triples* $(A(I), \mathcal{H}, D)$ *where* $D = G_0$ *such that* $A(I)$ *is a strongly dense unital* **-subalgebra of* $\pi_R(\mathcal{A}(I))$ *for every interval* $I \subset S^1$ *in* $\mathcal{I}$*.*
- *For the irreducible Ramond representation with* $h = c/24$*, we get a non-trivial even JLO-cocycle with* $\tau(1) = 1$*.*
- *For* $h \neq c/24$*, the irreducible representations are ungraded and only the odd JLO-cocycle may be non-trivial.*

Spectal triples from the Ramond algebra (2)

- lengthy proof, based on linear energy bounds and commutation relations for the Virasoro algebra; first considering the domain of G_0 in $\mathcal{H}$ and deducing then the domain of the associated graded derivation
- in fact, the (domain) *-algebra in this case contains as important elments the "resolvents"

$$G(f)(L(f)+\mathrm{i}\alpha)^{-1}, \quad (L(f)+\mathrm{i}\alpha)^{-1},$$

with non-negative $f \in C_c^\infty(I)$ and appropriate $\alpha \in \mathbb{R}$ depending on f; it is a local C^*-algebra
- remark on the cocycle: some arguments work as in the free-field example; non-trivilaity of odd cocycles (i.e., for $h \neq c/24$) open problem; if so, then distinguishable among each other?

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...in progress, esp. WZW...

Lessons from these examples

- whether even or odd cocycle – depending on the structure of the model's algebra
- restrict to rational theories, work on a suitable (global) universal C^*-algebra (which one?)
- two (different or cohomologous?) ways of associating the cocycles to the sectors
- many special or general "vanishing arguments", but the search for explicit cycles (as Chern characters coming from projections/unitaries in $\mathcal{M}_n(A)$) turns out to be exhausting
- abstract criterions to check non-triviality of τ, $\tau^{(\pi)}$, and $\rho_*\tau$ still missing and helpful

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Last but not least: ideas, hopes, and dreams...

- (when) do the DHR-endomorphisms preserve the domain: $\rho(\mathcal{D}(\delta)) \subset \mathcal{D}(\delta)$?
- if so, then possible to find special cycles (even or odd) x such that $[\rho_{1*}\tau] \cdot x \neq [\rho_{2*}\tau] \cdot x$?
- $\tau^\pi \simeq \rho^*\tau$ as expected above?
- special cases indicate $\rho_* \neq 1$ on $K_*(A)$, but is this really so? how to see in general? and if so, then $\rho^*[\tau] \neq [\tau]$ for this fixed τ?
- what properties does A have? suitable for KK-theory? helpful description of DHR-endomorphisms in terms of $KK(A, A)$-classes?
- what about the induced metrics?

Last but not least: ideas, hopes, and dreams...

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- if so, then possible to find special cycles (even or odd) x such that $[\rho_{1*}\tau] \cdot x \neq [\rho_{2*}\tau] \cdot x$?
- $\tau^\pi \simeq \rho^*\tau$ as expected above?
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