Planar Algebras and Subfactors

Arnaud Brothier

Université de Paris 7

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Planar Algebras and Subfactors

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What is a planar algebra?
Planar calculus
Tangle
Planar algebra

Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor

A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

Plan

1. What is a planar algebra?
   - Planar calculus
   - Tangle
   - Planar algebra
2. Connection with subfactor
   - Subfactor
   - An invariant
   - Planar algebra gives subfactor
3. A canonical MASA (maximal abelian self-adjoint subalgebra)
   - Description
   - Properties
   - Free group factor

Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

Planar calculus

Let $(E,+)$ be a set with an associative law.
We can compute elements on a line as follow:

$$a$$
$$a + b$$
$$c + a + b$$

We want to do the same but in the plane to give a sense to something like that:

$$\begin{matrix} & b & \\ & & c \\ a & & \end{matrix}$$

For this, I have to define what is the "+" in the plane.

Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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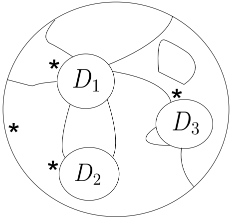
Tangle



Exterior disc

Disjoints subdiscs D_j

Strings

First segment for each disc denote by a star

Each disc cross an even number of times the strings, we denote by $2k_j$ the number of point of the boundary of the disc D_j which touch the strings and by $2k$ for the exterior disc.

Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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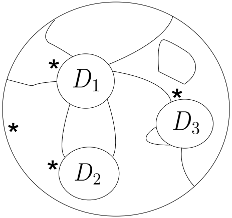

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Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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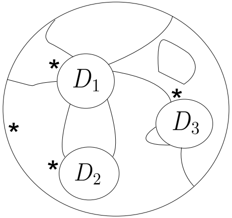

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Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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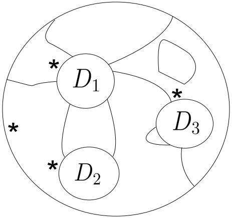

Exterior disc
Disjoints subdiscs D_j
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Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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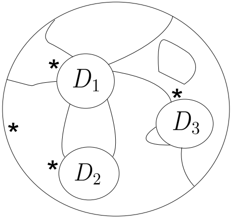

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Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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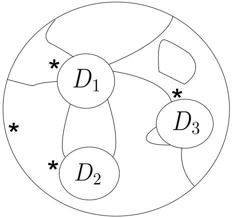

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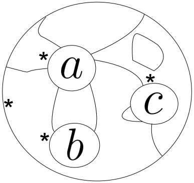
Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

Action of the tangles

Let $P = \{P_k\}_{k\in\mathbb{N}}$ be a sequence of sets.
We put an element of the P_{k_j} inside the subdisc D_j to obtain an element in P_k.
For example, the precedent tangle, let's call it T, is a map from $P_3 \times P_1 \times P_2$ into P_2.

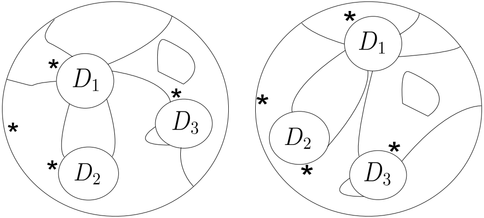
The element $T(a,b,c)$ is denoted by .

Remark: If the tangle doesn't have any subdisc, it is considered as an element of P_k.

Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

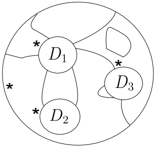
Be able to draw

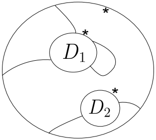
A tangle and its image under a homeomorphism preserving orientation act in the same way, like the two following:



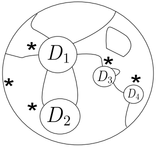
Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

Associativity

Consider T and $\tilde{T}$: 



We can put $\tilde{T}$ inside D_3 of T, we obtain a new tangle denoted by

$T \circ_3 \tilde{T}$: 

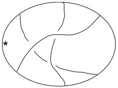
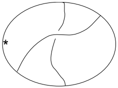
The analogue of the associativity is to say that $(T \circ_3 \tilde{T})(a,b,c,d) = T(a,b,\tilde{T}(c,d))$ for all a,b,c,d.

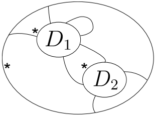
Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

Examples

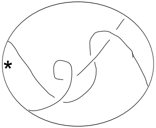
The set of tangles.

The set of knots with possibly close loops:

Consider the two following knots:  and  .

The action of the tangle 

on them is the

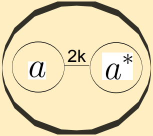
folowing knot:  .

A definition of a planar algebra

Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

Definition

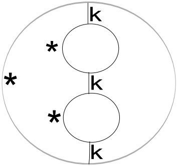
$P = \{P_k\}_{k\in\mathbb{N}}$ is a planar algebra if:
P_k is a $\mathbb{C}-$vector space with an involution $*$.
The tangles act multilinearly on the P_k and respect the involution $*$ in the following way:
$T(a_1,\dots,a_n)^* = \phi(T)(a_1{}^*,\dots,a_n{}^*)$ for any homeomorphism reversing the orientation ϕ, any tangle T and any element a_j of the P_{k_j}.
P is non degenerate:For any $a\in P_k$, $a = 0$ if and only if

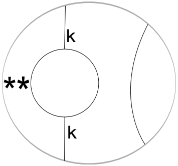


is zero.

Consequence

$\bigcup_k P_k$ has a structure of graded involutive $\mathbb{C}-$ algebra with the multiplication on P_k and the inclusion between P_k and P_{k+1} given by the two tangles





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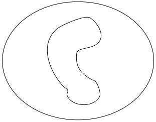
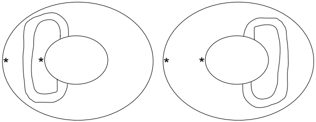
Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

Subfactor planar algebra

We add some other axioms:
$\forall k, P_k$ is finite dimensional.
$P_0 \simeq \mathbb{C}$.

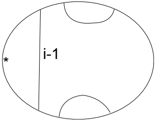
The close loop  is a real number strictly bigger than 1

P is spherically invariant,
i.e. We can deform tangle in the sphere S^2 instead of $\mathbb{R}^2$ without change the action. For example, the two following tangles act in the same way: 

Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

Example

The algebra of Temperley-Lieb which is generate by the set of projections $\{e_i, i \in \mathbb{N}\}$ such that:
$e_i e_j e_i = \delta^{-1} e_i$ if $\mid i-j \mid \leqslant 1$.
$e_i e_j = e_j e_i$ if $\mid i-j \mid \geqslant 2$.
We have the pictural representation of this algebra with δe_i

equal to 

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Remark:The axiom of non degeneracy of the planar algebra implies that we must have $\delta \in \{4\cos^2 \frac{\pi}{n}, n \leqslant 3\} \cup [4; \infty[$.

Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

Review

Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

A subfactor $N \subset M$ is a unital inclusion of II_1 factor (von Neumann algebra with a unique normal trace).

We suppose here that M is a finite $N-$module (i.e. the Jones-index $[M; N]$ is finite).

We suppose also that $N \subset M$ is extremal (i.e. the two traces of N' and M are equal on $N' \cap M$).

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Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

Planar algebra:an invariant for subfactor

Let $N \subset M$ be a subfactor with finite index and extremal.
Let $L^2(M)$ be the completion of M for the scalar product given by the trace.
The basic construction of $N \subset M$ is the von Neumann subalgebra of $\mathcal{L}(L^2(M))$ generated by M and the projection on $L^2(N)$ inside $L^2(M)$, we denote it by M_1.
M_1 is a II_1 factor $M \subset M_1$ is a subfactor with finite index.
We iterate this process and obtain what is called the Jones's tower:$N \subset M \subset M_1 \subset M_2 \subset \dots$.

Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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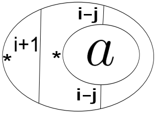
Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

Consider the set of the relative commutants $\{M_i' \cap M_j, i \leqslant j\}$. If we denote by P_k the algebra $N' \cap M_{k-1}$ then $P = \{P_k\}_k$ has a structure of a subfactor planar algebra and $M_i' \cap M_j$ correspond to element of this form:  .

Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

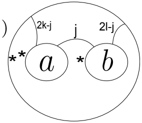
Planar algebra gives subfactor

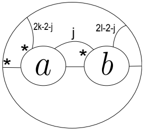
To a subfactor planar algebra P, we can associate a subfactor $N \subset M$ which has P as invariant.
To prove it we can taking out of P a strucure of a Popa-system and then reconstruct a subfactor.
But there is also a proof which uses only planar techniques

Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

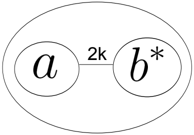
Idea of the proof

Let $P = \{P_k\}_k$ be a subfactor planar algebra.
Consider $\tilde{P}_0 = \bigoplus_k P_k$ and $\tilde{P}_1 = \bigoplus_{k>0} P_k$.
we put a structure of algebra on them:

$\forall (a,b) \in P_k \times P_l$, $a \star_0 b$ is equal to $\displaystyle\sum_{j=0}^{\min(2k,2l)}$  .

$\forall (a,b) \in P_k \times P_l$, $a \star_1 b$ is equal to $\displaystyle\sum_{j=0}^{\min(2k-2,2l-2)}$  .

Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

We put also a scalar product 

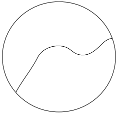
.
Let H_i be the completion of $\tilde{P}_i, i = 0{,}1$ for this scalar product.
The multiplication is continuous for the scalar product, so we can consider the left regular representation:

$$\begin{array}{cccc} \lambda_i & \tilde{P}_i & \longrightarrow & \mathcal{L}(H_i) \\ & a & \longmapsto & (b \mapsto a \star_i b) \end{array}$$

and take the bicomutant to obtain von Neumann algebra:$Q_i = \lambda_i(\tilde{P}_i)''$.
We get the subfactor $Q_0 \subset Q_1$ and it has P for invariant.

MASA generated by a Temperley-Lieb's element

Consider M the II_1 which we denoted by Q_0 in the previous page.
Inside M we can consider the algebra of Temperley-Lieb, which is the image of Tangles without subdiscs.

Consider the simplest element  .
Let A be the von Neumann algebra generated by it.
A is abelian because it is generated by a self-adjoint element and in fact, it is a MASA (maximal abelian self-adjoint subalgebra) of M.

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Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

Properties of $A \subset M$

$A \subset M$ is maximal injective.
So, this implies that for any subfactor planar algebra P, the corresponding M is not hyperfinite.
$A \subset M$ has got a Pukanszky invariant equal to infinity, in particular the MASA is singular and the action of A on $L^2(M)$ is reduced to the trivial and the coarse one,
so the Hilbert space $L^2(M)$ can be decomposed, in terms of $A - A$-bimodule, in sum of $L^2(A)$ and infinite copy of $L^2(A) \otimes L^2(A)$.

Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

Properties of $A \subset M$

With the planar expression of a dense subalgebra of M we can find explicit disintegrations of the regular left action of a dense C^*-subalgebra of M with respect to the right regular action of A.
In particular we can compute the invariant of Takesaki about equivalence class of representation inside disintegration, in that case the masa $A \subset M$ is said to be "simple" because the representations are all distinct almost everywhere.

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Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

Study of free group factor

The previous construction of factors gives sometime free group factor (interpolated or not).

So take a good planar algebra, you get a masa in the free group factor with all of the previous property.

In particular we can create the generator masa $L(\mathbb{Z}) * 1 \subset L(\mathbb{Z}) * L(\mathbb{Z})$.

The question of isomorphie between them seems not obvious because they've got all the same invariant.

But the planar point of view could suggest new invariant and maybe help to solve the problem of isomorphie between the generator and radial masa.

Planar Algebras and Subfactors
Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor

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Arnaud Brothier
What is a planar algebra?
Planar calculus
Tangle
Planar algebra
Connection with subfactor
Subfactor
An invariant
Planar algebra gives subfactor
A canonical MASA (maximal abelian self-adjoint subalgebra)
Description
Properties
Free group factor