Short Distance Analysis of Superselection Charges

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Recent Advances in Operator Algebras

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Motivations

- Intrinsic understanding of charge confinement concept in Quantum Field Theory.
- D. Buchholz: confined charges are charges of the scaling limit theory which are not also charges of the underlying theory.
- Problem: find canonical way to compare the charge structures of the two theories.
- In particular: identify charges of the underlying theory preserved in the scaling limit.
- Idea: characterize preservance of charges by the scaling behaviour of the associated charged fields.

Scaling Algebras

Data:

- $O \subset \mathbb{R}^4 \to \mathscr{A}(O) \subset B(\mathscr{H})$ net of observable algebras.
- $x \in \mathbb{R}^4 \to U(x)$ unitary representation of translations.
- $\Omega \in \mathscr{H}$ vacuum.

On C*-algebra of bounded functions $\lambda \in \mathbb{R}_+^\times \to \underline{A}_\lambda \in \mathscr{A}$

$$\underline{\alpha}_x(\underline{A})_\lambda := \mathrm{Ad}\, U(\lambda x)(\underline{A}_\lambda), \qquad x \in \mathbb{R}^4,$$

Definition ([Buchholz-Verch'95])

Local scaling algebra of O:

$$\underline{\mathfrak{A}}(O) := \left\{ \underline{A} \,:\, \underline{A}_\lambda \in \mathscr{A}(\lambda O), \lim_{x \to 0} \|\underline{\alpha}_x(\underline{A}) - \underline{A}\| = 0 \right\}$$

Scaling Algebras

φ locally normal state on $\mathscr{A} \rightsquigarrow \underline{\varphi}_\lambda(\underline{A}) := \varphi(\underline{A}_\lambda)$ states on $\underline{\mathfrak{A}}$,

$$\mathrm{SL}^{\mathscr{A}}(\varphi) := \{\text{weak* limit points of } (\underline{\varphi}_\lambda)_{\lambda>0} \text{ for } \lambda \to 0\}.$$

Theorem ([Buchholz-Verch'95])

- $\mathrm{SL}^{\mathscr{A}}(\varphi)$ *is independent of* φ.
- $\underline{\omega}_0 \in \mathrm{SL}^{\mathscr{A}}$ *with GNS representation* π_0. *Then* $\mathscr{A}_0(O) := \pi_0(\underline{\mathfrak{A}}(O))''$ *is a covariant net in vacuum representation.*

$O \to \mathscr{A}_0(O)$ is the scaling limit net of $\mathscr{A}$.
Physical interpretation: $\mathscr{A}_0$ describes the short-distance (i.e. high-energy) behaviour of $\mathscr{A}$.

Superselection Theory

Described by classes of localized endomorphisms:

$$\Delta(O) := \{\rho \in \mathrm{End}(\mathscr{A}) \ : \ \rho(A) = A \ \forall A \in \mathscr{A}(O')\}$$

Theorem ([Doplicher-Roberts'90])

$\exists O \to \mathscr{F}(O)$ field net, $g \in G \to V(g)$, G compact, such that:

- *$\mathscr{F}(O)^G = \mathscr{A}(O)$;*
- *$\forall \rho \in \Delta(O) \ \exists \psi_1, \dots, \psi_d \in \mathscr{F}(O)$ orthogonal isometries, $v_{[\rho]}$ d-dimensional irrep of G, with*

$$\mathrm{Ad}\, V(g)(\psi_i) = \sum_{j=1}^{d} v_{[\rho]}(g)_{ij}\psi_j, \quad \rho(A) = \sum_{j=1}^{d} \psi_j A \psi_j^*.$$

Scaling limit and preservation of DHR sectors

Field scaling algebra $\underline{\mathfrak{F}}$ and scaling limit field net $\mathscr{F}_0$ defined in analogy to $\underline{\mathfrak{A}}$, $\mathscr{A}_0$.
$\exists G_0 = G/N_0$ such that $\mathscr{F}_0(O)^{G_0} = \mathscr{A}_0(O)$.
General situation:

$$\begin{array}{ccc} \mathscr{A} & \xrightarrow{\text{DR}} & \mathscr{F} \\ {\scriptstyle\text{SL}}\downarrow & & \downarrow{\scriptstyle\text{SL}} \\ \mathscr{A}_0 & \xrightarrow[\text{DR}]{} & \mathscr{F}^0 \supseteq \mathscr{F}_0 \end{array}$$

$\mathscr{F}^0 \supsetneq \mathscr{F}_0 \implies \mathscr{A}$ has confined charges.
E.g. in the Schwinger model $\mathscr{F} = \mathscr{A} \implies \mathscr{F}_0 = \mathscr{A}_0 \subsetneq \mathscr{F}^0$

Scaling limit and preservation of DHR sectors

Which sectors *survive* the scaling limit?

Physical picture $\rightsquigarrow$ pointlike charges survive.

- $\psi_j(\lambda) \in \mathscr{F}(\lambda O)$ of class $[\rho] \implies \psi_j(\lambda)\Omega$ charge $[\rho]$ in λO.
- $[\rho]$ pointlike $\implies$ energy of $\psi_j(\lambda)\Omega \sim \lambda^{-1}$.

Theorem ([D'Antoni-M.-Verch'04])

With $\psi_j(\lambda)$ *as above and*

$$(\underline{\alpha}_h \psi_j)_\lambda := \int_{\mathbb{R}^4} dx\, h(x) \mathrm{Ad}\, U(\lambda x)(\psi_j(\lambda)),$$

there exists

$$\psi_j^0 = \mathrm{s}^*\text{-}\lim_{h \to \delta} \pi_0(\underline{\alpha}_h \psi_j) \in \mathscr{F}_0(O)$$

and ψ_j^0 *is a* G_0*-multilplet which implements a DHR sector of* $\mathscr{A}_0$*.*

Scaling limit and preservation of BF sectors

- For BF sectors $\mathscr{F}(O) \to \mathscr{F}(C)$, C spacelike cone.
- Asymptotically free theories $\rightsquigarrow$ charges in cone become localized in the scaling limit.
- $\underline{\mathfrak{F}}(C; O)$, $O \subset C$ defined by functions $\lambda \to \underline{F}_\lambda \in \mathscr{F}(\lambda C)$ such that $\lim_{\lambda \to 0} \sup_{\underline{A} \in \underline{\mathfrak{A}}(O')_1} \|[\underline{F}_\lambda, \underline{A}_\lambda]\| = 0$.
- $\mathscr{F}_0(O) := \bigcap_{C \supset O} \pi_0(\underline{\mathfrak{F}}(C; O))''$.
- Preservation notion similar to DHR case.

Scaling limit of tensor product theories

- Define $\Theta_{\beta,O} : \mathscr{F}(O) \to \mathscr{H}$, $\Theta_{\beta,O}(F) = e^{-\beta H} F\Omega$.
- $\mathscr{F}$ is asymptotically p-nuclear, $p \in (0,1]$, if

$$\limsup_{\lambda \to 0^+} \|\Theta_{\lambda\beta,\lambda O}\|_p < \infty$$

$\| \cdot \|_p$ = nuclear p-norm.

Theorem ([D'Antoni-M.'06])

$\mathscr{F}^{(i)}$ asymptotically p-nuclear, $\mathscr{F}_0^{(i)}$ with Haag duality, $i = 1, 2$, $\mathscr{F} := \mathscr{F}^{(1)} \otimes \mathscr{F}^{(2)}$. Then

$$\mathscr{F}_0^{(1)} \otimes \mathscr{F}_0^{(2)} \cong \mathscr{F}_0.$$

Construction of the Models

- G_1, G_2 compact Lie groups.
- ϕ_k G_1-multiplet of generalized free fields with $d\rho(m) = dm$.
- φ_k G_2-multiplet of free fields.
- $\mathscr{F}^{(1)}(O)$ generated by $\phi_k(\Box^{n(O)} f)$, $\operatorname{supp} f \subset O$, $n(O) \to +\infty$ as $O \to \{pt\}$.
- $\mathscr{F}^{(2)}(O)$ generated by $\varphi_k(f)$, $\operatorname{supp} f \subset O$.

Theorem ([Lutz'97, D'Antoni-M.'06])

Let $\mathscr{F} := \mathscr{F}^{(1)} \otimes \mathscr{F}^{(2)}$ with $G = G_1 \times G_2$. Then $\mathscr{F}_0^{(1)} = \mathbb{C}\mathbb{1}$ and $\mathscr{F}_0 = \mathscr{F}_0^{(2)}$, and all the G_2-sectors of $\mathscr{F}$ are preserved.

Summary

- Scaling algebras methods can be used to study the short distance properties of superselection charges.
- Intrinsic notion of confinement: A confined charge of $\mathscr{A}$ is a (DHR) sector of $\mathscr{A}_0$ which doesn't come from a preserved (DHR or BF) sector of $\mathscr{A}$.
- It is difficult to exclude the appearence of non-preserved sectors on the basis of standard assumptions.
- Outlook
    - Can Haag duality rule out non-preserved sectors?
    - Construct examples of preserved BF sectors.

References I

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