Scaling Algebras for Charge Carrying Quantum Fields and Superselection Structure at Short Distances

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Motivations

Main motivation: intrinsic understanding of charge confinement phenomenon in Quantum Field Theory.

Conventional notion of confinement is based on attaching a physical interpretation to unobservable degrees of freedom in the lagrangian.

D. Buchholz [Nucl. Phys. B **469** (1996)]: confined charges are those charges of the scaling limit theory which are not also charges of the underlying theory.

Intrinsic: scaling limit construction and superselection (i.e. charge) structure canonically determined by the net of observables.

Example [BV, RMP **11** (1998)]: Schwinger model (massless QED_2)

Problem: find a canonical way to identify charges of the scaling limit as coming from charges of the underlying theory which are *preserved* in the limit.

Then a confined charge is a charge of the scaling limit which doesn't come from a preserved charge.

Idea: try characterizing the preservance of charges by the scaling behaviour of the fields carrying them.

$\Longrightarrow$ study the scaling limit for charge carrying fields.

Scaling algebras and scaling limit

$(\mathscr{H}, \mathfrak{A}, U, \Omega)$ a Poincaré covariant local net, i.e.

(i) $\mathscr{O} \to \mathfrak{A}(\mathscr{O})$ local net of C*-algebras on $\mathscr{H}$;

(ii) $U : \mathscr{P}_+^\uparrow \to \mathscr{U}(\mathscr{H})$ unitary repn such that

$$U(s)\mathfrak{A}(\mathscr{O})U(s)^* = \mathfrak{A}(s \cdot \mathscr{O}), \qquad s \in \mathscr{P}_+^\uparrow,$$

and $\mathrm{Sp}\, U(\mathbb{1}, \cdot) \subseteq \overline{V}_+$;

(iii) $\Omega \in \mathscr{H}$ unique vacuum, $\overline{\mathfrak{A}\Omega} = \mathscr{H}$;

$(R_\lambda)_{\lambda>0}$ RG transformations. RG orbits $\lambda \to R_\lambda(A)$ occupy a fixed phase space volume at all scales $\iff$

$$\lim_{(\Lambda,x)\to(\mathbb{1},0)} \sup_{\lambda>0} \|\alpha_{(\Lambda,\lambda x)}(R_\lambda(A)) - R_\lambda(A)\| = 0$$

[BV, RMP **7** (1995)]

$\underline{A} : \lambda \to \underline{A}(\lambda) \in \mathfrak{A}$, morally $\underline{A}(\lambda) = R_\lambda(A)$

$$\|\underline{A}\| := \sup_{\lambda>0} \|\underline{A}(\lambda)\|$$

$$\underline{\alpha}_s(\underline{A})(\lambda) := \alpha_{s_\lambda}(\underline{A}(\lambda)), \quad (\Lambda, x)_\lambda := (\Lambda, \lambda x)$$

Definition. The *local scaling algebra* of $\mathscr{O}$ is

$$\underline{\mathfrak{A}}(\mathscr{O}) := \{\lambda \in \mathbb{R}_+ \to \underline{A}(\lambda) \in \mathfrak{A}(\lambda\mathscr{O}) \; : \; \|\underline{A}\| < +\infty, \; \lim_{s\to e} \|\underline{\alpha}_s(\underline{A}) - \underline{A}\| = 0\}.$$

φ (locally normal) state on $\mathfrak{A} \rightsquigarrow \left(\underline{\varphi}_\lambda\right)_{\lambda>0}$ states on $\underline{\mathfrak{A}}$:

$$\underline{\varphi}_\lambda(\underline{A}) := \varphi(\underline{A}(\lambda))$$

$$\mathrm{SL}_{\mathfrak{A}}(\varphi) := \left\{\text{weak* limit pts of } \left(\underline{\varphi}_\lambda\right)_{\lambda>0}\right\}.$$

Theorem.[BV95] $\mathrm{SL}_{\mathfrak{A}}(\varphi)$ is independent of φ. For $(\pi_0, \mathscr{H}_0, \Omega_0)$ and U_0 the GNS and $\mathscr{P}_+^\uparrow$ representations determined by $\underline{\omega}_0 \in \mathrm{SL}_{\mathfrak{A}}$, and with $\mathfrak{A}_0(\mathscr{O}) := \pi_0(\underline{\mathfrak{A}}(\mathscr{O}))$, $(\mathscr{H}_0, \mathfrak{A}_0, U_0, \Omega_0)$ is a Poincaré covariant local net (if $d = 2$ the vacuum may not be pure).

$\mathfrak{A}_0$ is called a scaling limit net of $\mathfrak{A}$. Possibilities:

(i) degenerate scaling limit: the various $\mathfrak{A}_0$ non-isomorphic (cfr. theories without ultraviolet fixed points);

(ii) unique (quantum) scaling limit: the various $\mathfrak{A}_0$ isomorphic and non-trivial (e.g. scalar field in $d = 3, 4$ [BV98], dilation invariant theories);

(iii) classical scaling limit: each $\mathfrak{A}_0(\mathscr{O}) = \mathbb{C}\mathbb{1}$, (linked to exceptional quantum behaviour of observables at small scales as in e.g. non-renormalizable theories).

Superselection theory

Superselection sectors are equivalence classes of irreps of $\mathfrak{A}$ describing localized excitations of the vacuum.

Definition.[DHR, CMP **23** (1971)] A representation π is DHR (or describes a *localizable charge*) if

$$\pi \restriction \mathfrak{A}(\mathscr{O}') \cong \iota \restriction \mathfrak{A}(\mathscr{O}'), \qquad \forall \mathscr{O}, \tag{1}$$

with ι the vacuum (identical) representation.

Excludes states carrying electric charge, due to Gauss' law. Also, in massive theories the most general localization property for charges is

Definition.[BF, CMP **84** (1982)] A representation π is BF (or describes a *topological charge*) if

$$\pi \restriction \mathfrak{A}(\mathscr{C}') \cong \iota \restriction \mathfrak{A}(\mathscr{C}')$$

for all spacelike cones $\mathscr{C} = a + \mathbb{R}_+\mathscr{O}$, $\mathscr{O} \subseteq \{0\}'$, $\mathscr{C}$ causally complete.

Expected to show up in nonabelian gauge theories (cone = flux string).

Superselection structure can be described by localized endomorphisms:

$$\Delta := \{\rho \in \mathrm{End}(\mathfrak{A}) \,:\, \rho(A) = A \,\forall A \in \mathfrak{A}(\mathscr{O}')\}$$

Charge carrying unobservable fields (e.g. fermionic fields) are encoded in $(\mathfrak{A}, \Delta)$.

Theorem.[DR, CMP **131** (1990)] $\exists\, \mathscr{O} \to \mathscr{F}(\mathscr{O})$, $g \in G \to V(g)$, G compact, such that

(i) $\mathscr{F}$ $\tilde{\mathscr{P}}_+^\uparrow$-covariant net with normal commutation relations;

(ii) $\mathscr{F}(\mathscr{O})^G = \mathscr{A}(\mathscr{O}) := \mathfrak{A}(\mathscr{O})''$;

(iii) $\forall\, \rho \in \Delta(\mathscr{O})$ irr. $\exists\, \psi_1, \dots \psi_d \in \mathscr{F}(\mathscr{O})$, $v_{[\rho]}$ d-dim. irrep of G, such that

$$\psi_i^* \psi_j = \delta_{ij} \mathbb{1}, \qquad \sum_{j=1}^{d} \psi_j \psi_j^* = \mathbb{1},$$

$$\beta_g(\psi_i) = \sum_{j=1}^{d} v_{[\rho]}(g)_{ij} \psi_j,$$

$$\rho(A) = \sum_{j=1}^{d} \psi_j A \psi_j^*, \qquad A \in \mathfrak{A}.$$

For BF sectors, $\mathscr{O} \to \mathscr{F}(\mathscr{O})$ replaced by $\mathscr{C} \to \mathscr{F}(\mathscr{C})$.

Scaling algebras for localized fields and preservance of DHR charges

Field scaling algebra $\underline{\mathfrak{F}}$, scaling limit field net $\mathscr{F}_0$ defined in analogy to $\underline{\mathfrak{A}}$, $\mathfrak{A}_0$, and $\mathscr{F}_0(\mathscr{O})^G = \mathscr{A}_0(\mathscr{O}) := \mathfrak{A}_0(\mathscr{O})''$.

Charges may disappear in the scaling limit. E.g. classical scaling limit $\implies$ all charges disappear.

Idea: study the behaviour, for $\lambda \to 0$, of *scaled multiplets* $\psi_1(\lambda), \ldots \psi_d(\lambda) \in \mathscr{F}(\lambda\mathscr{O})$ associated to a fixed sector ξ.

In general $\psi_j(\cdot) \notin \underline{\mathfrak{F}}(\mathscr{O})$.

Definition. DHR sector ξ is *preserved in the scaling limit* $\mathscr{F}_0$ if $\forall \mathscr{O}_1$, $\exists\, \psi_1(\lambda), \ldots, \psi_d(\lambda) \in \mathscr{F}(\lambda\mathscr{O}_1)$ multiplet of class ξ, such that $\forall \varepsilon > 0$, $\exists \underline{F}_j, \underline{F}'_j \in \underline{\mathfrak{F}}$ such that

$$\begin{aligned}\limsup_{\lambda\to 0} & \|[\psi_j(\lambda) - \underline{F}_j(\lambda)]\Omega\| + \\ & \|[\psi_j(\lambda) - \underline{F}'_j(\lambda)]^*\Omega\| < \varepsilon.\end{aligned}$$

Energy of $\psi_j(\lambda)^*\Omega$ restricted only by uncertainty principle $\implies$ ξ "pointlike" charge $\implies$ ξ survives the scaling limit.

Satisfied in the Majorana-Dirac free field and in dilation invariant theories.

$h \in C_c(\tilde{\mathscr{P}}_+^\uparrow)$, $h \geq 0$, $\int_{\tilde{\mathscr{P}}_+^\uparrow} h = 1$

$$\underline{\alpha}_h \psi_j(\lambda) := \int_{\tilde{\mathscr{P}}_+^\uparrow} ds\, h(s) \alpha_{s_\lambda}(\psi_j(\lambda)),$$

$\underline{\alpha}_h \psi_j \in \underline{\mathfrak{F}}(\mathscr{O})$, $\forall \mathscr{O} \supset \overline{\mathscr{O}_1}$.

Theorem. There exists

$$\boldsymbol{\psi}_j := {}^*s\text{-}\lim_{h\to\delta} \pi_0(\underline{\alpha}_h \psi_j) \in \mathscr{F}_0(\mathscr{O})$$

for each $\mathscr{O} \supset \overline{\mathscr{O}}_1$, and

$$\boldsymbol{\psi}_i^* \boldsymbol{\psi}_j = \delta_{ij} \mathbb{1}, \qquad \sum_{j=1}^d \boldsymbol{\psi}_j \boldsymbol{\psi}_j^* = \mathbb{1},$$

$$V_0(g) \boldsymbol{\psi}_i V_0(g)^* = \sum_{j=1}^d v_\xi(g)_{ij} \boldsymbol{\psi}_j,$$

Furthermore $\boldsymbol{\rho}(\boldsymbol{a}) := \sum_{j=1}^d \boldsymbol{\psi}_j \boldsymbol{a} \boldsymbol{\psi}_j^*$, $\boldsymbol{a} \in \mathfrak{A}_0$, is a localized endomorphism of $\mathfrak{A}_0$.

Scaling algebras for fields in cones and preservance of BF charges

If quarks are non-confined, they are localized in cones $\Longrightarrow$ above analysis too narrow

Spacelike cones not affected by rescaling $\Longrightarrow$ unclear how to implement the RG phase space.

In asymptotically free theories charges in cones become localized in the scaling limit (the flux string vanishes) $\Longrightarrow$ phase space recoverd asymptotically.

$\mathscr{C} \to \mathscr{F}(\mathscr{C})$ canonical DR field net determined by BF sectors.

A bounded function $F : \mathbb{R}_+^\times \to \mathscr{F}$ is asymptotically localized in $\mathscr{O}$ if

$$\lim_{\lambda \to 0} \sup_{A \in \underline{\mathfrak{A}}(\mathscr{O}')_1} \|[F(\lambda), \underline{A}(\lambda)]\| = 0.$$

Definition. We introduce the C*-algebras, for $\mathscr{O} \subset \mathscr{C}$,

$$\begin{aligned} \underline{\mathfrak{F}}(\mathscr{C}, \mathscr{O}) := \{\lambda \in \mathbb{R}_+^\times \to \underline{F}(\lambda) \in \mathscr{F}(\lambda\mathscr{C}) \; : \; &\|\underline{F}\| < +\infty \\ &\lim_{s \to e} \|\underline{\alpha}_s(\underline{F}) - \underline{F}\| = 0, \\ &\underline{F} \text{ asymptotically localized in } \mathscr{O}\}, \end{aligned}$$

φ normal state on $B(\mathscr{H}_{\mathscr{F}}) \rightsquigarrow \mathrm{SL}_{\underline{\mathfrak{F}}}(\varphi)$ defined as above.

Theorem. $\mathrm{SL}_{\underline{\mathfrak{F}}}(\varphi)$ is independent of φ. For $\underline{\omega}_0 \in \mathrm{SL}_{\underline{\mathfrak{F}}}$, let $(\pi_0, \mathscr{H}_0, \Omega_0)$ be the GNS representation. Then

$$\mathscr{F}_0(\mathscr{O}) := \bigcap_{\mathscr{C} \supset \mathscr{O}} \pi_0(\underline{\mathfrak{F}}(\mathscr{C}, \mathscr{O}))'',$$

defines a field net with normal commutation relations, called a *scaling limit* net of $\mathscr{F}$.

The direction of the cone is irrelevant $\implies$ each choice of direction should give the same states in the limit.

Definition. BF sector ξ is *preserved in the scaling limit* $\mathscr{F}_0$ if $\forall\, \mathscr{O}_1$, $\mathscr{C}_1 \supset \mathscr{O}_1$, $\exists\, \psi_1(\lambda), \dots, \psi_d(\lambda) \in \mathscr{F}(\lambda \mathscr{C}_1)$ multiplet of class ξ asymptotically localized in $\mathscr{O}_1$, and $\forall\, \mathscr{C} \supset \mathscr{O}_1$ $\exists\, \lambda \to \psi_j^{\mathscr{C}}(\lambda) \in \mathscr{F}(\lambda \mathscr{C})$ asymptotically localized in $\mathscr{O}_1$, such that

(i) $\lim_{\lambda \to 0} \|[\psi_j^{\mathscr{C}}(\lambda) - \psi_j(\lambda)]\Omega\| + \|[\psi_j^{\mathscr{C}}(\lambda) - \psi_j(\lambda)]^*\Omega\| = 0;$

(ii) $\forall \varepsilon > 0$, $\exists \underline{F}_j^{\mathscr{C}}, \underline{F}_j^{\mathscr{C}\prime} \in \underline{\mathfrak{F}}^{\times}$ such that

$$\limsup_{\lambda \to 0} \|[\psi_j^{\mathscr{C}}(\lambda) - \underline{F}_j^{\mathscr{C}}(\lambda)]\Omega\| + \|[\psi_j^{\mathscr{C}}(\lambda) - \underline{F}_j^{\mathscr{C}\prime}(\lambda)]^*\Omega\| < \varepsilon.$$

Theorem. There exists

$$\boldsymbol{\psi}_j := {}^*\boldsymbol{s}\text{-}\lim_{h\to\delta} \pi_0(\underline{\alpha}_h \psi_j^{\mathscr{C}})$$

independent of $\mathscr{C}$, so that $\boldsymbol{\psi}_j \in \mathscr{F}_0(\mathscr{O})$ for each $\mathscr{O} \supset \overline{\mathscr{O}}_1$, and $\boldsymbol{\psi}_j$ is a G-multiplet of class ξ of orthogonal isometries of support $\mathbb{1}$. Furthermore if

$$\omega_\xi(\boldsymbol{a}) := \sum_{j=1}^{d} (\Omega_0 | \boldsymbol{\psi}_j \boldsymbol{a} \boldsymbol{\psi}_j^* \Omega_0), \qquad \boldsymbol{a} \in \mathfrak{A}_0,$$

and if $\mathscr{H}_0$ is separable, the GNS representation π_ξ is DHR.

Conclusions and outlook

"Pointlike" charges have no scale $\Longrightarrow$ should appear both at finite scale and in scaling limit.

According to uncertainty principle, they require energy of order λ^{-1} to be localized in regions of radius λ.

Can be expressed by conditions on scaled multiplets $\lambda \to \psi_j(\lambda) \Longrightarrow$ multiplet of the same class appear in the scaling limit $\Leftrightarrow$ charge is preserved.

Further developements:

- mathematical structure of preserved sectors;
- anomalous charge scaling;
- study of BF sectors in lattice gauge models.