On the Scaling Limit of Local Nets Arising from Factorizing S-Matrices

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joint work with H. Bostelmann, G. Lechner
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Modern Trends in Algebraic Quantum Field Theory

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Outline

Introduction 1/2

Scaling algebra: model independent, intrinsic approach to the analysis of UV behaviour of QFT within the algebraic approach [Buchholz-Verch '95].

Main results:

- General analysis of scaling behavior of superselection charges and intrinsic concept of charge confinement [Buchholz '96, D'Antoni-M.-Verch '04, Conti-M. '09]
- Model independent understanding of pointlike field renormalization [Bostelmann-D'Antoni-M. '09]
- Connections with quantum Gromov-Hausdorff metric [Bostelmann-Guido-Suriano, in progress] and with Connes-Higson asymptotic morphisms [Conti-M., in progress]
- Applications to concrete models: free scalar field of mass $m > 0$ in $d = 3, 4$, Schwinger model $\iff$ free scalar field of mass $m > 0$ in $d = 2$ [Buchholz-Verch '97], certain generalized free fields [Lutz '97, Mohrdieck '02, D'Antoni-M. '07]

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What about (more interesting) interacting models?
Problem: main tool of constructive theory is field-theoretic Euclidean approach, relation with algebraic Minkowskian approach is rather indirect

But:
an approach to construction of interacting models directly in the algebraic framework has been proposed in [Buchholz-Lechner '04], and the construction of a class of 2d integrable models has been performed in [Lechner '08].
Interesting: 2d sigma models (integrable, but not directly covered here) share several features with QCD, e.g. asymptotic freedom

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Main Idea

Natural data for a constructive approach in AQFT: particle spectrum and S-matrix.
In 2d there exists a family of simple S-matrices (factorizing S-matrices) described by an appropriate single analytic function S, the scattering function. Arise in integrable models (Sin(h)-Gordon, Ising, Thirring...)
Form factor program: form factors (i.e. matrix elements) of local fields determined by axioms, n-point functions obtained as series. But convergence is not under control.

Strategy of algebraic approach:

1. Define auxiliary fields and algebras associated to unbounded regions (wedges)
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Scattering Functions

Definition

A scattering function is a bounded $S : \mathbb{R} + i[0, \pi] \to \mathbb{C}$, analytic in the interior, such that

$$\overline{S(\theta)} = S(\theta)^{-1} = S(\theta + i\pi) = S(-\theta), \qquad \theta \in \mathbb{R}$$

S is regular if it is analytic and bounded in $\mathbb{R} + i(-\kappa, \pi + \kappa)$, $\kappa > 0$.
S regular has limit if $\lim_{\theta \to +\infty} S(\theta) = \lim_{\theta \to -\infty} S(\theta)$.

$\mathcal{S}_0 :=$ {regular S}, $\mathcal{S}_\infty :=$ {regular S with limit}

Example of $S \in \mathcal{S}_\infty$ (essentially all):

$$S(\theta) = \pm \prod_{k=1}^{N} \frac{\sinh\theta - ib_k}{\sinh\theta + ib_k}, \qquad b_k > 0$$

Result: $S \in \mathcal{S}_\infty \implies S(\infty) := \lim_{\theta \to +\infty} S(\theta) = \lim_{\theta \to -\infty} S(\theta) = \pm 1$.

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Scaling of Scattering Functions

Rapidity θ related to 2-momentum by $p(\theta) = m(\cosh\theta, \sinh\theta)$ for $m > 0$ (mass)

In order to study scaling $p \to \lambda^{-1}p$ define

$$S_m(p,q) := S\big(\sinh^{-1}(p/m) - \sinh^{-1}(q/m)\big) \qquad p,q \in \mathbb{R}$$
$$S_m(\lambda^{-1}p, \lambda^{-1}q) = S_{\lambda m}(p,q)$$

Lemma

If $S \in \mathcal{S}_\infty$

$$S_0(p,q) := \lim_{\lambda\to 0^+} S_{\lambda m}(p,q) = \begin{cases} S(\infty) = \pm 1 & pq < 0 \\ S(0) & p = q = 0 \\ S\big(\log p - \log q\big) & p > 0, q > 0 \\ S\big(\log(-q) - \log(-p)\big) & p < 0, q < 0 \end{cases}$$

Guess: Chiral theory in the limit with some "interaction" (see later)

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Wedge Local Fields and Algebras 1/2

S-symmetric Fock space $\mathscr{H}_m := \bigoplus_{n=0}^{+\infty} \mathscr{H}_{m,n}$, where $\mathscr{H}_{m,n} \subset L^2(\mathbb{R}, dp/\omega_m(p))^{\otimes n}$ is defined by ($m \geq 0$)

$$\Psi_n(p_1, \dots, p_{k+1}, p_k, \dots, p_n) = S_m(p_k, p_{k+1})\Psi_n(p_1, \dots, p_n)$$

Creation-annihilation operators satisfy Zamolodchikov-Faddeev algebra (as distributions):

$$z_m(p)z_m(q) = S_m(p,q)z_m(q)z_m(p)$$
$$z_m(p)z_m^\dagger(q) = S_m(q,p)z_m^\dagger(q)z_m(p) + \omega_m(p)\delta(p-q)$$

Used in [Schroer '97] to define a quantum field

$$\phi_m(x) = \int_{\mathbb{R}} \frac{dp}{2\pi\omega_m(p)}\left(e^{-i(\omega_m(p),p)\cdot x} z_m(p) + e^{i(\omega_m(p),p)\cdot x} z_m^\dagger(p)\right)$$

(to be smeared with appropriate test functions $f \in \mathscr{S}(\mathbb{R}^2)$).

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Wedge Local Fields and Algebras 2/2

ϕ_m is Poincaré covariant but non-local if $S \neq 1$:

$$[\phi_m(f), \phi_m(g)] \neq 0 \qquad \mathrm{supp}\, f \subset (\mathrm{supp}\, g)'$$

Define right/left wedge $W_{R/L} := \{x = (x_0, x_1) \in \mathbb{R}^2 \; : \; x_1 > \pm|x_0|\}$

Algebra associated to ϕ_m and left wedge:

$$\mathscr{M}_{m,L} := \{e^{i\phi_m(f)} \; : \; f \in \mathscr{S}_{\mathbb{R}}(W_L)\}''$$

Theorem ([Lechner '03, Buchholz-Lechner '04])

- *Ω cyclic and separating for $\mathscr{M}_{m,L}$*
- *If (Δ, J) are modular objects for $(\mathscr{M}_{m,L}, \Omega) \implies \Delta^{it}$ are boosts and*

$$(J\Psi)_n(p_1, \ldots, p_n) = \overline{\Psi_n(p_n, \ldots, p_1)}$$

space-time reflection.

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Local Algebras for $m > 0$

Algebra associated to the right wedge:

$$\phi'_m(f) := J\phi_m(f^j)J, \qquad f^j(x) := \overline{f(-x)}$$

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Algebra of double cone $O_{x,y} = (W_L + x) \cap (W_R + y)$, $y - x \in W_L$:

$$\mathscr{A}_m(O_{x,y}) := \alpha^m_x(\mathscr{M}_{m,L}) \cap \alpha^m_y(\mathscr{M}_{m,R})$$

Then $O \mapsto \mathscr{A}_m(O)$ is a local net, i.e. $\mathscr{A}_m(O_1) \subset \mathscr{A}_m(O_2)'$ if $O_1 \subset O'_2$, but it could be that $\mathscr{A}_m(O) = \mathbb{C}1$

Theorem ([Lechner '08])

If $S \in \mathcal{S}_0$ and $m > 0$ then

- *Ω is cyclic and separating for $\mathscr{A}_m(O)$*
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Linked to spectral properties of Δ

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Buchholz-Verch Scaling Limit of $\mathscr{A}_m$

On C*-algebra of bounded functions $\lambda \in \mathbb{R}_+^\times \mapsto \underline{A}_\lambda \in \mathscr{A}_m$ define:

- $\underline{\alpha}_{(\Lambda,x)}(\underline{A})_\lambda := \mathrm{Ad}\, U(\Lambda, \lambda x)(\underline{A}_\lambda)$, $(\Lambda, x) \in \mathscr{P}_+^\uparrow$
- Local scaling algebra of O:

$$\underline{\mathfrak{A}}_m(O) := \left\{ \underline{A} \, : \, \underline{A}_\lambda \in \mathscr{A}_m(\lambda O), \lim_{\gamma \to e} \|\underline{\alpha}_\gamma(\underline{A}) - \underline{A}\| = 0 \right\}$$

- $\underline{\omega}_\lambda(\underline{A}) := \omega(\underline{A}_\lambda)$, $\underline{A} \in \underline{\mathfrak{A}}_m$
- scaling limit states: $\underline{\omega}_0$ weak* limit point of $(\underline{\omega}_\lambda)_{\lambda>0}$ for $\lambda \to 0$
- scaling limit theory: $\mathscr{A}_{m,0}(O) := \pi_0(\underline{\mathfrak{A}}_m(O))''$, with π_0 GNS represenation of $\underline{\omega}_0$

Buchholz-Verch scaling limit always exists and is a local theory (possibly trivial), but actual computation is complicated because local observables are difficult to exhibit explicitly.

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Scaling of Wedge Local Fields

Given $f \in \mathscr{S}(\mathbb{R}^2)$ set $f_\lambda(x) = \lambda^{-2} f(\lambda^{-1} x)$

Theorem

If $f_j \in \mathscr{S}(\mathbb{R}^2)$ are derivatives of test functions then

$$\lim_{\lambda \to 0} \langle \Omega_m, \phi_m^{[l]}(f_{1,\lambda}) \dots \phi_m^{[l]}(f_{n,\lambda}) \Omega_m \rangle = \langle \Omega_0, \phi_0^{[l]}(f_1) \dots \phi_0^{[l]}(f_n) \Omega_0 \rangle$$

Condition $\hat{f}_j(0) = 0$ due to logarithmic infrared divergence of measure $\frac{dp}{\omega_m(p)}$ as $m \to 0$ in $d = 2$

Question: if $\hat{f}(0) \neq 0$, does $|\log \lambda|^{-1/2} \phi_m(f_\lambda)$ tends to a multiple of the identity, as in 2d free scalar field?

Anyway massless model, defined by ϕ_0, is at least a subnet (tensor factor?) of the complete BV scaling limit

Also, massless model is interesting in its own right

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Half-Line Local Fields and Algebras 1/2

Since $S_0(p,q) = \pm 1$ for $pq < 0$, the massless 2d theory has a (twisted) chiral structure

S-symmetric Fock space $\mathscr{H} := \bigoplus_{n=0}^{+\infty} \mathscr{H}_n$, where $\mathscr{H}_n \subset L^2(\mathbb{R}_+, dp/p)^{\otimes n}$ is defined by

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Associated Zamolodchikov operators:

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$$z(p)z^\dagger(q) = S_0(q,p)z^\dagger(q)z(p) + \frac{1}{p}\delta(p-q) \qquad p, q > 0$$

and translation-dilation covariant quantum field

$$\varphi(x) = \int_0^{+\infty} dp\,(e^{-ipx}z(p) + e^{ipx}z^\dagger(p)), \qquad x \in \mathbb{R}$$

to be smeared with test functions $f \in \mathscr{S}(\mathbb{R})$ such that $\hat{f}(0) = 0$.

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$$z(p)z^\dagger(q) = S_0(q,p)z^\dagger(q)z(p) + \frac{1}{p}\delta(p-q) \qquad p, q > 0$$

and translation-dilation covariant quantum field

$$\varphi(x) = \int_0^{+\infty} dp \big(e^{-ipx}z(p) + e^{ipx}z^\dagger(p)\big), \qquad x \in \mathbb{R}$$

to be smeared with test functions $f \in \mathscr{S}(\mathbb{R})$ such that $\hat{f}(0) = 0$.

Half-Line Local Fields and Algebras 1/2

Since $S_0(p,q) = \pm 1$ for $pq < 0$, the massless 2d theory has a (twisted) chiral structure

S-symmetric Fock space $\mathscr{H} := \bigoplus_{n=0}^{+\infty} \mathscr{H}_n$, where $\mathscr{H}_n \subset L^2(\mathbb{R}_+, dp/p)^{\otimes n}$ is defined by

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Wedge Local Fields and Algebras 2/2

Right half-line algebra:

$$\mathscr{M}_+ := \{e^{i\varphi(f)} \,:\, f \in \mathscr{S}_{\mathbb{R}}(0, +\infty)\}''$$

Theorem

- *Ω cyclic and separating for $\mathscr{M}_+$*
- *(Δ, J) modular objects of $(\mathscr{M}_+, \Omega) \implies \Delta^{it} =$ dilation by $e^{2\pi t}$, J reflection*

$$(J\Psi)_n(p_1, \dots, p_n) = \overline{\Psi_n(p_n, \dots, p_1)}$$

Left field and algebra:

$$\varphi'(f) := J\varphi(f^j)J, \qquad f^j(x) := \overline{f(-x)}$$

$$\mathscr{M}_- := \{e^{i\phi'(f)} \,:\, f \in \mathscr{S}_{\mathbb{R}}(-\infty, 0)\}'' = \mathscr{M}_+'$$

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Chiral Local Algebras 1/2

We can define local algebras for finite intervals $I = (a, b)$:

$$\mathscr{A}(I) := \alpha_a(\mathscr{M}_+) \cap \alpha_b(\mathscr{M}_-).$$

This gives a consistent local net ($\mathscr{A}(I) \subset \mathscr{A}(J)'$ if $I \cap J = \emptyset$) of von Neumann algebras on $\mathbb{R}$, translation-dilation-reflection covariant.

- Question 1: How large are the $\mathscr{A}(I)$?
- Question 2: Is this a conformally covariant net?

First result:

Theorem

$A \in \mathscr{A}(I) \implies [A, S(\infty)^N] = 0$, i.e. local operators are even with respect to the particle number

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Chiral Local Algebras 2/2

Proof:

- Take $A \in \mathscr{A}(-1,1)$, $g \in \mathscr{S}(1,+\infty)$, $g' \in \mathscr{S}(-\infty,1)$
- With $g_\lambda(x) := g(e^\lambda x)$ (dilatation of g), there holds for all $\lambda > 0$,

$$\begin{aligned} 0 &= [A, \varphi(g_\lambda)\varphi'(g'_\lambda)] \\ &= [A, z^\dagger(g_\lambda)z^\dagger(g'_\lambda)' + z^\dagger(g_\lambda)z(g'_\lambda)' + z^\dagger(g'_\lambda)'z(g_\lambda) + z(g_\lambda)z(g'_\lambda)'] \\ &+ [A, [z(g_\lambda), z^\dagger(g'_\lambda)']] \end{aligned}$$

- weakly of finite particle vectors:

$$\lim_{\lambda\to\infty} z^\dagger(g_\lambda)z^\dagger(g'_\lambda)' + z^\dagger(g_\lambda)z(g'_\lambda)' + z^\dagger(g'_\lambda)'z(g_\lambda) + z(g_\lambda)z(g'_\lambda)' = 0$$

$$\lim_{\lambda\to\infty} [z(g_\lambda), z^\dagger(g'_\lambda)'] = S(\infty)^N$$

Local Algebras vs. Conformal Symmetry

Theorem

- *The space* $\mathscr{H}_{\mathrm{loc}} := \overline{\mathscr{A}(I)\Omega}$ *is independent of* I.
- *The representation of the translation-dilation-reflection group extends to a representation of the conformal group on* $\mathscr{H}_{\mathrm{loc}}$ *and the net extends to a net on* $\mathbb{S}^1$ *which is covariant under it.*

Follows from [Guido-Longo-Wiesbrock '98]

This leaves the two alternatives:

- Local observables + conformal symmetry, or
- No conformal symmetry and no local observables ($\mathscr{H}_{\mathrm{loc}} = \mathbb{C}\Omega$, $\mathscr{A}(I) = \mathbb{C}\mathbb{1}$).

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Chiral decomposition of 2d Models 1/3

$x_{l/r} := x_0 \pm x_1$ light ray coordinates
$\mathscr{H}_{l/r}, z_{l/r}, \varphi_{l/r}, \mathscr{A}_{l/r}$ copies of $\mathscr{H}, z, \varphi, \mathscr{A}$ associated to left/right light ray

- splitting p integration in ϕ_0 in $(-\infty, 0)$ and $(0, +\infty)$ and making $p \to -p$ in first integral:

$$\phi_0(x) = \frac{1}{2\pi}\Big(\underbrace{\int_0^{+\infty} \frac{dp}{p}(e^{-ipx_l} z_0(-p) + e^{ipx_l} z_0^\dagger(-p))}_{\varphi_l'(x_l)\otimes 1} + \underbrace{\int_0^{+\infty} \frac{dp}{p}(e^{-ipx_r} z_0(p) + e^{ipx_r} z_0^\dagger(p))}_{S(\infty)^{N_l}\otimes\varphi_r(x_r)}$$

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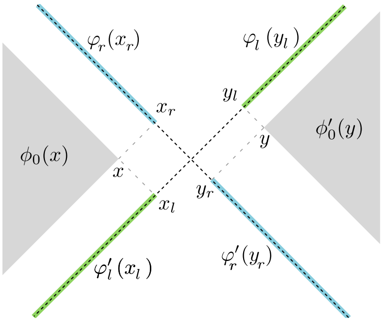
Chiral decomposition of 2d Models 2/3

Theorem

- $\mathscr{H}_0 \cong \mathscr{H}_l \otimes \mathscr{H}_r$
- $z_0^\sharp(-p)/p \cong z_l^\sharp(p)' \otimes \mathbb{1}$, $z_0^\sharp(p)/p \cong S(\infty)^{N_l} \otimes z_r^\sharp(p)$, $p > 0$
- $\phi_0(x) \cong \frac{1}{2\pi}\big(\varphi_l'(x_l) \otimes \mathbb{1} + S(\infty)^{N_l} \otimes \varphi_r(x_r)\big)$



Chiral decomposition of 2d Models 3/3

- $A_{e/o}$ even/odd parts of $A \in B(\mathscr{H})$ w.r.t. $S(\infty)^N$
- for $\mathscr{R}_{l/r}$ vNa on $\mathscr{H}_{l/r}$ define twisted tensor products

$$\mathscr{R}_l \hat{\otimes} \mathscr{R}_r = \mathscr{R}_l \otimes \mathscr{R}_{r,e} + S(\infty)^{N_l} \mathscr{R}_l \otimes \mathscr{R}_{r,o}$$
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Last equality follows from first two and the fact that $\mathscr{A}_{l/r}(I)_o$ is trivial

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Outline

$S = 1$: free U(1) current

For $S = 1$ we have the free U(1) current:

- $\mathscr{H}_{\mathrm{loc}} = \mathscr{H}$
- Ω cyclic and separating for $\mathscr{A}(I)$ (large local algebras)
- conformal symmetry with $c = 1$

$S = -1$: critical Ising model 1/2

If $S = -1$ the Zamolodchikov's algebra is the usual CAR algebra, so one can expect:

Theorem

For $S = -1$ ("critical Ising model"):

- $\mathscr{H}_{\mathrm{loc}} = \{$*states of even particle number*$\} \subsetneq \mathscr{H}$
- $\mathscr{A}(I)$ *generated by energy density of a free Fermi field*

$$\psi(x) := \frac{1}{2\pi} \int_0^{+\infty} dp \sqrt{p} \left(\sqrt{i}\, e^{ipx} z^\dagger(p) + \frac{1}{\sqrt{i}} e^{-ipx} z(p) \right)$$

- *conformal symmetry on $\mathscr{H}_{\mathrm{loc}}$ with $c = 1/2$*

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$S = -1$: critical Ising model 2/2

Proof:

- $\operatorname{supp} f \subset (a,b) \implies \psi(f) \in \mathscr{A}(a,+\infty)$, since $\psi(f) = \varphi(k)$ with $\operatorname{supp} k \subset (a,+\infty)$
- $\{\psi(f), \varphi(g)\} = 0$ if $\operatorname{supp} g \subset (b,+\infty)$
- then $\mathscr{P}_e(a,b)$ (even polynomials in ψ) $\subset \mathscr{A}(a,b)$ and $\mathscr{H}_e := \overline{\mathscr{P}_e(a,b)\Omega} = \mathscr{H}_{\mathrm{loc}}$ (therefore local algebras are non-trivial)
- $T(x) := :\psi\partial\psi:(x)$ is translation-dilations covariant, local, relatively local to $\mathscr{P}_e$ and (weakly on suitable states)

$$\int_{\mathbb{R}} dx\, T(x) = H$$

- then T has Lüscher-Mack commutation relations with $c = 1/2$
- then $Vir_{1/2} \subset \mathscr{A}$ (on $\mathscr{H}_e$) and therefore they coincide by Haag duality

What about non-constant S?

Conjecture

For non-constant S:

- $\mathscr{H}_{\mathrm{loc}} = \mathbb{C}\Omega$
- $\mathscr{A}(I) = \mathbb{C}\mathbb{1}$

Form factors of local observables $A \in \mathscr{A}(I)$

$$F_n(p, q_1, \ldots, q_{n-1}) := \langle z^\dagger(p) z^\dagger(q_1) \ldots z^\dagger(q_{n-1})\Omega, A\Omega\rangle$$

have analytic continuation in p to the upper and lower complex plane, and the "jump" on $\{p < 0\}$ is a distribution determined by S.
But conformal symmetry seems to require F_n to be analytic across the cut.
Compatible only for $A = c\mathbb{1}$.

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Summary & Outlook

Results:

- Massless scattering models can be defined.
  - Observables localized in half-lines
  - Translation-dilation-reflection symmetry
- In the cases $S = \pm 1$, there are local observables and conformal symmetry, maybe on a proper subspace.
- In particular, the model depends on S!
  - Important to know, since scattering theory is not available.

Open points:

- Prove/disprove conjecture about non constant S
- In which sense are the models interacting? Can one measure S?
- Relation to literature (models are often considered in a thermodynamical context – Thermodynamic Bethe Ansatz)
- Relation to the Buchholz-Verch scaling limit of the massive factorizing models

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