Scaling Algebras and Short Distance Analysis of Superselection Charges

Gerardo Morsella

Università di Roma "Tor Vergata"

Göttingen, 4/6/2004

Contents

1. Motivations.
2. The algebraic approach to QFT.
3. Scaling algebras and scaling limits.
4. Examples and applications.
5. Superselection charges and the reconstruction of fields.
6. Scaling algebras for fields and scaling limits of charges.
7. Conclusions and outlook.

References

[BV95] D. Buchholz, R. Verch, *Rev. Math. Phys.* **7** (1995).

[Buc96] D. Buchholz, *Nucl. Phys.* **B469** (1996).

[BV98] D. Buchholz, R. Verch, *Rev. Math. Phys.* **10** (1998).

[DMV04] C. D'Antoni, R. Verch, G. M., `math-ph/0307048`, to appear in *Ann. Henri Poincaré* (2004).

1. Motivations

A great deal of information about short distance properties of QFT is obtained through the RG.

RG formulted in terms of unobservable fields: conceptually unsatisfactory, fields are just a "coordinatization" of observables (Borchers classes, Schwinger model, Seiberg-Witten dualities in SUSY YM).

Prominent example: confinement. Based on attaching a physical interpretation to unobservable degrees of freedom in the lagrangian (e.g. quark and gluon fields in QCD).

Algebraic approach to QFT provides a framework for an intrinsic description of the ultraviolet behaviour: scaling algebras allow to compute the scaling (short distance) limit of a theory entirely in terms of observables.

Analysis of charges and particles of the scaling limit theory, through DHR theory of superselection sectors, gives the *ultracharges* and *ultraparticles* described by the given theory at short distances.

Unambiguous notion of confinement obtained through comparison of charge and ultracharge content of the theory.

Necessary to find a canonical way of comparing the two charge structures. Identify ultracharges which are short distance remnants of (finite scales) charges.

Leads to a generalization of scaling algebras to charge carrying (unobservable) fields.

2. The algebraic approach to QFT

Theory completely characterized by assignment

$$\mathscr{O} \to \mathfrak{A}(\mathscr{O})$$

$\mathscr{O} \subset \mathbb{R}^4$ open double cone, $\mathfrak{A}(\mathscr{O}) \subset B(\mathscr{H})$ C*-algebra generated by observables measurable in $\mathscr{O}$.

Assumptions:

- *isotony*: $\mathscr{O}_1 \subset \mathscr{O}_2 \implies \mathfrak{A}(\mathscr{O}_1) \subset \mathfrak{A}(\mathscr{O}_2)$;
- *locality*: $\mathscr{O}_1 \subset \mathscr{O}_2' \implies \mathfrak{A}(\mathscr{O}_1) \subset \mathfrak{A}(\mathscr{O}_2)'$;
- *covariance*: $\exists\ U : \mathscr{P}_+^\uparrow \to \mathscr{U}(\mathscr{H})$ unitary repn such that

  $$U(\Lambda,x)\mathfrak{A}(\mathscr{O})U(\Lambda,x)^* = \mathfrak{A}(\Lambda\mathscr{O}+x), \quad (\Lambda,x) \in \mathscr{P}_+^\uparrow,$$

  and $\mathrm{Sp}\, U(\mathbb{1},\cdot) \subseteq \overline{V}_+$;
- *existence of vacuum*: $\exists!\ \Omega \in \mathscr{H}$ translation invariant unit vector, $\overline{\mathfrak{A}\Omega} = \mathscr{H}$.

$\mathfrak{A} := \overline{\bigcup_{\mathscr{O}} \mathfrak{A}(\mathscr{O})}$ *quasi-local algebra.*

Notation. $\alpha_{(\Lambda,x)}(A) := U(\Lambda,x)AU(\Lambda,x)^*$.

3. Scaling algebras and scaling limits

Conventional RG transformations defined by

$$R_\lambda(\phi(x)) = Z_\lambda \phi(\lambda x), \quad \lambda > 0,$$

$$\lim_{\lambda \to 0} \langle \Omega, R_\lambda(\phi(x)) R_\lambda(\phi(y)) \Omega \rangle \quad \text{finite.}$$

Characteristic feature: scale length by λ and 4-momentum by λ^{-1} $\implies$ phase space occupation of orbits is fixed.

Momentum scaling equivalent to:

$$\lim_{(\Lambda,x)\to(\mathbb{1},0)} \sup_{\lambda>0} \|\alpha_{(\Lambda,\lambda x)}(R_\lambda(A)) - R_\lambda(A)\| = 0.$$

For $\underline{A} : \mathbb{R}_+ \to \mathfrak{A}$ bounded function:

$$\|\underline{A}\| := \sup_{\lambda>0} \|\underline{A}(\lambda)\|,$$

$$\underline{\alpha}_{(\Lambda,x)}(\underline{A})(\lambda) := \alpha_{(\Lambda,\lambda x)}(\underline{A}(\lambda)).$$

Definition.[BV95] The *local scaling algebra* of $\mathscr{O}$ is

$$\underline{\mathfrak{A}}(\mathscr{O}) := \{\lambda \in \mathbb{R}_+ \to \underline{A}(\lambda) \in \mathfrak{A}(\lambda\mathscr{O}) : \lim_{(\Lambda,x)\to(\mathbb{1},0)} \|\underline{\alpha}_{(\Lambda,x)}(\underline{A}) - \underline{A}\| = 0\}.$$

Morally $\underline{A}(\lambda) = R_\lambda(A) \implies \underline{\mathfrak{A}}$ orbits of observables under *all possible RG transformations.*

If φ (locally normal) state on $\underline{\mathfrak{A}}$:

$$\underline{\varphi}_\lambda(\underline{A}) := \varphi(\underline{A}(\lambda))$$

$$\mathrm{SL}_{\mathfrak{A}}(\varphi) := \left\{\text{weak* limit pts of } \left(\underline{\varphi}_\lambda\right)_{\lambda>0}\right\}.$$

Theorem.[BV95] $\mathrm{SL}_{\mathfrak{A}}(\varphi)$ *is independent of* φ*. For* (π_0, Ω_0) *the GNS representation of* $\underline{\omega}_0 \in \mathrm{SL}_{\mathfrak{A}}$, $\mathscr{O} \to \mathfrak{A}_0(\mathscr{O}) := \pi_0(\underline{\mathfrak{A}}(\mathscr{O}))$ *is a covariant local net of observables with vacumm* Ω_0 *(if* $d = 2$ *the vacuum may not be pure).*

$\mathfrak{A}_0$ *scaling limit net* of $\mathfrak{A}$. Possibilities:

- *degenerate scaling limit*: the various $\mathfrak{A}_0$ non-isomorphic;
- *unique (quantum) scaling limit*: the various $\mathfrak{A}_0$ isomorphic and non-trivial;
- *classical scaling limit*: each $\mathfrak{A}_0(\mathscr{O}) = \mathbb{C}\mathbb{1}$.

4. Examples and applications

- Examples of *unique* scaling limit:

  Theorem.[BV98] *Each scaling limit theory of the free scalar field in* $d = 3, 4$ *spacetime dimensions, is isomorphic to the massless free scalar field.*

- Example of *classical* scaling limit [Lutz, Diploma (1997)]: ϕ generalized free field with constant mass measure, $\mathfrak{A}(\mathscr{O})$ generated by $\Box^{n(\mathscr{O})}\phi(x)$, $x \in \mathscr{O}$, $n(\mathscr{O}) \to +\infty$ as radius $\mathscr{O} \to 0$.

- Schwinger model (massless QED$_2$): $\mathfrak{A}(\mathscr{O})$ = massive free scalar field in $d = 2$ $\implies$ no charged states $\implies$ conventional interpretation: confined electrons.

  Intrinsic? Scaling limit has nontrivial charged states [Buc96, BV98] $\implies$ $\mathfrak{A}$ has *ultracharges*, intrinsically confined.

ϕ field on (M, g) globally hyperbolic. Φ satisfies *local stability* at $p \in M$ if (x normal coordinates at p):

$$\lim_{\lambda \to 0} \langle \Phi, R_\lambda(\phi(x_1)) \dots R_\lambda(\phi(x_n))\Phi \rangle$$
$$= \langle \Omega_0, \phi_0(x_1) \dots \phi_0(x_n)\Omega_0 \rangle,$$

ϕ_0 field on $\mathbb{R}^4$ [HNS, CMP **94** (1984)].

- substitute for spectrum condition (no symmetries on M);
- drawback: depends on ϕ.

Algebraic approach:

- local net $\mathscr{R} \subset M \to \mathfrak{A}(\mathscr{R})$;
- $F : \mathbb{R} \times \Sigma \to M$ Cauchy foliation; *propagator family*: $\alpha^{(F)}_{t,s} \in \mathrm{Aut}(\mathfrak{A})$, $t \geq s$,

$$\alpha^{(F)}_{t,s}\alpha^{(F)}_{s,u} = \alpha^{(F)}_{t,u}, \quad \alpha^{(F)}_{t,t} = \mathrm{id}_{\mathfrak{A}},$$

$$\alpha^{(F)}_{t,s}(\mathfrak{A}(F(\{s\} \times G)'')) = \mathfrak{A}(F(\{t\} \times G)''), \quad G \subset \Sigma.$$

Dynamics at p: $\{\alpha^{(F)}\}_F$ with $\{\partial_t F(p)\}_F = V_p^+$. Exists for linear models [Kay, CMP **71** (1980)].

Scaling algebra at $p \in M$, $\underline{\mathfrak{A}}_p(\mathscr{O})$, $\mathscr{O} \subset T_pM$: bounded functions $\lambda \in \mathbb{R}_+ \to \underline{A}(\lambda) \in \mathfrak{A}$ with

- $\underline{A}(\lambda) \in \mathfrak{A}(\exp_p(\lambda\mathscr{O}))$;
- $\lim_{t,s\to 0} \limsup_{\lambda\to 0} \|\alpha^{(F)}_{\lambda t,\lambda s}(\underline{A}(\lambda)) - \underline{A}(\lambda)\| = 0$ $\forall\, \alpha^{(F)}$.

$\mathrm{SL}_{\mathfrak{A}}(\varphi)$ defined as before.

Proposition.[Verch, PhD (1996)] *Let* $\underline{\omega}_0 \in \mathrm{SL}_{\mathfrak{A}}(\varphi)$ *and* π_0 *its GNS representation. Then* $\mathscr{O} \to \mathfrak{A}_{p,0}(\mathscr{O}) := \pi_0(\underline{\mathfrak{A}}_p(\mathscr{O}))$ *is a local net on* $\mathbb{R}^4$*, and in favourable cases* $\exists\, \alpha^{(F,0)}_{t,s}$ *propagator family such that*

$$\alpha^{(F,0)}_{t,s} \circ \pi_0 = \pi_0 \circ \underline{\alpha}^{(F)}_{t,s}\,.$$

Algebraic version of local stability: φ state on $\mathfrak{A}$ satisfies local stability at p if $\forall\, \underline{\omega}_0 \in \mathrm{SL}_{\mathfrak{A}}(\varphi)$, π_0 is irreducible, and $\exists\, x \in \mathbb{R}^4 \to \alpha^{(0)}_x \in \mathrm{Aut}(\mathfrak{A}_{p,0})$ which satisfies the spectrum condition and such that

$$\alpha^{(0)}_{(t-s)v} \circ \pi_0 = \pi_0 \circ \underline{\alpha}^{(F)}_{t,s}, \quad v = \partial_t F(p).$$

5. Superselection charges and reconstruction of fields

Interesting states in particle physics describe localized excitations of the vacuum $\implies$ criteria needed to select the relevant states.

Superselection sectors: equivalence classes of corresponding irreducible representations.

Two main criteria:

- a representation π is DHR (or describes a *localizable charge*) if

$$\pi \restriction \mathfrak{A}(\mathscr{O}') \cong \iota \restriction \mathfrak{A}(\mathscr{O}'), \qquad \forall \mathscr{O},$$

  ι vacuum representation [DHR, CMP **23** (1971)];

- a representation π is BF (or describes a *topological charge*) if

$$\pi \restriction \mathfrak{A}(\mathscr{C}') \cong \iota \restriction \mathfrak{A}(\mathscr{C}')$$

  for all spacelike cones $\mathscr{C}$ [BF, CMP **84** (1982)].

Theorem. π *describes massive particles* $\implies$ $\pi \in$ BF*.*

DHR (resp. BF) charges can be described by classes of *localized endomorphisms*: $\rho : \mathfrak{A} \to \mathfrak{A}$,

$$\rho(A) = A, \quad A \in \mathfrak{A}(\mathscr{O}')$$

(resp. $A \in \mathfrak{A}(\mathscr{C}')$).

Global intertwiners between ρ, σ:

$$(\rho : \sigma) := \{T \in \mathfrak{A} \,:\, T\rho(A) = \sigma(A)T,\, A \in \mathfrak{A}\}$$

Local intertwiners, $(\rho : \sigma)_{\mathscr{O}}$, defined by $T\rho(A) = \sigma(A)T$, $A \in \mathfrak{A}(\mathscr{O})$.

Mathematical structure of $\Delta := \{\rho\}$ encodes physical properties of charges:

- composition law;
- exchange statistics (Bose-Fermi alternative);
- conjugates.

In models:

$$\{\text{charges}\} \longleftrightarrow \left\{ \begin{array}{l} \text{irreducible representations} \\ \text{of global gauge group } G \end{array} \right\}.$$

More precisely:

- $\mathscr{O} \to \mathfrak{F}(\mathscr{O}) \subset B(\mathscr{H}_{\mathfrak{F}})$ generated by (unobservable) fields;
- $V : G \to U(\mathscr{H}_{\mathfrak{F}})$ unitary representation of G (compact), $V(g)\mathfrak{F}(\mathscr{O})V(g)^* = \mathfrak{F}(\mathscr{O})$;
- $\mathfrak{A}(\mathscr{O}) = \mathfrak{F}(\mathscr{O})^G := \{F \in \mathfrak{F}(\mathscr{O}) : V(g)FV(g)^* = F \,\forall\, g \in G\}$, observables.

Correspondence defined by:

$$\mathscr{H}_{\mathfrak{F}} = \bigoplus_{\xi} \mathscr{H}_{\xi},$$

$$V(g) \restriction \mathscr{H}_{\xi} = v_{\xi}(g) \otimes \mathbb{1}_{\mathscr{H}_{\xi}^{\text{mult}}}, \quad A \restriction \mathscr{H}_{\xi} = \mathbb{1}_{d_{\xi}} \otimes \pi_{\xi}(A),$$

v_{ξ} d_{ξ}-dimensional irreducible G-representation, $\pi_{\xi} \in$ DHR.

Notation. $\beta_g(F) := V(g)FV(g)^*$.

This is already encoded in observables:

Theorem.[DR, CMP **131** (1990)] *Given* $\mathfrak{A}, \Delta$ *there exist unique* $\mathfrak{F}, V$ *as above, such that:*

- $\mathfrak{F}$ *has normal commutation relations;*
- $\forall \rho \in \Delta(\mathscr{O})$ *irreducible* $\exists\, \psi_1, \dots, \psi_d \in \mathfrak{F}(\mathscr{O})$ *and* $v_{[\rho]}$ *d-dimensional irreducible G-representation such that*

$$\psi_i^* \psi_j = \delta_{ij} \mathbb{1}, \qquad \sum_{j=1}^{d} \psi_j \psi_j^* = \mathbb{1},$$

$$\beta_g(\psi_i) = \sum_{j=1}^{d} v_{[\rho]}(g)_{ij} \psi_j,$$

$$\rho(A) = \sum_{j=1}^{d} \psi_j A \psi_j^*, \qquad A \in \mathfrak{A};$$

- $\mathfrak{F}(\mathscr{O})$ *is generated by* $\mathfrak{A}(\mathscr{O})$ *and the multiplets* ψ_j*.*

Analogous result for BF charges, with net $\mathscr{C} \to \mathfrak{F}(\mathscr{C})$, i.e. fields localized in cones.

5. Scaling algebras for fields and scaling limit of charges

Reference [DMV04].

$\mathscr{O} \to \mathfrak{F}(\mathscr{O})$ field net. Construction parallel to the observable case. For $\underline{F} : \mathbb{R}_+ \to \mathfrak{F}$ bounded:

$$\underline{\beta}_g(\underline{F})(\lambda) := \beta_g(\underline{F}(\lambda)), \quad g \in G.$$

Scaling field algebra $\underline{\mathfrak{F}}(\mathscr{O})$: bounded functions $\underline{F} : \mathbb{R}_+ \to \mathfrak{F}$ such that

- $\underline{F}(\lambda) \in \mathfrak{F}(\lambda\mathscr{O})$;
- $\lim_{(\Lambda,x)\to(\mathbb{1},0)} \|\underline{\alpha}_{(\Lambda,x)}(\underline{F}) - \underline{F}\| = 0$;
- $\lim_{g\to e} \|\underline{\beta}_g(\underline{F}) - \underline{F}\| = 0$.

$\implies$ We restrict to "dimensionless" charges.

Clearly: $\underline{\mathfrak{A}}(\mathscr{O}) \subset \underline{\mathfrak{F}}(\mathscr{O})$.

φ state on $\mathfrak{F}$: $\left(\underline{\varphi}_\lambda\right)_{\lambda>0}$ and $\mathrm{SL}_{\mathfrak{F}}(\varphi)$ defined as for observables.

Theorem. $\mathrm{SL}_{\mathfrak{F}}(\varphi)$ *is independent of* φ*. For* (π_0, Ω_0) *the GNS representation of* $\underline{\omega}_0 \in \mathrm{SL}_{\mathfrak{F}}$*,* $\mathscr{O} \to \mathfrak{F}_0(\mathscr{O}) := \pi_0(\underline{\mathfrak{F}}(\mathscr{O}))$ *is a covariant normal field net with an action* V_0 *of* $G_0 := G/N_0$ *such that*

$$\mathfrak{A}_0(\mathscr{O}) := \pi_0(\underline{\mathfrak{A}}(\mathscr{O})) = \mathfrak{F}_0(\mathscr{O})^G.$$

General situation:

$$\begin{tikzcd}
& \mathfrak{A} \arrow[dl, "\mathrm{SL}"'] \arrow[dr, "\mathrm{DR}"] & \\
\mathfrak{A}_0 \arrow[dr, "\mathrm{DR}"'] & & \mathfrak{F} \arrow[dl, "\mathrm{SL}"] \\
& \mathfrak{F}^0 \supseteq \mathfrak{F}_0 &
\end{tikzcd}$$

$\mathfrak{F}_0 \subsetneq \mathfrak{F}^0 \implies \mathfrak{A}$ has *confined ultracharges* [Buc96]: charges appearing at short distances but not at finite distances.

Intrinsic: everything fixed by the observable net.

Example. Schwinger model: $\mathfrak{F} = \mathfrak{A} \implies \mathfrak{F}_0 = \mathfrak{A}_0 \subsetneq \mathfrak{F}^0$.

"Converse" problem: which charges *survive* to the scaling limit.

Some may disappear: if scaling limit classical, $G^0 = \{e\}$ (canonical DR gauge group of $\mathfrak{A}_0$) $\implies$ all charges disappear.

Conditions needed to select preserved charges.

Physical picture: "pointlike" objects survive.

$\psi_j(\lambda) \in \mathfrak{F}(\lambda\mathscr{O})$ of class $\xi \implies \psi_j(\lambda)\Omega$ charge ξ localized in $\lambda\mathscr{O}$. If ξ pointlike, Heisenberg $\implies$ energy$(\psi_j(\lambda)\Omega) \sim \lambda^{-1}$.

Note: $\psi_j(\cdot) \notin \underline{\mathfrak{F}}(\mathscr{O})$. But

$$\underline{\alpha}_h\psi_j(\lambda) := \int_{\tilde{\mathscr{P}}_+^\uparrow} d\Lambda d^4x \; h(\Lambda, x)\alpha_{(\Lambda,\lambda x)}(\psi_j(\lambda)),$$

$h \in L^1(\tilde{\mathscr{P}}_+^\uparrow)$. Then $\underline{\alpha}_h\psi_j(\lambda) \in \underline{\mathfrak{F}}(\mathscr{O}_1)$, $\mathscr{O}_1 \supset \overline{\mathscr{O}}$.

Requirement above equivalent to:

$$\limsup_{\lambda\to 0} \|[\psi_j(\lambda) - \underline{\alpha}_h\psi_j(\lambda)]\Omega\| + \|[\psi_j(\lambda) - \underline{\alpha}_h\psi_j(\lambda)]^*\Omega\| < \varepsilon,$$

for small $\operatorname{supp} h$.

Theorem. *With ξ and $\psi_j(\lambda)$ as above, there exists*

$$\boldsymbol{\psi}_j = {}^*s\text{-}\lim_{h\to\delta} \pi_0(\underline{\alpha}_h \psi_j) \in \mathfrak{F}_0(\mathscr{O}_1)''$$

for each $\mathscr{O}_1 \supset \overline{\mathscr{O}}$, and $\boldsymbol{\psi}_j$ is a G_0-multiplet of class ξ, with $v_\xi^{(0)}(gN_0) := v_\xi(g)$ a well-defined irrep of G_0. Furthermore,

$$\boldsymbol{\rho}(\boldsymbol{a}) := \sum_{j=1}^{d} \boldsymbol{\psi}_j \boldsymbol{a} \boldsymbol{\psi}_j^*, \qquad \boldsymbol{a} \in \mathfrak{A}_0,$$

is a localized irreducible DHR endomorphism of $\mathfrak{A}_0$.

Example. Majorana free field ϕ: $\psi(\lambda) = \phi(f_\lambda)$.

If all sectors are preserved, much of the superselection structure can be determined locally.

Theorem. *If all the $\mathfrak{F}(\mathscr{O})$ are factors, $\mathfrak{F}(\mathscr{O}) \cap \mathfrak{F}(\mathscr{O})' = \mathbb{C}\mathbb{1}$, and each sector is preserved in some scaling limit, then*

$$(\rho : \sigma) = (\rho : \sigma)_{\mathscr{O}}, \qquad \forall \mathscr{O}.$$

Generalization of result of Roberts for dilation invariant theories [Rob, CMP **37** (1974)], useful on curved spacetimes.

BF charges expected in nonabelian gauge theories (cone $\leftrightarrow$ flux string), which also exhibit confinement $\Longrightarrow$ above analysis too narrow: if quarks are non-confined, they are localized in cones.

Problem: spacelike cones not affected by rescaling $\Longrightarrow$ how to implement RG phase space?

In asymptotically free theories charges in cones should become localized in the scaling limit (flux string vanishes) $\Longrightarrow$ phase space recovered asymptotically.

Define $\underline{\mathfrak{F}}(\mathscr{C}, \mathscr{O})$, $\mathscr{O} \subset \mathscr{C}$: bounded functions $\underline{F} : \mathbb{R}_+ \to \mathfrak{F}$ such that:

- $\underline{F}(\lambda) \in \mathfrak{F}(\lambda\mathscr{C})$;
- $\lim_{(\Lambda,x)\to(\mathbb{1},0)} \|\underline{\alpha}_{(\Lambda,x)}(\underline{F}) - \underline{F}\| = 0$;
- $\lim_{g\to e} \|\underline{\beta}_g(\underline{F}) - \underline{F}\| = 0$;
- $\lim_{\lambda\to 0} \sup_{A\in\underline{\mathfrak{A}}(\mathscr{O}')_1} \|[\underline{F}(\lambda), \underline{A}(\lambda)]\| = 0$.

$\underline{\mathfrak{F}}$ C*-algebra generated by all $\underline{\mathfrak{F}}(\mathscr{C}, \mathscr{O})$.

φ normal state on $B(\mathscr{H}_{\mathfrak{F}})$: states $\big(\underline{\varphi}_\lambda\big)_{\lambda>0}$ on $\underline{\mathfrak{F}}$ and $\mathrm{SL}_{\mathfrak{F}}(\varphi)$ defined as above.

Theorem. $\mathrm{SL}_{\mathfrak{F}}(\varphi)$ *is independent of* φ*. For* (π_0, Ω_0) *the GNS representation of* $\underline{\omega}_0 \in \mathrm{SL}_{\mathfrak{F}}$*,*

$$\mathscr{O} \to \mathfrak{F}_0(\mathscr{O}) := \bigcap_{\mathscr{C} \supset \mathscr{O}} \pi_0(\underline{\mathfrak{F}}(\mathscr{C}, \mathscr{O}))''$$

is a covariant normal field net with an action V_0 *of* $G_0 := G/N_0$*.*

We obtain a net indexed by *double cones* in the scaling limit, as expected in the asymptotically free case.

Study of preservation similar to DHR case: fix $\psi_j(\lambda) \in \mathfrak{F}(\lambda\mathscr{C})$; if $\forall\, \hat{\mathscr{C}} \supset \mathscr{O}$, $\exists\, \hat{\psi}_j(\lambda) \in \mathfrak{F}(\lambda\hat{\mathscr{C}})$ asymptotically localized in $\mathscr{O}$, and such that for small $\operatorname{supp} h$

$$\limsup_{\lambda\to 0} \|[\psi_j(\lambda) - \underline{\alpha}_h \hat{\psi}_j(\lambda)]\Omega\| + \|[\ldots]^*\Omega\| < \varepsilon,$$

then

$$\boldsymbol{\psi}_j = {}^*s\text{-}\lim_{h\to\delta} \pi_0(\underline{\alpha}_h \hat{\psi}_j) \in \mathfrak{F}_0(\mathscr{O}_1)''$$

is independent of $\hat{\psi}_j$, and is a multiplet inducing a DHR charge.

To summarize:

$$\begin{tikzcd}
\mathfrak{A} \arrow[rr, "\mathrm{SL}"] \arrow[d] & & \mathfrak{A}_0 \arrow[d] \\
\mathrm{BF}(\mathfrak{A}) & \mathrm{BF}_0(\mathfrak{A}) \arrow[l, hook'] \arrow[r, dashed] & \mathrm{DHR}(\mathfrak{A}_0)
\end{tikzcd}$$

Provides another confinement notion:

$\xi \in \mathrm{DHR}(\mathfrak{A}_0) \setminus \mathrm{BF}_0(\mathfrak{A})$ is a confined ultracharge of $\mathfrak{A}$.

7. Conclusions and outlook

AQFT provides tools for model-independent, intrinsic analysis of short-distance properties of QFT and classification of possible ultra-violet behaviour.

It makes also possible the formulation of unambiguous confinement criteria, as illustrated by the Schwinger model example.

It has also applications to the problem of selecting physical states in curved spacetime.

Scaling algerba methods can be generalized to charge carrying fields, in order to analyse the short-distance properties of superselection charges, and in particular to characterize their preservation.

Future developments:

- study of models (non-preserved charges, preserved BF charges);
- anomalous charge scaling;
- scaling of charges without DR theorem.