Curved Quantum Spacetime and Cosmology

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joint work with S. Doplicher, N. Pinamonti
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Outline

Introduction 1/2

- Generally accepted view: continuous description of spacetime should break down at distance of the order of the Planck length λ_P
- Analysis of [Doplicher, Fredenhagen, Roberts '95]: QM + GR $\Rightarrow$ accuracies Δq^μ on spacetime coordinates of an event in Minkowski satisfy Spacetime Uncertainty Relations (STUR), implemented by commutation relations between the q^μ's (more later)
- Minkowski spacetime replaced by a Quantum (noncommutative) Spacetime $\mathcal{E}$ (C*-algebra generated by q^μ's)
- QFT on QST has interesting properties (more later)

Introduction 2/2

Problems with DFR argument:

- it works only on Minkowski spacetime
- it uses GR in linearized approximation
- it relies on a rough criterion for formation of trapped surfaces
- it employs the notion of energy, ill-defined on curved backgrounds

Questions

1. Is it possible to generalize DFR argument to (some) curved spacetime, in a more rigourous way?
2. Is it possible to define QFT on the resulting model of curved Quantum Spacetime?
3. Does it have interesting physical consequences?

Here: positive answer to 1, and partially to 2 and 3

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Operational limitations to localizability of events

QM: need energy $E \simeq 1/L$ to prepare a quantum state localized in a small region of size L

GR: large energy E creates a trapped surface of radius $r \simeq E$ around localization region

$$\Downarrow$$

localization has operational meaning only if $L \geq r$ i.e. $L \gtrsim 1 = \lambda_P$ (in natural units)

Principle of gravitational stability against localization

The gravitational field generated by the concentration of energy required by the Heisenberg Uncertainty Principle to localize an event in spacetime should not be so strong to hide the event itself to any distant observer

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Spacetime Uncertainty Relations

If only one coordinate is localized with high precision, TS will not form: transferred energy density goes to zero

[DFR] analysis:

- quantum state localized in region of sizes Δq^μ, $\mu = 0, \ldots, 3$, has energy $E \simeq 1/\min_\mu\{\Delta q^\mu\} \implies$ energy density ρ
- solution of linearized Einstein equations with source ρ given by retarded potential
- condition of non formation of TS: $g_{00} > 0$

Spacetime Uncertainty Relations (STURs)

$$\Delta q^0 \sum_{j=1}^{3} \Delta q^j \geq \lambda_P^2, \qquad \sum_{i<j=1}^{3} \Delta q^i \Delta q^j \geq \lambda_P^2$$

Necessary conditions imposed by the principle of gravitational stability

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Quantum Spacetime

STURs can be realized by assuming that Δq^μ's are indeterminacies of quantum operators q^μ satisfying suitable commutation relations, as for Heisenberg uncertainty relations

Quantum Conditions

$$[q^\mu, q^\nu] = i\lambda_P^2 Q^{\mu\nu}, \quad [q^\rho, Q^{\mu\nu}] = 0,$$

$$Q_{\mu\nu} Q^{\mu\nu} = 0, \quad \left(\frac{1}{4} Q^{\mu\nu} (*Q)_{\mu\nu}\right)^2 = 1$$

- Noncommutative C*-algebra $\mathcal{E}$ of Quantum Spacetime (QST) generated by q^μ's replaces algebra of functions on Minkowski
- It is equipped with action of the Poincaré group $q^\mu \to \Lambda^\mu_\nu q^\nu + a^\nu$
- $\mathcal{E}$ has nontrivial center $Z(\mathcal{E})$ = functions on a manifold $\Sigma \simeq T\mathbb{S}^2 \times \mathbb{Z}_2$ and $\mathcal{E} \simeq C_0(\Sigma, \mathcal{K})$, $\mathcal{K}$ = compact operators

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Optimal localization on QST

In an irreducible representation q^μ is a Lorentz transform of Schroedinger's (x_1, x_2, p_1, p_2)

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There exists states of optimal localization ω on $\mathcal{E}$, minimizing

$$\sum_\mu (\Delta q^\mu)^2 = (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta p_1)^2 + (\Delta p_2)^2$$

given by translates of the harmonic oscillator ground states

They are the best approximation of points on QST

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Free quantum fields on QST

ϕ free (scalar) field on Minkowski can be defined on QST through Weyl-von Neumann-Moyal quantizazion

$$\phi(q) = \int d^4k\, \check{\phi}(k) \otimes e^{ikq}$$

(formal) element of $\mathfrak{F} \otimes \mathcal{E}$, $\mathfrak{F}$ field algebra

- it satisfies Klein-Gordon equation (derivatives on $\mathcal{E}$ defined by $\partial_\mu \phi(q) := \frac{\partial}{\partial x^\mu} \phi(q + x\mathbb{1})$)
- ω_x, ω_y optimally localized states around $x, y \implies [\mathrm{id} \otimes \omega_x(\phi(q)), \mathrm{id} \otimes \omega_y(\phi(q))]$ falls off as a Gaussian of width λ_P for large spacelike $x - y$

Locality is lost at distances small w.r.t. λ_P, but recovered as $\lambda_P \to 0$

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(Perturbative) interacting fields on QST

Several (inequivalent) possibilities of defining perturbative interacting fields

- Hamiltonian approach (interaction picture) with interaction Lagrangian defined by $:\phi(q)^n:$ [DFR]
- Yang-Feldman equation and quasi-planar Wick products [Bahns, Doplicher, Fredenhagen, Piacitelli '02 & '04]
- Hamiltonian approach with interaction defined by quantum Wick product $:\phi^n(q):_Q$, which yields UV-finite (IR-cutoff) theory to all orders [Bahns, Doplicher, Fredenhagen, Piacitelli '03]

$:\phi^n(q):_Q$ defined by generalizing point-splitting to QST: e.g., for $n = 2$

$$:\phi^2:(x) := \lim_{y \to x} \phi(x)\phi(y) - \langle \Omega, \phi(x)\phi(y)\Omega \rangle$$

limit $y \to x$ has to be performed in a way compatible with the STURs

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Quantum Wick product

- Introduce quantum coordinates of independent events

$$q_1^\mu := q^\mu \otimes \mathbb{1}, \qquad q_2^\mu := \mathbb{1} \otimes q^\mu$$

and identify commutators $[q_1^\mu, q_1^\nu] = i\lambda_P^2 Q^{\mu\nu} = [q_2^\mu, q_2^\nu]$

- introduce center of mass and relative coordinates

$$\bar{q}^\mu := \frac{1}{2}(q_1^\mu + q_2^\mu), \qquad \xi^\mu := \frac{1}{\lambda_P}(q_1^\mu - q_2^\mu)$$

identification of commutators $\implies [\bar{q}^\mu, \xi^\nu] = 0$

- evaluating optimally localized state on ξ^μ yields a map $E^{(2)} : \mathcal{E} \otimes_Z \mathcal{E} \to \mathcal{E} \simeq C^*(\bar{q}^\mu)$

Quantum Wick product

$$\begin{aligned} :\phi^2(\bar{q}):_Q &:= E^{(2)}(:\phi(q_1)\phi(q_2):) \\ &= \int d^4k_1 d^4k_2 :\check{\phi}(k_1)\check{\phi}(k_2): e^{-\frac{\lambda_P^2}{4}|k_1-k_2|^2} e^{i(k_1+k_2)\bar{q}} \end{aligned}$$

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Overview

Aim: produce a rigorous version of DFR argument on curved spacetime
Strategy:

1. prepare a localized state ω_f of (scalar massless) free quantum field ϕ in equilibrium with background
2. evaluate change to expectation value of $T_{\mu\nu}$ after localization (CCR of ϕ as substitute for energy and Heisenberg principle)
3. estimate backreaction on metric and formation of TS by Raychauduri equation (no linearization of gravity and more precise criterion than $g_{00} > 0$)
4. impose principle of gravitational stability

Step 3 (and 4) only under assumption of spherical symmetry of background metric

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Change of $\omega(T_{\mu\nu})$ under localization

- ϕ field on curved background $(M, g_{\mu\nu})$ and ω quasifree Hadamard state of ϕ (positive linear expectation functional on *-algebra $\mathcal{A}$ generated by Wick monomials in ϕ and its derivatives) satisfying

$$\Box\phi = 0, \qquad G_{\mu\nu} = 8\pi\omega(T_{\mu\nu})$$

- $f \in C_c^\infty(M)$, state ω_f after localization (measurement of $\phi(f)$)

$$\omega_f(A) := \frac{\omega(\phi(f)A\phi(f))}{\omega(\phi(f)\phi(f))}, \qquad A \in \mathcal{A}$$

Proposition

If μ is a lightlike direction and Δ the causal propagator of ϕ,

$$\langle T_{\mu\mu}(x)\rangle_{f,0} := (\omega_f - \omega)(T_{\mu\mu}(x)) \geq \frac{1}{2}\frac{|\partial_\mu\Delta(f)(x)|^2}{\omega(\phi(f)\phi(f))}$$

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Spherical symmetry

To evaluate backreaction, we should solve

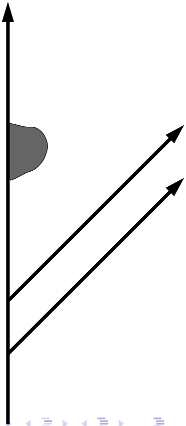
$$G_{\mu\nu} = 8\pi\, \omega_f(T_{\mu\nu})$$

It is **very** difficult. Assume spherical symmetry

- Spacetime is $\mathbb{R}^2 \times \mathbb{S}^2$, retarded coordinates:
- spanned by future null geodesic emanated from the center of the sphere
  - u proper time on the worldline γ of center
  - s retarded distance: affine parameter along the null geodesics with $s(0) = 0$ and $\dot{s}(0) = 1$
- The general spherically symmetric metric is

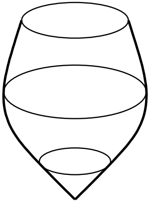
$$ds^2 := -A(u,s)du^2 - 2ds\,du + r(u,s)^2 d\mathbb{S}^2$$

- Fix u, the family of null geodesics forms a cone $\mathcal{C}_u$



Expansion of null geodesics

- For every $\mathcal{C}_u$ consider the expansion parameter θ of that family
  θ measures the rate of change of $4\pi r^2$ along $\mathcal{C}_u$
  - $\theta > 0$ expansion
  - $\theta = 0$ trapped surface
  - $\theta < 0$ contraction



- Its evolution along $\mathcal{C}_u$ is governed by the Raychaudhuri equation

$$\dot{\theta} = -\frac{\theta^2}{2} - R_{ss}, \qquad \lim_{s\to 0^+} s\theta = 2$$

- We solve this equation semiclassically, namely:

$$R_{ss} = 8\pi\,\omega_f(T_{ss}) = 8\pi\,\omega(T_{ss}) + 8\pi\langle T_{ss}\rangle_{f,0} = R_{ss}^{(0)} + 8\pi\langle T_{ss}\rangle_{f,0}$$

$R_{\mu\nu}^{(0)}$ is the *"curvature"* without the influence of the measurement.

Backreaction and trapped surfaces

Theorem

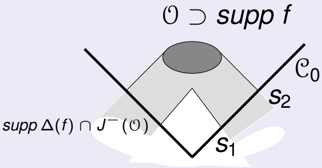
M spherically symmetric and ω as before. Assume:

1. *semiclassical Einstein equations are satisfied by ω and M*
2. $R^{(0)}_{ss} = 8\pi\omega(T_{ss})$ *is positive on* $\mathcal{C}_0$
3. *for every f supported in* $J^+(\mathcal{C}_0)$

$$|\omega(\phi(f)\phi(f))| \leq C\|s\Delta(f)\|_{L^2(\mathcal{C}_0)}\|\partial_s(s\Delta(f))\|_{L^2(\mathcal{C}_0)}$$

Consider f as in figure with

$$s_1 < s_2 < \frac{3}{2}s_1, \quad (s_2)^2 < \bar{s}^2 \ , \quad \bar{s}^2 := \frac{1}{6\,C}$$



Hence θ vanishes in $\mathcal{C}_0$ and thus $J^+(\mathcal{C}_0)$ contains a trapped surface.

Comments

- Solutions of the semiclassical Einstein equation do exists at least in cosmology [Pinamonti '11]
- $R^{(0)}_{ss} \geq 0$ is realized in every reasonable cosmological model
- The required continuity for ω occurs in many concrete examples (Minkowski vacuum, many other Hadamard states of interest [Dappiaggi, Pinamonti, Porrmann '11])

If we impose principle of gravitational stability:

Extension of DFR result

On a spherically symmetric spacetime, it has no operational meaning to localize an event in a spherical region of size smaller than some minimal length λ_P

For a full set of STURs in curved spacetime, one would need to treat the non-symmetric case

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Question

Does the quantum structure of spacetime have consequences of cosmological interest?

General strategy: evaluate $\omega(T_{\mu\nu})$ on QST modeled on cosmological (FRW) spacetime and solve Einstein equation

Obvious (big) problems:

- no general definition of curved QST
- no extension of QFT to it

Our (more modest) strategy here:

- evaluate $\omega(T_{00})$ on Minkowski QST $\mathcal{E}$
- make an ansatz for $\omega(T_{00})$ on flat FRW QST using conformal embedding in Minkowski
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Energy density on Minkowski QST

On Minkowski QST:

- energy density defined by the quantum Wick product:

$$: \rho(\bar{q}) :_Q := E^{(2)} \Big(: \partial_0 \phi(q_1) \partial_0 \phi(q_2) - \frac{1}{2} \eta_{\mu\nu} \partial_\mu \phi(q_1) \partial^\nu \phi(q_2) : \Big)$$

- KMS state ω_β (β inverse temperature) modeling the CMB

$$\omega_\beta(\check{\phi}(k_1)\check{\phi}(k_2)) = \delta(k_1 + k_2)\delta(k_1^2) \frac{\varepsilon(k_1^0)}{1 - e^{-\beta k_1^0}}.$$

Result:

$$\omega_\beta(: \rho :_Q) = 4\pi \int_0^{+\infty} k^3 \frac{e^{-\lambda_P^2 k^2}}{e^{\beta k} - 1} dk \sim \frac{C}{\beta \lambda_P^3}, \qquad \frac{\lambda}{\beta} >> 1$$

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Energy density on flat FRW QST

On flat FRW spacetime, with metric

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2 = a(\tau)^2[-d\tau^2 + d\mathbf{x}^2]$$

- consider conformally coulpled massless scalar field ϕ
- consider ω_β^M conformal KMS state for ϕ
- temperature scales with a: $\beta = \beta_0 a(t)$
- the length scale λ_P is scale invariant

A *"reasonable expression"* for the expectation value of the energy density is then

$$\rho_\beta(t) := \omega_\beta^M(:\rho:_Q) = 4\pi \int_0^{+\infty} dk\, k^3 \frac{e^{-\lambda_P^2 k^2}}{e^{\beta(t)k} - 1} \sim \frac{C}{\beta_0 a(t) \lambda_P^3} \qquad \frac{\lambda_P}{\beta} >> 1$$

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Solution of Friedmann equation near BB

Friedmann equation for small a becomes

$$H^2(t) = \frac{C}{a(t)}.$$

Solution in terms of conformal time

$$\tau_0 - \tau = \int_t^{t_0} \frac{1}{a(t')} dt' = \int_a^{a_0} \frac{1}{a'^2 H(a')} da' = \frac{2}{\sqrt{C}} \Big(\frac{1}{\sqrt{a}} - \frac{1}{\sqrt{a_0}} \Big)$$

- **Classical solution** (radiation dominated): $\tau \to \tau_0$ for $a \to a_0$
  Horizon problem
- **Quantum NC Corrections**:
  $\tau \to -\infty$ for $a \to 0$
  Singularity is light like, no Horizon Problem
  Power law inflation with Null Big Bang $\mathfrak{I}^- \cup i^-$

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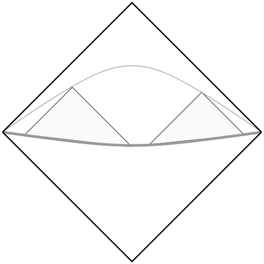

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Outline

Summary and Outlook

Summary:

- Using
  - free field CCR instead of Heisenberg principle
  - semiclassical backreaction instead of linearized Einstein equations
  - Raychauduri equation instead of $g_{00} > 0$
  - spherical symmetry

  it is possible to find a generalization to curved spacetime of the case of STURs in which $\Delta q^0 \simeq \Delta q^1 \simeq \ldots \Delta q^3$, yielding a minimal localization length λ_P
- Using this we can estimate the influence of non commutativity on the curvature
- In a cosmological model the Horizon problem disappears

Open questions:

- Can we extend the argument to non-symmetric spacetimes obtaining a generalization of the full set of STURs?
- Can we implement these STURs through a noncommutative spacetime $\mathcal{E}$ and construct a full fledged QFT on it?

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