Present and future research interests

1. STATISTICAL PHYSICS

A. Universality in 2D statistical physics

Several two dimensional statistical models like Ising models with interactions quartic in the spins, Vertex models, Ashkin-Teller models and quantum spin chains are described by the same quantum field theory in the continuum limit. The critical exponents can be computed however only in certain special exactly solvable cases, like the Eight Vertex and the XYZ models, and they appear to be non trivial functions of the hamiltonian parameters. However for the majority of models there are no exact solutions and the exponents are therefore unknown. In the continuum limit the exponents verify certain relations and the universality principle suggests that they are generically true for all the models in this class. Such universal relations are used in the interpretation of experimental results for compounds described by models in this class, like carbon nanotubes; indeed they allow to express the exponents in terms of a few measurable parameters. Despite several attempts based on a number of methods ranging from bosonization to operator product expansion, a proof of the validity of such universal relations in non solvable models was lacking. Indeed all such methods use as a starting point the continuum limit, where extra Lorentz and Gauge symmetries are verified; however, lattice effects destroy such symmetries and change the exponents, and their effect has never been analyzed.

1. Present results

1. In [5],[6] I have introduced a new approach for the analysis of such models, based on the techniques of Constructive Quantum Field Theory, and in particular Grassmanian Renormalization Group combined with Ward Identities; the lattice produces additional terms in the Ward Identities which can be rigorously taken into account. By this methods I could express the energy, crossover, specific heat and correlation length exponents (for Ising models with interactions quartic in the spins, Vertex models, isotropic Ashkin-Teller models and quantum spin chains) in terms of a convergent renormalized expansion; this allows to compute the above exponents with arbitrary precision, by explicit computation of lowest orders and rigorous bounds on the rest. This is actually the only way in which the exponents can be rigorously computed in non solvable models, like the isotropic Ashkin

Teller models or spin chain with next to nearest neighbor interaction; this is true also for the density and crossover exponents in the Eight vertex, which cannot be derived by the exact solution.

2. In the case of the anisotropic Ashkin-Teller model, it was proved [3],[4] by the above methods that there are two critical temperatures which coalesces to one in the isotropic limit. Close to the critical temperatures the specific heat has the same logarithmic behaviour than in the Ising model; however the difference between the critical temperatures scales with an anomalous critical exponents, previously unknown, which is a continuous function of the coupling.

3. The critical exponents are expressed as convergent series in the coupling, but their complexity makes the direct verification of universal relation between exponents essentially impossible. In [1] it is derived a number of exact universal relation between exponents; one of them was unknown, others were proposed in the seventies but, despite several attempts, no one was able to prove their validity in non solvable models. The proof is based on the use of Ward Identities, with extra terms due to the presence of the lattice; the non perturbative analogue of the Adler-Bardeen property of the non renormalization of the quantum anomalies plays a crucial role in the analysis.

2. Future directions

1. So far, spin-spin correlations in the above models seem to be beyond the reach of the method. Note that already in the integrable case the computation of such correlations is very tricky: it is based on an asymptotic analysis of a Toeplitz determinant or, alternatively, on the derivation of highly non trivial non linear finite difference equations, whose scaling limit is related to the third Painleve' equation. In order to progress on this problem, it is necessary to combine Renormalization Group and Ward Identities techniques with different powerful methods, such as non-linear recursive finite difference equations or bosonization (see [41] for partial results).

2. Another important direction is to use the above methods to take into account the lattice and mass effects in other two dimensional statistical models which can be mapped in the scaling limit in conformal field theory models.

2. MANY-BODY THEORY

A. Two-dimensional Hubbard models on the square and the honeycomb lattice

The Hubbard model plays essentially the same role in the problem of the electron correlations as the Ising model in the problem of spin-spin correlations, that is it the the simplest model displaying many real world features. In two dimension with a square lattice, it is considered the basic model for the physics of high T_c superconductors. Recently a large interest has been devoted also to the properties of the $2D$ Hubbard model on the honeycomb lattice, as a basic model for the description of graphene. The theoretical analysis of such models is of extraordinary difficulty and most of their basic properties are unknown.

1. *Present results*

1. The proof that the $2D$ Hubbard model on the square lattice is a *Fermi liquid* in the sense of Landau, above exponentially small temperatures and in non half filled band case [14],[19]; in particular, it is shown that the wave function renormalization and the Fermi velocity are essentially temperature-independent. The derivation of such a property requires a detailed analysis of the interaction effect on the convexity and regularity properties of the Fermi surface, and it is based on the convergence of the renormalized series at weak coupling . It was proposed in the physical literature the possibility that the lack of convergence could explain some aspects of the physics of high T_c superconductors, and such a result excludes this possibility.

2. Close to the half-filled band case, the above result does not apply (for the lack of convexity) and this opens the way to a different non Fermi liquid behavior. For certain Fermi surfaces and with long range interactions, it has been proved that Luttinger liquid behavior emerges [9],[10].

3. In the case of the Hubbard model on the honeycomb lattice, it has been computed [7],[8] the asymptotic behaviour of the 2-point functions in the half filled band case, proving the convergence of the perturbative expansion up to zero temperature and ruling out the presence of superconducting or magnetic instabilities at weak coupling and proving the isotropy of the Dirac cones even in presence of interaction. This is one of the very few examples in which the ground state properties (including correlations) of interacting 2D fermions can be obtained.

1. In the Hubbard model on the square lattice, I plan to investigate the behaviour close to the half filled band case. In this region the crossover between Fermi and non Fermi liquid behavior is expected to produce a very interesting and non trivial behavior. As non-perturbative RG methods have produced a complete understanding in the normal phase in the not half filled base case [14] (but with a convergence radius vanishing closer and closer to the half filled band case) and in the filled band case (where non Fermi liquid behavior is found), I expect that by such methods it is possible to clarify such a crossover regime, which is still very poorly understood and it appears to be of interest in the study of the so-called pseudo-gap phase in cuprates superconductors.

2. In the Hubbard model in the honeycomb lattice, I plan to study the behavior for densities close to one; this situation is relevant for the physics of doped graphene and requires the combination of the methods developed in [14] and [9] as the Fermi surface is not anymore point-like.

3. Finally, I plan to study the Hubbard model on the honeycomb lattice with long range interaction. In this case the interaction should dramatically modify the behaviour with respect to the free case, and possibly the presence of Luttinger liquid behavior can be established. This problem can be faced by combining the methods in [14] and the ones developed for one dimensional models in [16].

3. LUTTINGER LIQUID CONSTRUCTION

One dimensional fermionic models are widely investigated, but only in very few exactly solvable cases, like the Luttinger model, the correlation can be computed. According to the predictions of phenomenological "Luttinger liquid theory" the exponents and other thermodynamic quantities verify certain exact relation allowing the determination of such quantities in terms of a few parameters; such predictions have been verified in certain solvable cases but an analytical derivation in more realistic models was unknown; methods like bosonization cannot be applied in presence of a lattice or non linear bands.

3. Present results

1. The observables and other physical properties have been expressed in terms of convergent series (so that they can be computed with arbitrary precision with a lower order computation and a rigorous bound on the rest) in a number of non solvable model, like interacting spinless fermions on a chain with short range interactions [25], repulsive 1D Hubbard model [12],[15], interacting fermions with periodic or quasi periodic potentials [29]; in all such cases bosonization or exact solutions does not apply. One of the main problems in Wilsonian Renormalization Group is the conflict between regularizations and Ward Identities; the regularizations introduced in the multiscale analysis, together with the lattice or the non linear bands, break the local symmetries on which Ward Identities are based. Recently [16],[21] new methods have been developed which allow the solution of this problem; the cut-offs produce in the Ward identities extra terms which can be rigorously taken into account. By such methods Luttinger liquid behavior in all the above non solvable models has been established.

2. The exponents and other quantities like the Fermi or sound velocity are expressed in terms of convergent expansions depending from all the microscopic details; according to the phenomenological Luttinger liquid theory, such quantities should verify a set of universal relations, allowing their complete determination in terms of a few parameters. The complexity of the expansions make however the direct verification of such relations essentially impossible, and other methods, like bosonization or operator product expansions, could not be applied in generic non solvable models. In [75] a proof of such universal relations has been given for the first time in non solvable lattice fermionic models.

4. Future directions

1. The interplay of the intrachain and interchain interaction in models of coupled chains model is of deep interest, also as a starting point in the understanding of non Fermi liquid behavior in higher dimensions.

2. Interacting spinning fermions with attractive interaction are one of the most important open problem in the physics of one dimensional interacting fermions, and I plan to study them by an approach based on Ward Identities and Renormalization Group.

4. QUANTUM FIELD THEORY

Non perturbative constructive results in Quantum field theory have been obtained, at low dimensions, only the case of superrenormalizable or asymptotically free models, like the Yukawa or the Gross-Neveu model with $N \geq 2$. However, in such models Ward Identities plays no role; on the other hand, it is well known that Ward Identities plays a major role even in the perturbative renormalizability proof of any realistic quantum field theory, starting from quantum electrodynamics in $4D$. However non perturbative constructive results are based on momentum regularizations, which violate the symmetries on which Ward Identities are based; dimensional regularization cannot at the moment be used beyond perturbation theory.

A. 2D Quantum Field Theory

1. Present results

1. The complete construction of the massive Thirring model in 2D, which is the simplest model requiring the implementation of Ward Identities for its construction [40]; the results are uniform in the fermionic mass, which can be taken arbitrarily small and whose presence destroys the exact solvability. The use of a lattice regularization allows the axioms verification.
2. A very important property of quantum anomalies is the validity of their *non-renormalization*, proved for QED in the perturbative Adler-Bardeen theorem. I have proven [42] a non perturbative version of the Adler-Bardeen result in 2D, provided that the interaction is superrenormalizable. In the renormalizable case the situation is more complex: the anomaly renormalization holds if the fermionic cut-off is removed before the bosonic one [43], while in the opposite situation the anomaly is renormalized by radiative corrections [40].
3. The first proof of the Coleman conjecture on the duality between boson and fermions [41].

B. 4D Quantum Field Theory

1. Present results

1. The non perturbative construction of the (infrared) Quantum Electrodynamics in four dimensions QED_4, with a small or vanishing electron mass,

a photon mass and an ultraviolet cut-off. The cut-off breaks the local symmetries but the Ward Identity is essentially insensitive to the ultraviolet cut-offs, in the sense that it has corrections which are however negligible, as far as momenta far from the ultraviolet scale are considered

2. Future directions

1. The derivation of the Ward Identity for chiral currents in infrared QED_4, and the non perturbative properties of the chiral anomalies.
2. The analysis of infrared QED_4 with no photon mass.
3. The renormalizability, at least order by order, of non Abelian gauge theories with momentum regularization instead than with the dimensional one.