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Tautological cycles on Jacobian varieties

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Abstract

In this paper we study the algebraic structure of the Tautological ring of a Jacobian: by the use of hard-Lefschetz-primitive classes we construct convenient generators that allow us to list and describe all the possible structures that may occur (the explicit list is given for $g \leq 9$ and for a few special curves).

§1. Introduction

Let $\mathcal{C}$ be a complex curve of genus g and let $J(\mathcal{C})$ denote its Jacobian. We consider the group of rational cycles modulo algebraic equivalence

$$\mathcal{A}_\bullet(J(\mathcal{C}))_{\mathbb{Q}}$$

and the so called "tautological ring" $\mathcal{R}(\mathcal{C})$, that is, the subgroup of $\mathcal{A}_\bullet(J(\mathcal{C}))_{\mathbb{Q}}$ containing $\mathcal{C}$ and stable with respect to the Fourier transform, the intersection product, the Pontryagin product, pull-backs and push-forwards of multiplication maps by integers. The tautological ring has a mysterious algebraic structure, which has not been completely understood so far. Beauville proved that $\mathcal{R}(\mathcal{C})$ is finite dimensional as a $\mathbb{Q}$-vector space [4]. He proved that the W^d's and their intersections generate $\mathcal{R}(\mathcal{C})$ as a vector space (this gives a rough bound for its dimension). Polishchuk found a set of equivalence relations between tautological cycles and also show compatibility with the action of the Fourier transform [19], Colombo-van Geemen [7] found relations for curves that are a m-cover of $\mathbb{P}^1$, Herbaut [12] and Kouvidakis-Van Der Geer [16] found relations for curves carrying a g^r_d. The difficulty of understanding the structure of $\mathcal{R}(\mathcal{C})$ comes from the fact that for a Jacobian, the cycles occurring in nature, such

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