Coupled dynamics of two extended bodies

Leon "Lee Lee" Simpson
Early-stage researcher with Astronet
University of Zielona Góra, Poland

Zielona Góra

- It's a very interesting city.
- Widely regarded as Europe's capital of art, architecture, music, shopping, sport and more.
- You should visit!

Celestial bodies as spheres

- Celestial bodies are not spheres, but modelling them as such is convenient as the resulting equations of motion are integrable.
- However, certain phenomena, such as **gravitational torque**, are not described by this model.
- This is of importance to the description of double asteroids...
  - ...and furthermore for planning missions to asteroids.

Extended bodies

- As a first approximation, take bodies to be rigid.
- One body may be well-approximated by a point-mass, depending on its geometry.
- Extended bodies may be well-approximated by **dumbbells**.
  - There are several variants of the dumbbell approach.
  - Only the very simplest variant is discussed in this talk.

The dumbbell model: motivation

- Extended rigid bodies are modelled by **two** spheres separated by an inflexible massless rod.
- This model explains gravitational torque.
- (Weakness) Can only model **prolate** bodies, not **oblate** bodies, **unless** complex masses and complex rod lengths are used.

Point-mass and solid dumbbell

- One way in which dumbbells can be classified is by whether the two spheres with which the dumbbell is composed have finite or zero radius.
- The zero radius case sometimes results in fewer equations of motion; however, it must be treated differently as the inertia tensor of such a body is singular in $\mathbb{R}^3$. This is the case that this discussion concerns.

$$\mathrm{rk}(I) = 2$$

State of literature

- Equations of motion are derived.
- Relative equilibria are identified.
  However. . .
- Bifurcation analysis of libration points is convoluted and incomplete.
- Stability analysis of libration points is similarly deficient.
- Invariant manifolds are not determined.
- Question of integrability remains unanswered.

Our contribution

- A novel global analysis of the point-mass dumbbell + point-mass problem, including:
  - An exhaustive analysis of the libration points, including their stability and any bifurcations that arise.
  - Proofs of the (non-)integrability of both the general point-mass dumbbell problem and the special case where the dumbbell is symmetric.
- Eventually, a similarly complete analysis for the more complex(!) dumbbell systems.

Equations of motion

$$\dot{R}_1 = P_1 - I^{-1}K_1R_3, \tag{1}$$

$$\dot{R}_3 = P_3 + I^{-1}K_1R_1, \tag{2}$$

$$\dot{P}_1 = \frac{P_2^2}{R_1} - I^{-1}\left(\frac{K_2R_3P_2}{R_1} + K_1P_3\right) - \frac{\mu(1+\mu)^2R_1}{\left\|(1+\mu)\mathbf{R} - \left(\begin{smallmatrix}0\\0\\1\end{smallmatrix}\right)\right\|^3} - \frac{(1+\mu)^2R_1}{\left\|(1+\mu)\mathbf{R} + \mu\left(\begin{smallmatrix}0\\0\\1\end{smallmatrix}\right)\right\|^3}, \tag{3}$$

$$\dot{P}_2 = -\frac{P_1P_2}{R_1} + I^{-1}K_2\frac{R_3P_1 - R_1P_3}{R_1}, \tag{4}$$

$$\dot{P}_3 = I^{-1}\left(K_1P_1 + K_2P_2\right) - \frac{\mu(1+\mu)(-1+R_3+R_3\mu)}{\left\|(1+\mu)\mathbf{R} - \left(\begin{smallmatrix}0\\0\\1\end{smallmatrix}\right)\right\|^3} - \frac{(1+\mu)(R_3+\mu+R_3\mu)}{\left\|(1+\mu)\mathbf{R} + \mu\left(\begin{smallmatrix}0\\0\\1\end{smallmatrix}\right)\right\|^3}, \tag{5}$$

$$\dot{K}_1 = \frac{-I^{-1}K_2^2R_3 + K_2P_2}{R_1} + \frac{\mu(1+\mu)R_1}{\left\|(1+\mu)\mathbf{R} - \left(\begin{smallmatrix}0\\0\\1\end{smallmatrix}\right)\right\|^3} - \frac{\mu(1+\mu)R_1}{\left\|(1+\mu)\mathbf{R} + \mu\left(\begin{smallmatrix}0\\0\\1\end{smallmatrix}\right)\right\|^3}, \tag{6}$$

$$\dot{K}_2 = I^{-1}K_1\frac{K_2R_3 - IP_2}{R_1}. \tag{7}$$

QUESTIONS?