Jet Transport and applications

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Numerical detection of invariant manifolds in Dynamical Systems using:

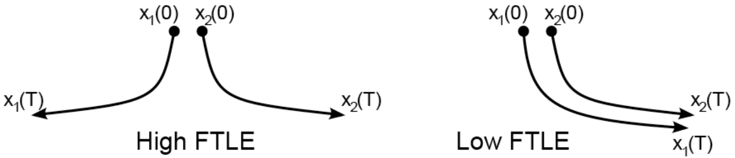
- Lagrangian Coherent Structures (LCS) by means of Finite Time Lyapunov Exponents (FTLE)
- Jet Transport

Let $\varphi_{t_0}^{t_f}(\vec{x})$ the flow map from time t_0 to time t_f with initial conditions $\vec{x}$, then:

Definition

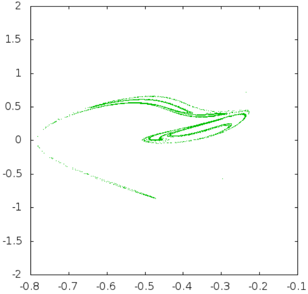
The FTLE at $\vec{x}$ from $t = t_0$ up to $t = T$ is defined as:

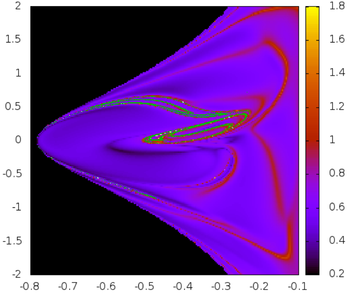
$$\sigma_{t_0}^{t_f}(\vec{x}) := \frac{1}{|T|} \ln \left\| \frac{d\varphi_{t_0}^{t_0+T}(\vec{x})}{dx} \right\|_2 ,$$



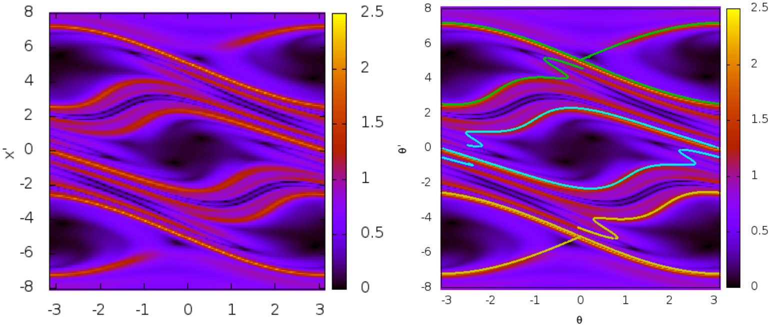
Therefore the invariant FTLE are able to detect hyperbolic invariant manifolds.

Numerical FTLE results. The Circular Restricted Three Body Problem





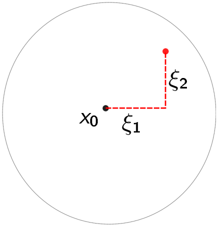
$$\ddot{x} = \left(\frac{5}{2}\cos(t) - 1\right)\sin(x).$$



Given a dynamical system $\dot{x} = f(t, x)$ and the associated flow map:

$$\varphi(t; t_0, x_0)$$

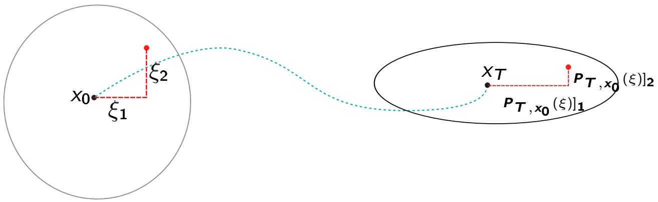
Goal: Compute $\varphi(t; t_0, U)$ where U is a certain neighbourhood of x_0.

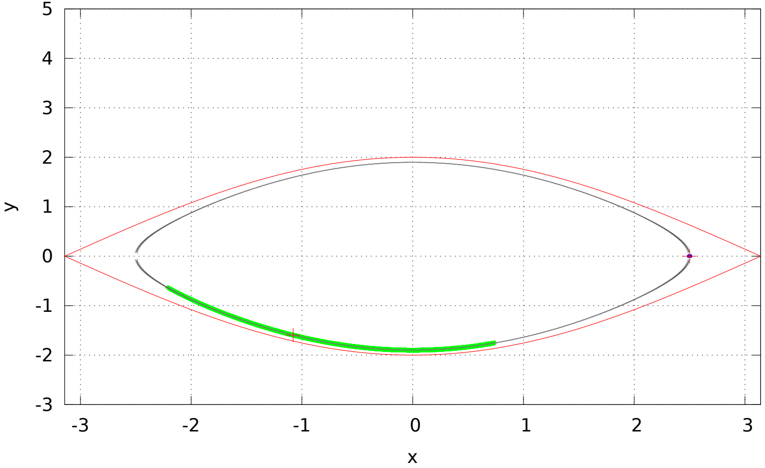


parametrized by the polynomial $P_{t_0,x_0}(\xi) = x_0 + \xi = x_0 + (\xi_1, ..., \xi_n)^T$.

We can use any integration method (RK, Taylor's, ...) but taking into account that all the operations that usually are done with **real numbers** now must be done with **polynomials**, using polynomial arithmetic.

$\varphi(t; t_0, U)$ will be a polynomial, $P_{T,x_0}(\xi)$, giving the positions at time T as a function ξ.




5
4
3
2
1
0
-1
-2
-3
y
-3
-2
-1
0
1
2
3
x

Jet Transport Applications (in progress and foreseen)

- Computation of Lagrangian Coherent Structures and other indicators
- Propagation of uncertainties and prediction of space debris location
- Formation Flight (in col. with Fabrizio Paita)

Thanks for your attention