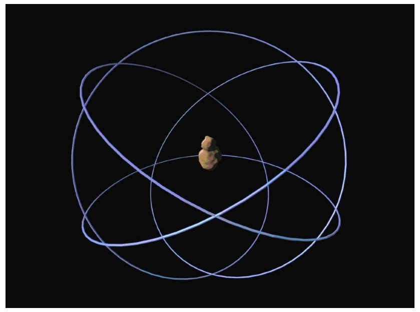


The Dynamical Environment about Asteroids: Orbit mechanics of particles and spacecraft

What are the Challenges?

- Much work has been focused on the *pathways* to small bodies, but not so much on what to do on arrival...
- ... but that is where things get interesting, as the small body dynamical environment is one of the most perturbed environments found in the solar system
  - Gravity and rotational effects can destabilize an orbit, causing impact or escape on time scales of less than a day.
  - Solar radiation pressure perturbations can strip a spacecraft out of orbit or cause an impact.
  - Coupled effects from these perturbations can cause chaotic orbit dynamics.

What are the Challenges?

Gravitational Effects:

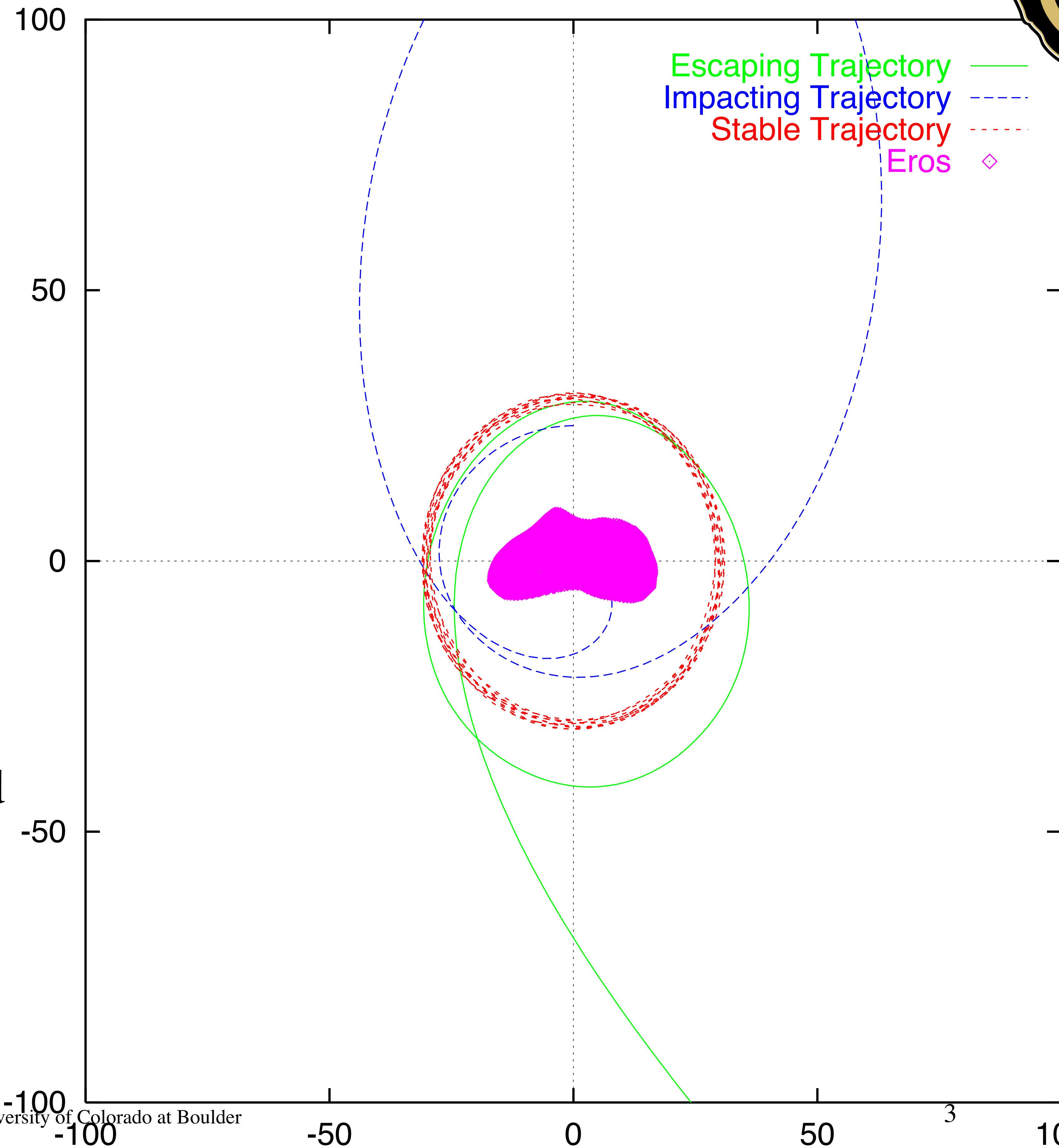
Simulation of spacecraft orbits at 433 Eros.

Stable Orbit

Impacting Orbit

Escaping Orbit

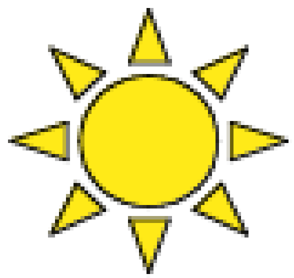
Small changes in initial conditions yield large variations in outcome.



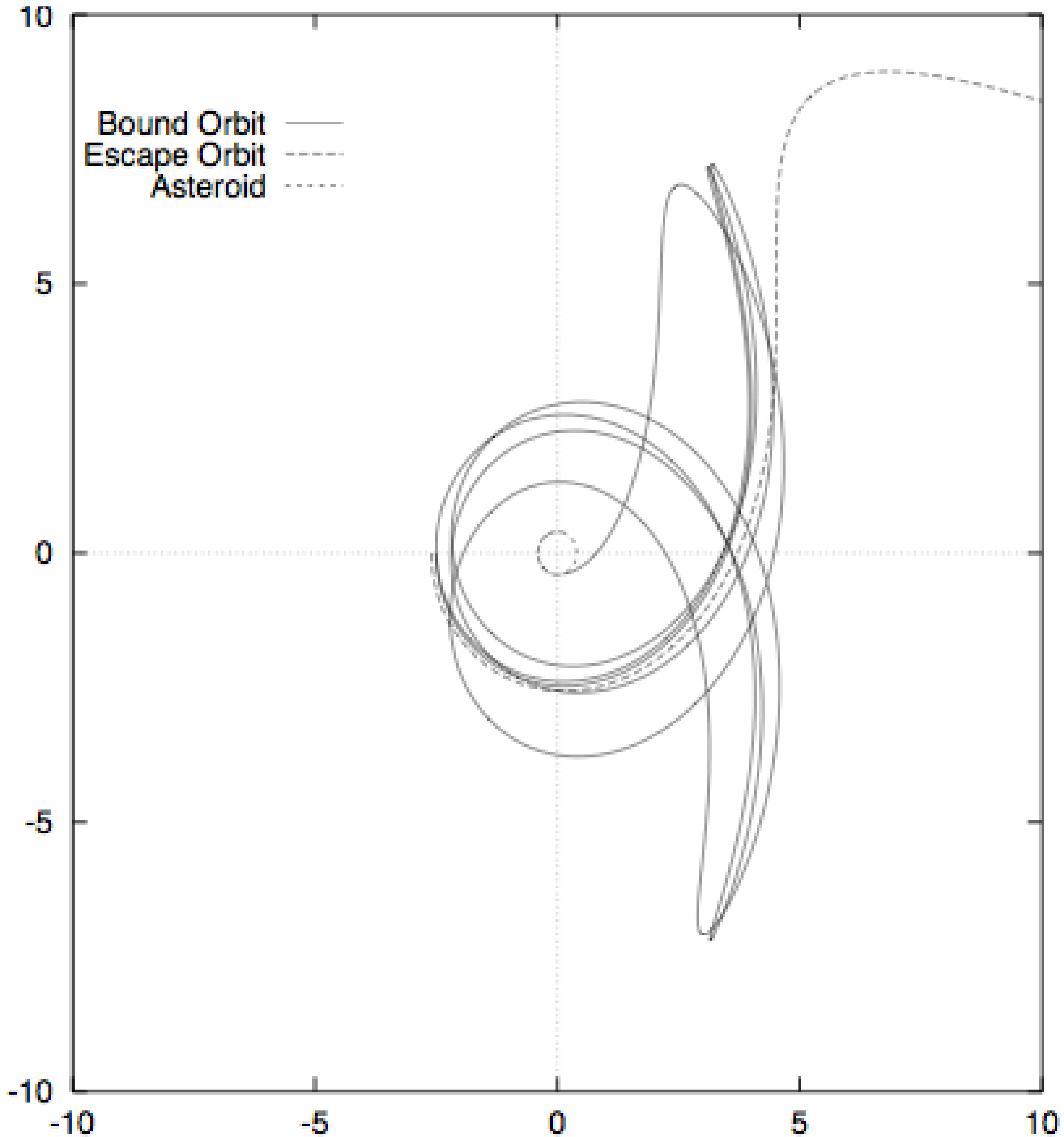
What are the Challenges?

Solar Radiation Pressure Effects:

S/C orbits about a small point mass

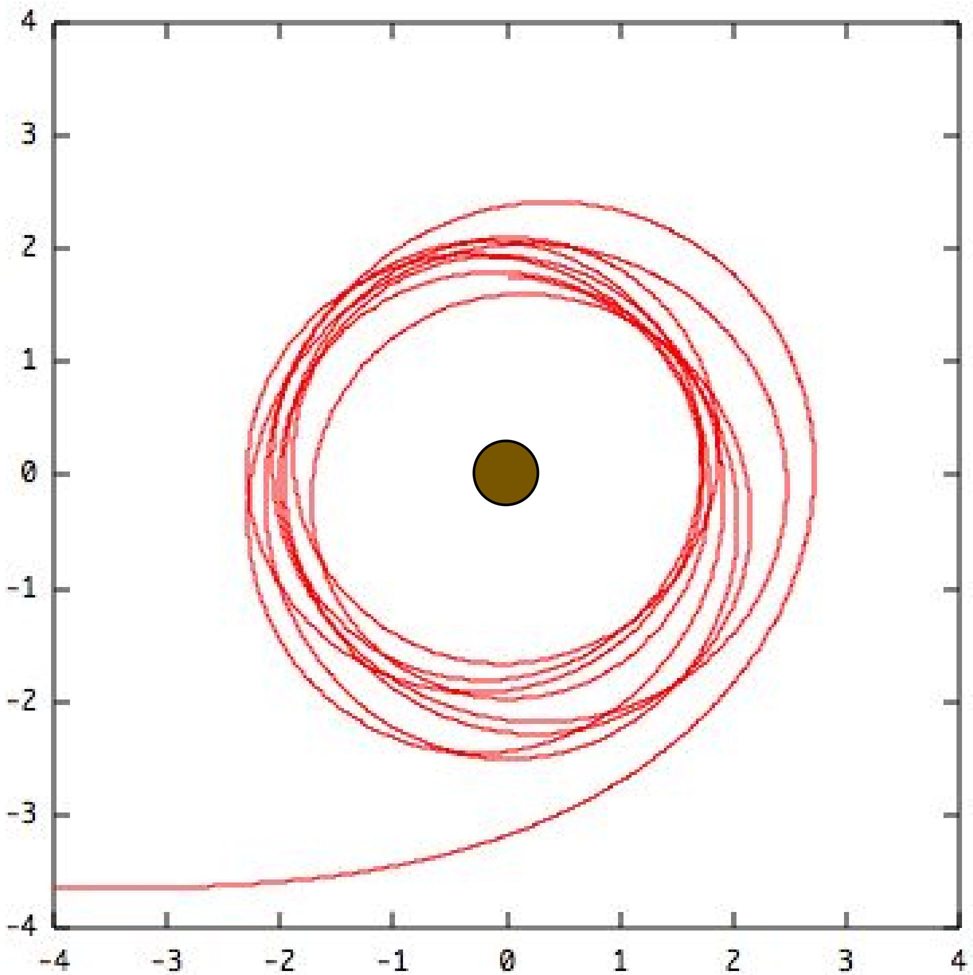


A 100 meter difference in initial conditions can change escape to impact

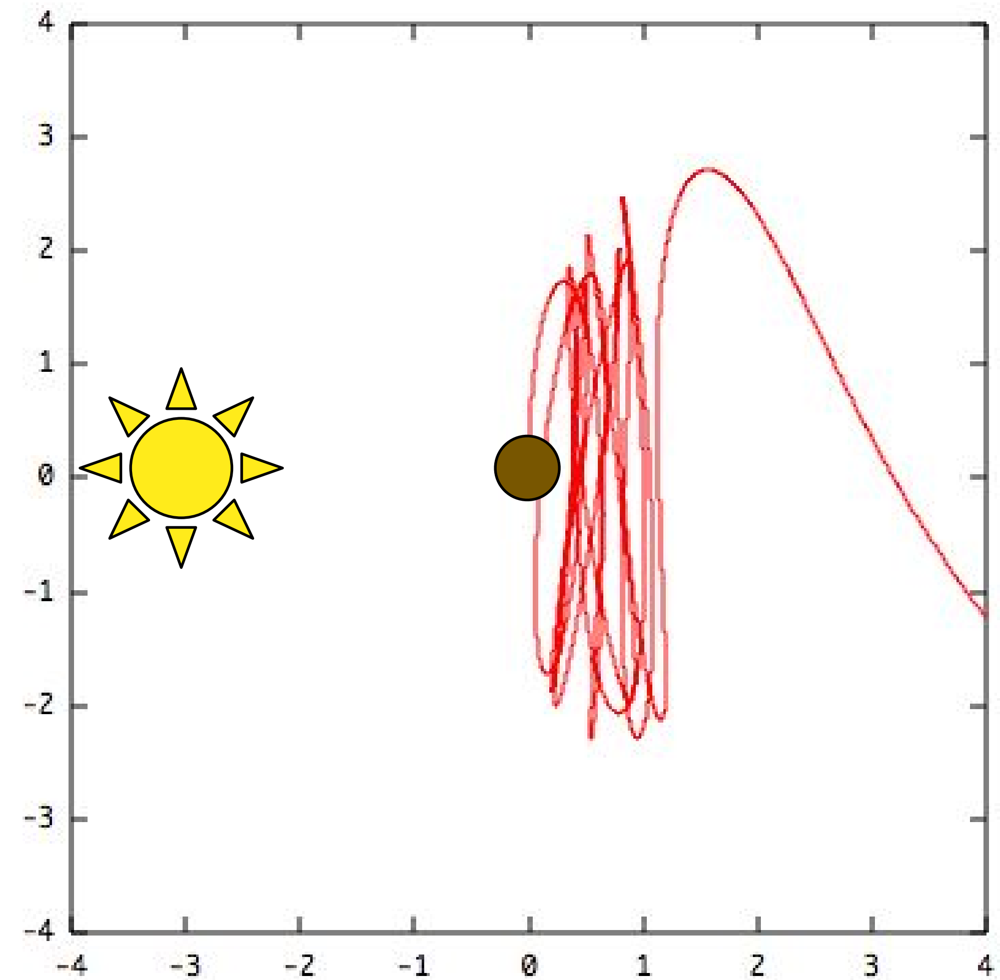


What are the Challenges?

Escape due to SRP



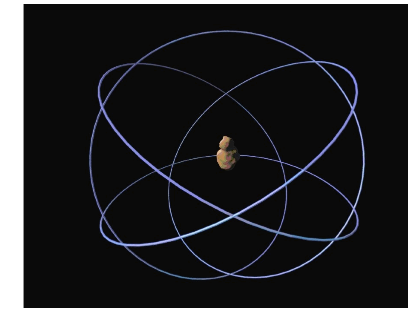
View from the Sun



View in the terminator plane

a ~ constant in orbit perturbed only by SRP

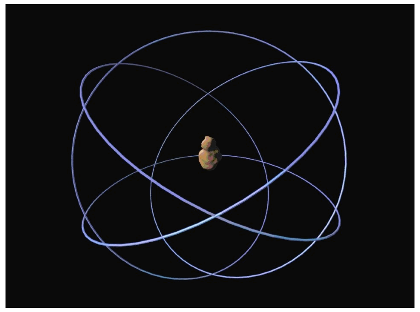
S/C escapes once body travels too close to the sun



Significant Forces

- Asteroid gravitational attraction:
  - Non-spherical mass distributions create a non-Keplerian field
- Rotation of the object
  - Creates centrifugal forces that act on the asteroid surface
  - Create resonances between orbital motion and the rotating gravity field
- Solar radiation pressure
  - Acts on particles and spacecraft to create a drag force
- Solar gravity
  - Acts on both the particle and the asteroid to create a tidal force

Simple Surface Relationships



- Surface gravity: $$\frac{4\pi\mathcal{G}}{3}\rho R$$
- Surface speed: $$\sqrt{\frac{4\pi\mathcal{G}\rho}{3}}R$$
- Surface period: $$\sqrt{\frac{3\pi}{\mathcal{G}\rho}}$$
- R is asteroid radius, ρ is asteroid density

Characteristic Values

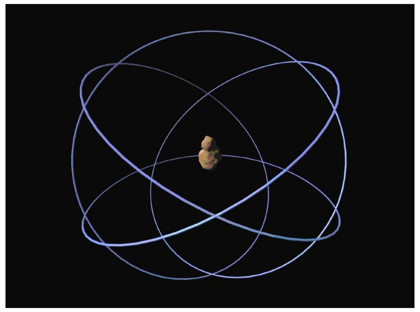
Asteroid	Mean Radius (km)	Density (g/cm^3)	GM (km^3/s^2)	Surface Gravity (g)	Surface Speed (m/s)	Surface Period (h)
Itokawa	0.162	2	2.4E-09	9E-06	0.12	2.3
	1.0	2	5.6E-07	6E-5	0.75	2.3
Eros	8.97	2.5	5E-04	6E-4	7.5	2.0
Mathilde	26.5	1.3	6.8E-03	1E-3	16	2.8
Vesta	265	3.3	17	2E-2	250	1.8
Earth	6400	~5	4E+05	1	7900	1.4

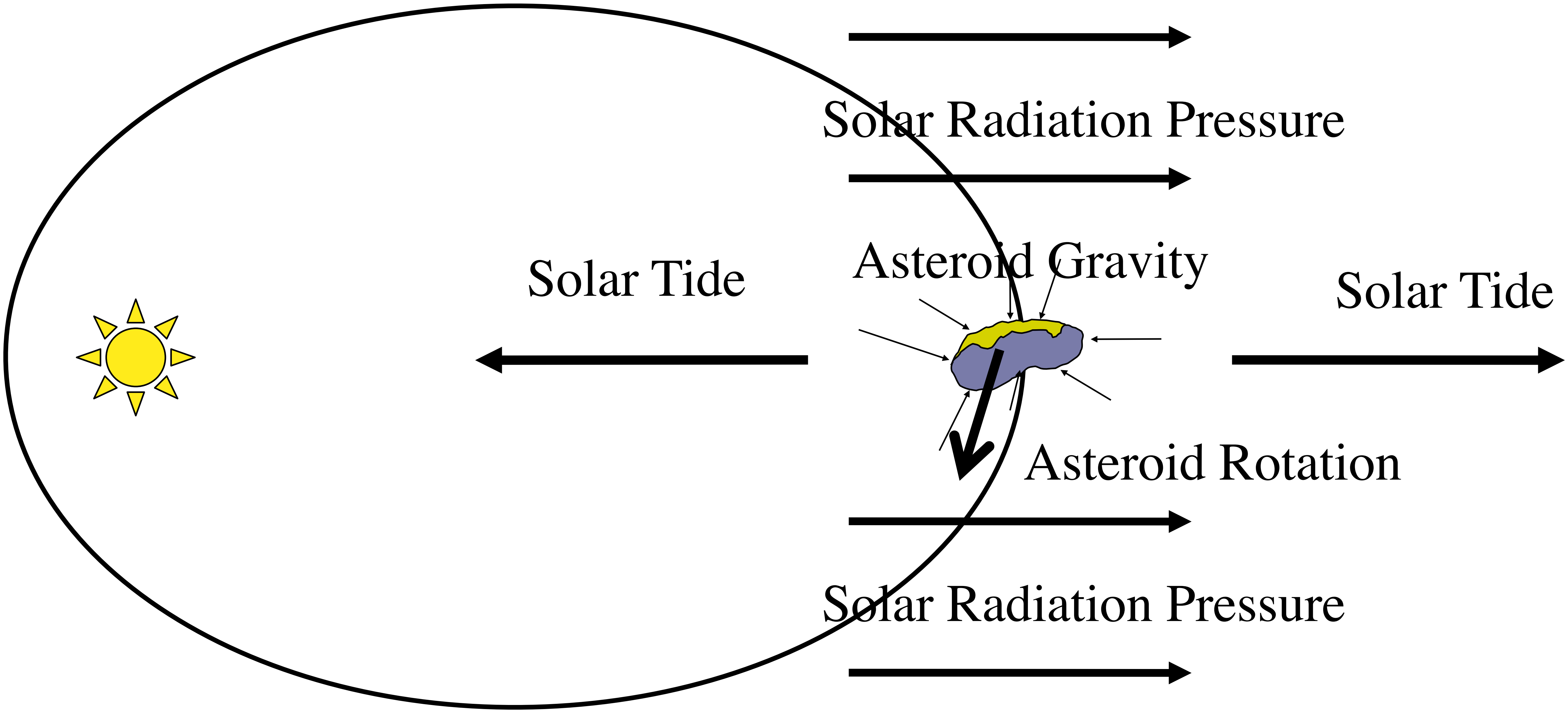
Surface Gravity vs ...

- Solar Radiation Pressure:
  - Equal when $r_{part} \sim \dfrac{1E-6}{3R_{ast}}$

Asteroid	Particle
0.1 km	3 mm
1 km	300 um
10 km	30 um
100 km	3 um

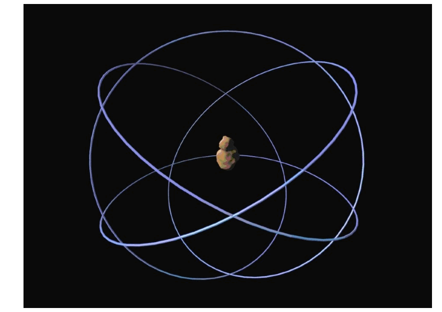
- Solar Gravity:
  - Ratio ~ 1E-7, negligible on the surface
- Both relative perturbations scale with distance from asteroid $\propto r^2$





Solar Radiation Pressure
Asteroid Gravity
Solar Tide
Solar Tide
Asteroid Rotation
Solar Radiation Pressure

Equations of Motion



- Inertial frame:

$$\ddot{\boldsymbol{r}}_I \quad = \quad T \cdot \frac{\partial U}{\partial \boldsymbol{r}} + \frac{\mathcal{G}_s}{d^2 B}\hat{\boldsymbol{d}} + \frac{\mu_s}{d^3}\left[3(\hat{\boldsymbol{d}} \cdot \boldsymbol{r})\hat{\boldsymbol{d}} - \boldsymbol{r}\right]$$

- Body-Fixed frame:

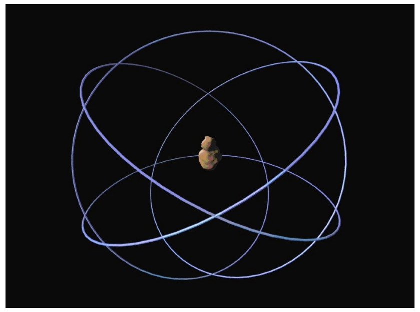
$$\ddot{\boldsymbol{r}} + 2\tilde{\boldsymbol{\omega}} \cdot \dot{\boldsymbol{r}} + \dot{\tilde{\boldsymbol{\omega}}} \cdot \boldsymbol{r} + \tilde{\boldsymbol{\omega}} \cdot \tilde{\boldsymbol{\omega}} \cdot \boldsymbol{r} \quad = \quad \frac{\partial U}{\partial \boldsymbol{r}} + \frac{\mathcal{G}_s}{d^2 B}\boldsymbol{T}^T \cdot \hat{\boldsymbol{d}} + \frac{\mu_s}{d^3}\left[3(\hat{\boldsymbol{d}} \cdot \boldsymbol{r})\boldsymbol{T}^T \cdot \hat{\boldsymbol{d}} - \boldsymbol{r}\right]$$

- Orbit-Fixed frame:

$$\ddot{\boldsymbol{r}} + 2\frac{\sqrt{\mu_s p_s}}{d^2}\tilde{\boldsymbol{z}} \cdot \dot{\boldsymbol{r}} - \frac{\mu_s}{d^3}\left[2e_s \sin\nu\,\tilde{\boldsymbol{z}} + (1 + e_s\cos\nu)\left(\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}}\hat{\boldsymbol{y}}\right)\right] \cdot \boldsymbol{r} =$$

$$\frac{\mathcal{G}_s}{d^2 B}\hat{\boldsymbol{x}} + \frac{\mu_s}{d^3}\left[3(\hat{\boldsymbol{x}} \cdot \boldsymbol{r})\hat{\boldsymbol{x}} - \boldsymbol{r}\right] + \boldsymbol{T}_o^T \cdot \boldsymbol{T} \cdot \frac{\partial U}{\partial \boldsymbol{r}}$$

Tide + SRP

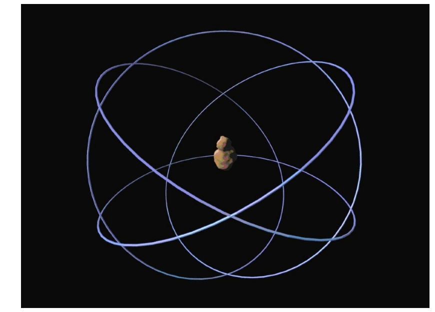


- For asteroids, the solar tide effects extend beyond the influence of the body's higher-order gravity fields, allowing us to treat it as a point mass
- Changing the independent parameter in the sun-frame orbit to true anomaly and scaling by the sun/asteroid distance yields a "pulsating" frame model:

$$
\begin{aligned}
\mathbf{r}'' + 2\hat{\mathbf{z}} \times \mathbf{r}' + (\hat{\mathbf{z}} \cdot \mathbf{r})\,\hat{\mathbf{z}} &= \frac{1}{1 + e_S \cos\nu} \frac{\partial U}{\partial \mathbf{r}} \\
U &= \frac{1}{|\mathbf{r}|} + \beta \hat{\mathbf{d}} \cdot \mathbf{r} + \frac{3}{2}\left(\hat{\mathbf{d}} \cdot \mathbf{r}\right)^2
\end{aligned}
$$

- NEAR at Eros has a value of $\beta \sim 1$, Hayabusa and Rosetta have $\beta \sim 30$

Jacobi-like integral

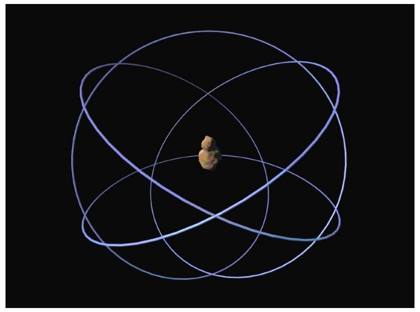


- Due to the pulsating frame (eccentric orbit) a Jacobi integral does not exist for this problem
  - Can define a related, non-conserved quantity:

$$\Gamma = 2 \quad U(\mathbf{r}) - (1 + e_S \cos\nu)\left[v^2 + z^2\right]$$
$$\Gamma' = e_S \sin\nu \left[v^2 + z^2\right]$$

  - Zero-velocity curves can be defined with Γ when $z = 0$
- Provides a restriction: $\Gamma \leq 2U(\boldsymbol{r})$
  - which can be used to develop necessary conditions for escape

Tide + SRP cont.



- Despite the pulsation, this system has fixed equilibrium points:

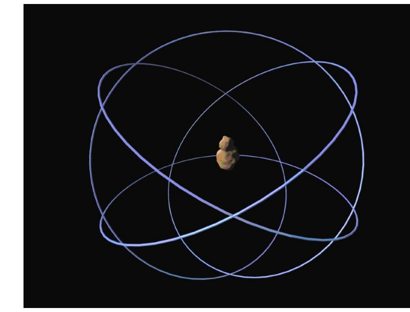
$$x^* \quad \sim \quad \pm\left(\frac{1}{3}\right)^{1/3} - \frac{\beta}{9} \pm \frac{3^{1/3}}{81}\beta^2 + \dots \;\; \text{if } \beta \ll 1$$

$$x^* \quad \sim \quad \begin{cases} -\frac{1}{3}\beta - \frac{9}{\beta^2} + \dots & x^* < 0 \\ \frac{1}{\sqrt{\beta}} - \frac{3}{2\beta^2} + \dots & x^* > 0 \end{cases} \quad \text{if } \beta \gg 1$$

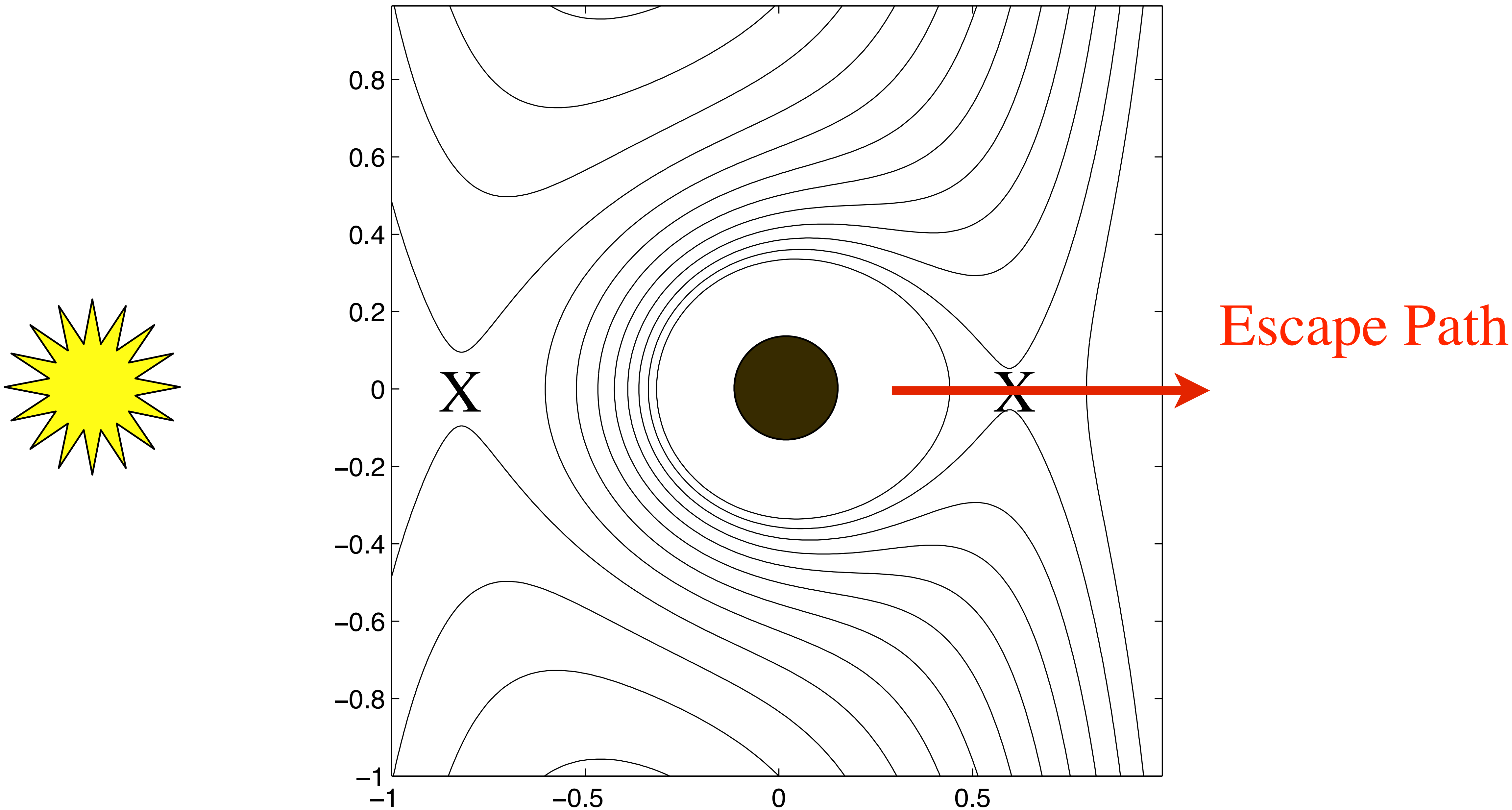
  – The sun-ward equilibrium point can be used as a monitoring site for a comet when passing through perihelion

  – The anti-sun point provides a sufficient condition for escape

Tide + SRP cont.

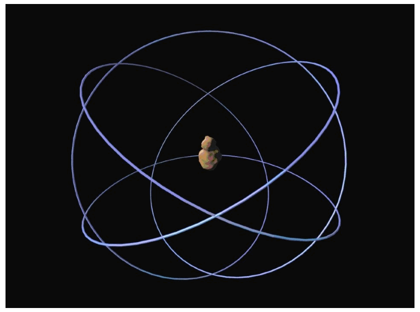


Curves of constant Γ



Dynamics has been studied for gravity-only case by Burns and Hamilton in the early 1990’s

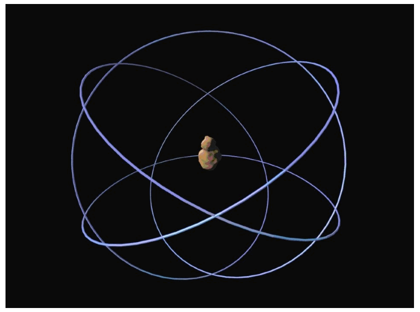
SRP only perturbations



- For small asteroids the SRP perturbations can dominate over solar tidal effects
  - Especially true for spacecraft dynamics at small bodies
  - Preferred frame of analysis is the Inertial frame, subsequently transported into the orbit-fixed frame
  - Can determine limits for captured orbits and perform an averaging analysis:

$$\ddot{\boldsymbol{r}}_I = -\frac{\mu}{r^3}\boldsymbol{r} + T \cdot \frac{\partial R}{\partial \boldsymbol{r}} + \frac{\mathcal{G}_s}{d^2 B}\hat{\boldsymbol{d}} + \cancel{\frac{\mu_s}{d^3}\left[3(\hat{\boldsymbol{d}}\cdot\boldsymbol{r})\hat{\boldsymbol{d}} - \boldsymbol{r}\right]}$$

Orbit Mechanics Issues

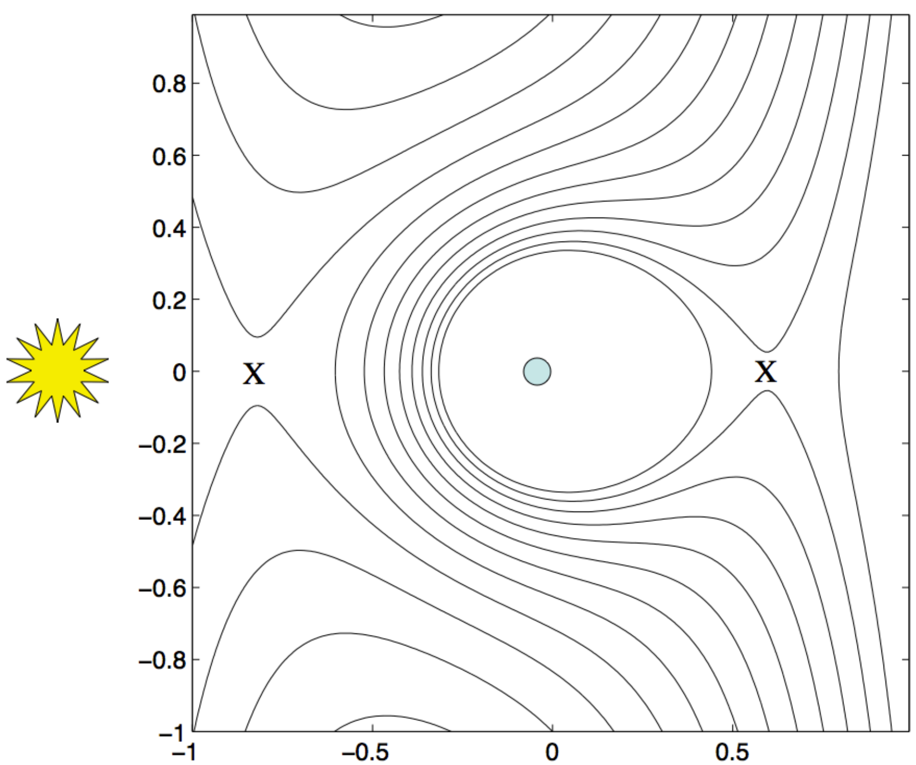


- Escape
  - Solar radiation pressure (SRP) can strip a S/C out of orbit
- Impact
  - SRP can cause eccentricity to approach unity, leading to impact
- Coupled perturbations
  - Joint perturbations from asteroid mass distribution and SRP can cause impact or escape
- An understanding of these issues can be described as a function of S/C and system parameters
- This allows the robust design of S/C missions to these bodies

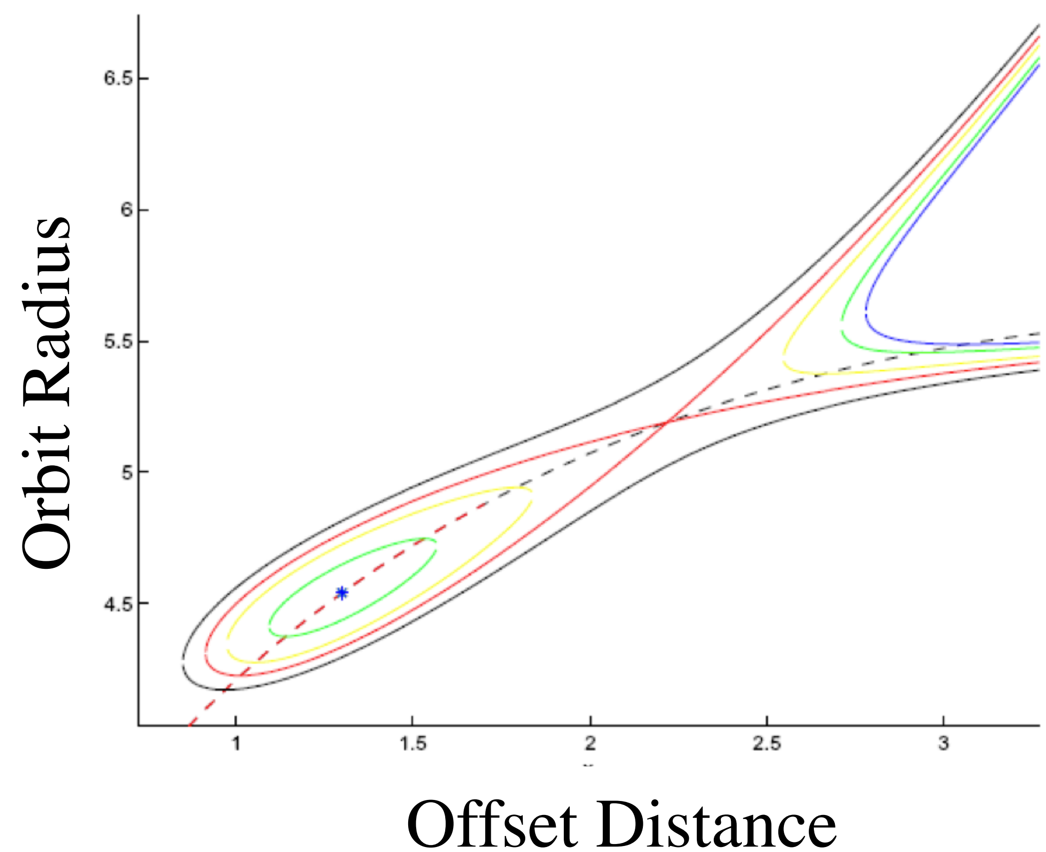
Escape Limits

Maximum semi-major axis for bound orbits: $a_{max} \sim \frac{\sqrt{3}}{4}\sqrt{\frac{\mu}{(A/m)P_\Phi}}d$

Semi-major axis remains constant until $a > a_{max}$ and then escapes.
Orbiter traveling towards perihelion can be lost as d decreases.

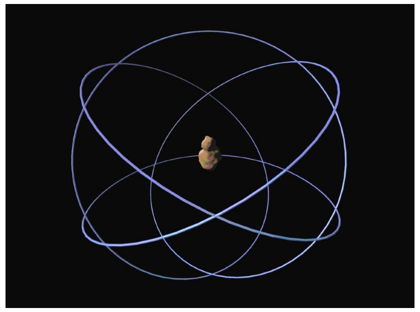


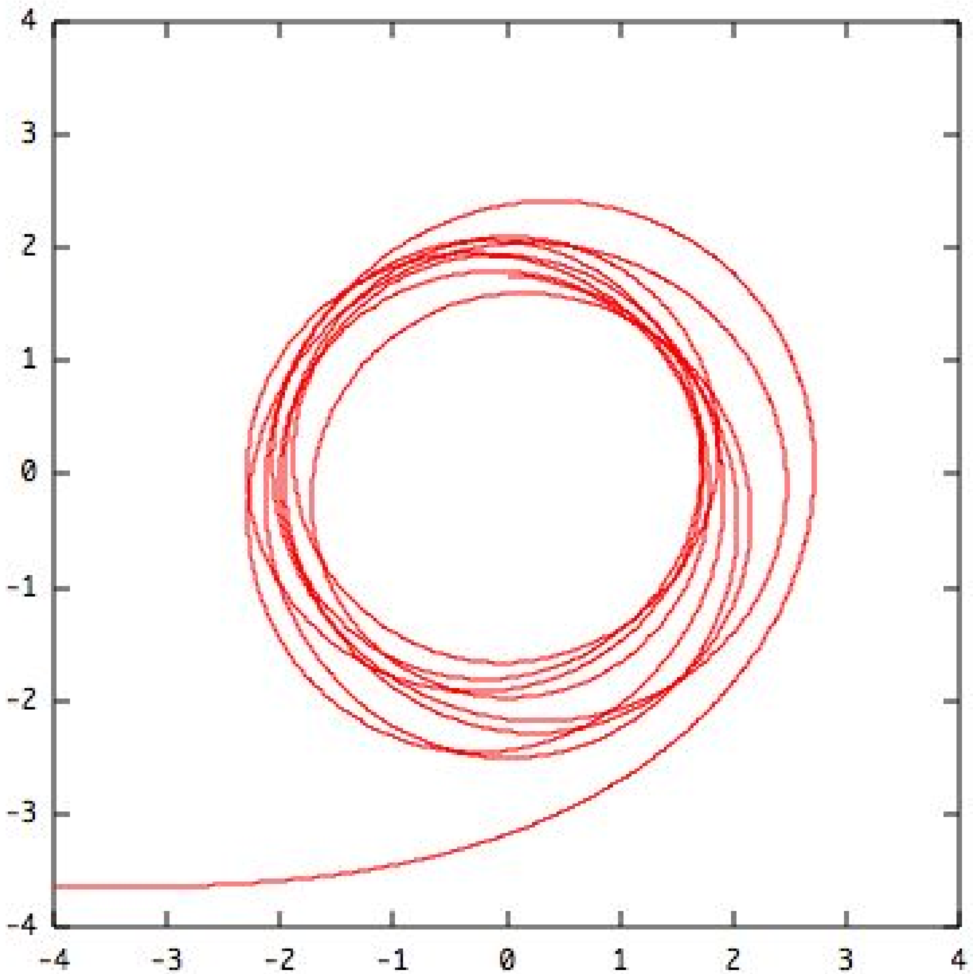
Zero-Velocity Curves in the Elliptic-Restricted SRP Problem



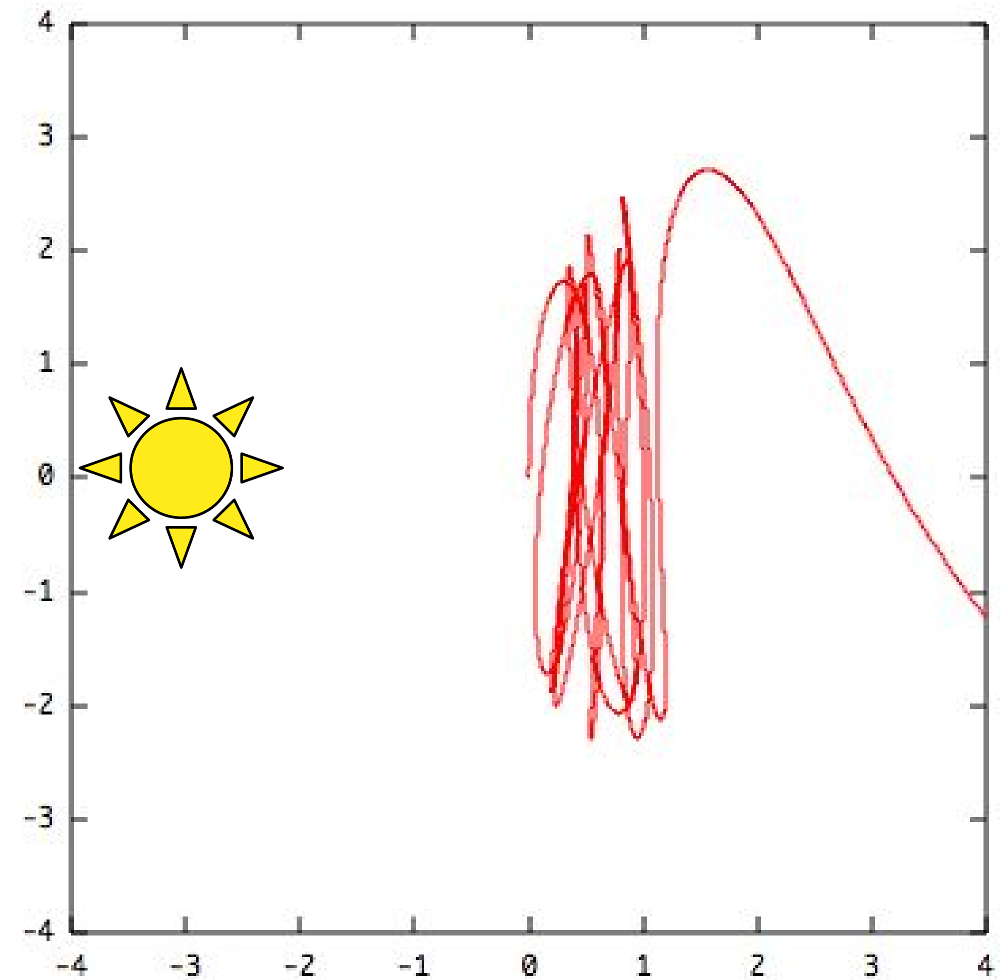
Zero-Velocity Curves in the Non-Rotating SRP Problem

Escape example due to SRP





View from the Sun



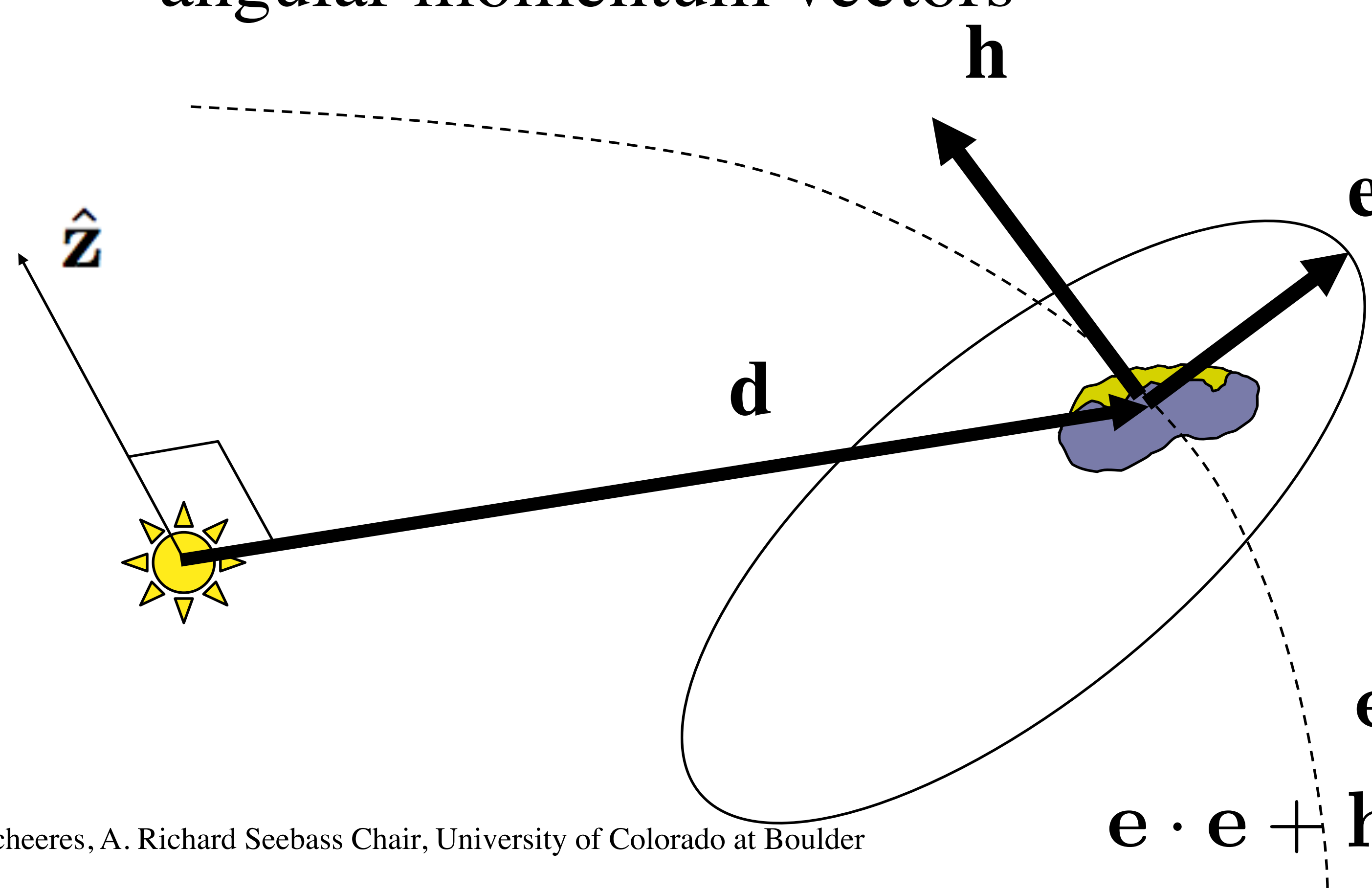
View in the terminator plane

a ~ constant in orbit perturbed only by SRP

S/C escapes once $a > a_{max}$ as d decreases

Averaged Orbit Mechanics for SRP

- If $a < a_{max}$ averaging can be applied
  - Semi-major axis a is constant on average
  - The secular equations can be solved in closed form, assuming a point mass (Mignard and Henon, 1984 and Richter and Keller, 1995), and generalized to the case of an asteroid orbiting the sun on an elliptic orbit (Scheeres 2009).
  - Solution is simplest to state using the osculating eccentricity and angular momentum vectors



$$\overline{\mathcal{R}} = -\frac{3\mathcal{G}_S a}{2Bd^2}\mathbf{e}\cdot\hat{\mathbf{d}}$$

$$\begin{aligned} a &= \text{Constant} \\ |\mathbf{h}| &= \sqrt{1-e^2} \\ |\mathbf{e}| &= e \\ \mathbf{e}\cdot\mathbf{h} &= 0 \\ \mathbf{e}\cdot\mathbf{e}+\mathbf{h}\cdot\mathbf{h} &= 1 \end{aligned}$$

Averaged SRP Equations

- In a frame rotating with the sun-line, with the heliocentric orbit true anomaly as the independent parameter:

$$\begin{bmatrix} \mathbf{e}' \\ \mathbf{h}' \end{bmatrix} = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -\tan\Lambda \\ 0 & 0 & 0 & 0 & \tan\Lambda & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\tan\Lambda & -1 & 0 & 0 \\ 0 & \tan\Lambda & 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} \mathbf{e} \\ \mathbf{h} \end{bmatrix}$$

$$\tan\Lambda = \frac{3\mathcal{G}_S}{2B}\sqrt{\frac{a}{\mu\mu_{sun}a_{sun}(1-e_{sun}^2)}}$$

  - For a strong perturbation, Λ -> π/2
  - For a weak perturbation, Λ -> 0
  - Hayabusa at Itokawa, Λ ~87° NEAR at Eros, Λ ~13°

Solution to the Eqns

- A Linear, Time Invariant System, its solution can be expressed as:

$$\begin{bmatrix} \mathbf{e} \\ \mathbf{h} \end{bmatrix} = \Phi(\nu/\cos\Lambda) \begin{bmatrix} \mathbf{e}_o \\ \mathbf{h}_o \end{bmatrix}$$

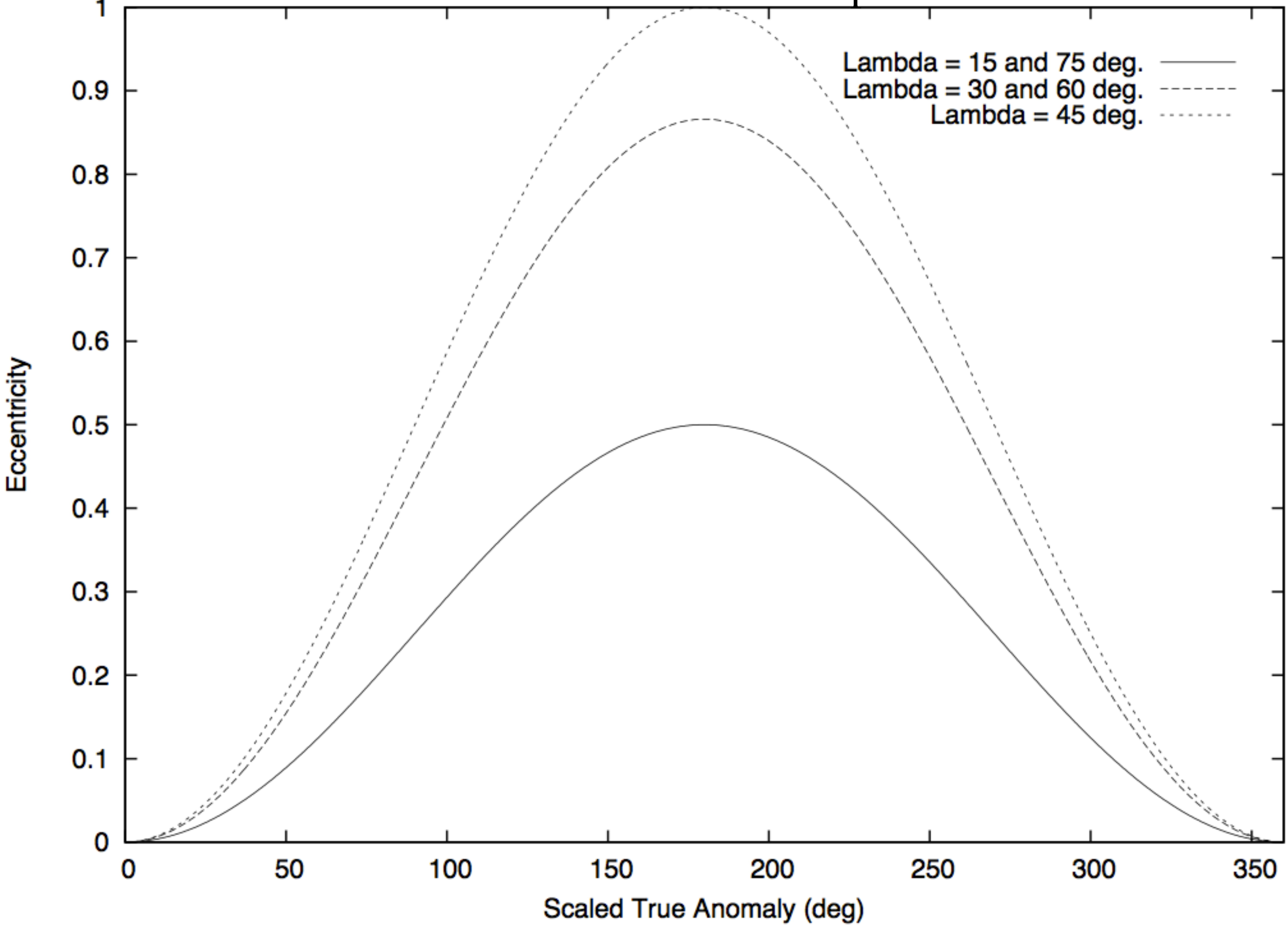
  – Φ is a 6x6 orthonormal rotation matrix, periodic with period $2\pi/\cos(\Lambda)$

$$\begin{aligned}\Phi(\psi) &= \cos(\psi) I_{6\times 6} + \\ & [1-\cos(\psi)] \begin{bmatrix} \cos^2\Lambda\hat{\mathbf{z}}\hat{\mathbf{z}} + \sin^2\Lambda\hat{\mathbf{d}}\hat{\mathbf{d}} & -\sin\Lambda\cos\Lambda\left(\hat{\mathbf{z}}\hat{\mathbf{d}} + \hat{\mathbf{d}}\hat{\mathbf{z}}\right) \\ -\sin\Lambda\cos\Lambda\left(\hat{\mathbf{z}}\hat{\mathbf{d}} + \hat{\mathbf{d}}\hat{\mathbf{z}}\right) & \cos^2\Lambda\hat{\mathbf{z}}\hat{\mathbf{z}} + \sin^2\Lambda\hat{\mathbf{d}}\hat{\mathbf{d}} \end{bmatrix} \\ & + \sin(\psi) \begin{bmatrix} -\cos\Lambda\tilde{\hat{\mathbf{z}}} & \sin\Lambda\tilde{\hat{\mathbf{d}}} \\ \sin\Lambda\tilde{\hat{\mathbf{d}}} & -\cos\Lambda\tilde{\hat{\mathbf{z}}} \end{bmatrix}\end{aligned}$$

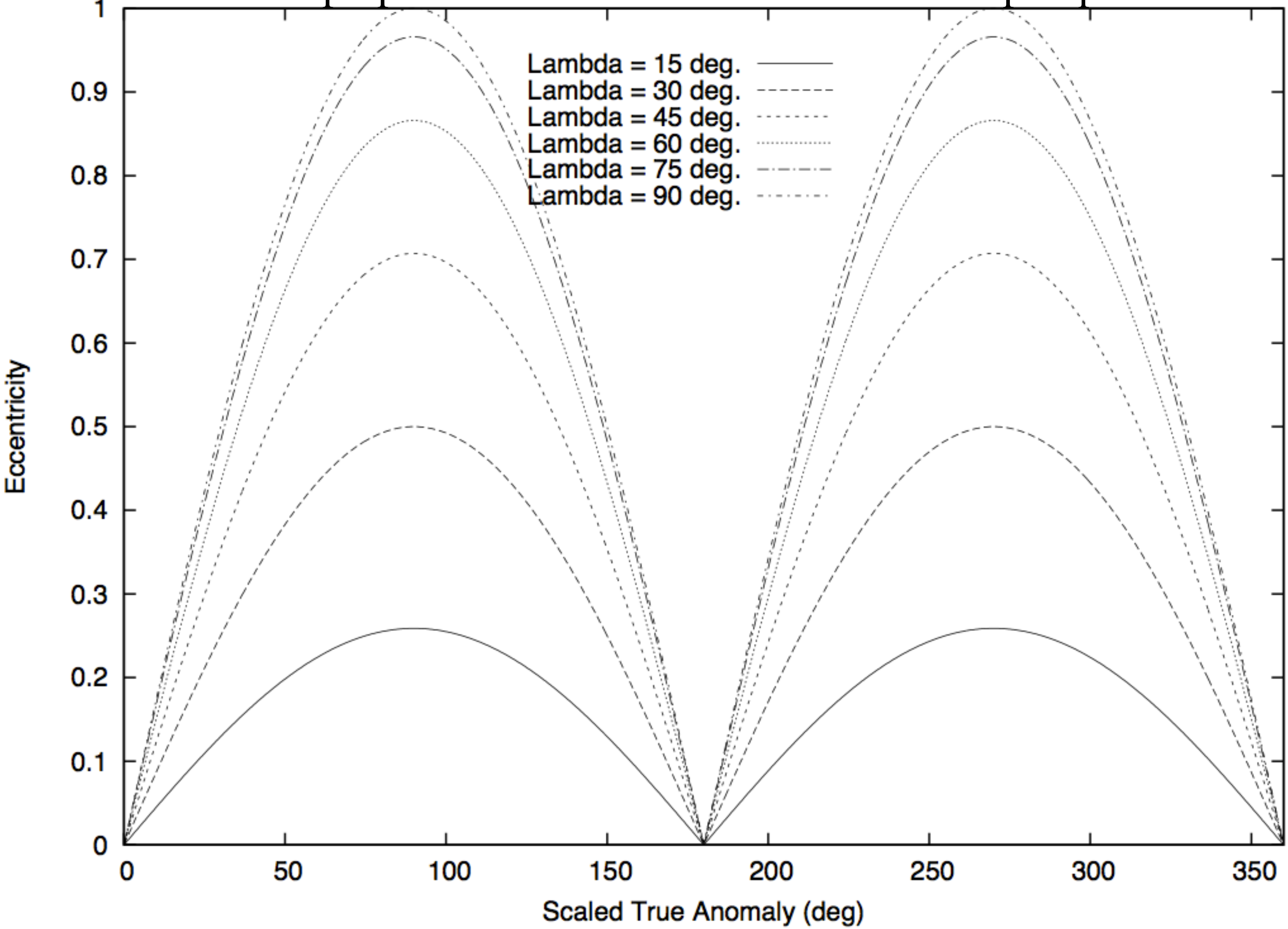
  – Secular motion is periodic in true anomaly with period $2\pi/\cos(\Lambda)$

  – Orbital evolution changes drastically as a function of Λ

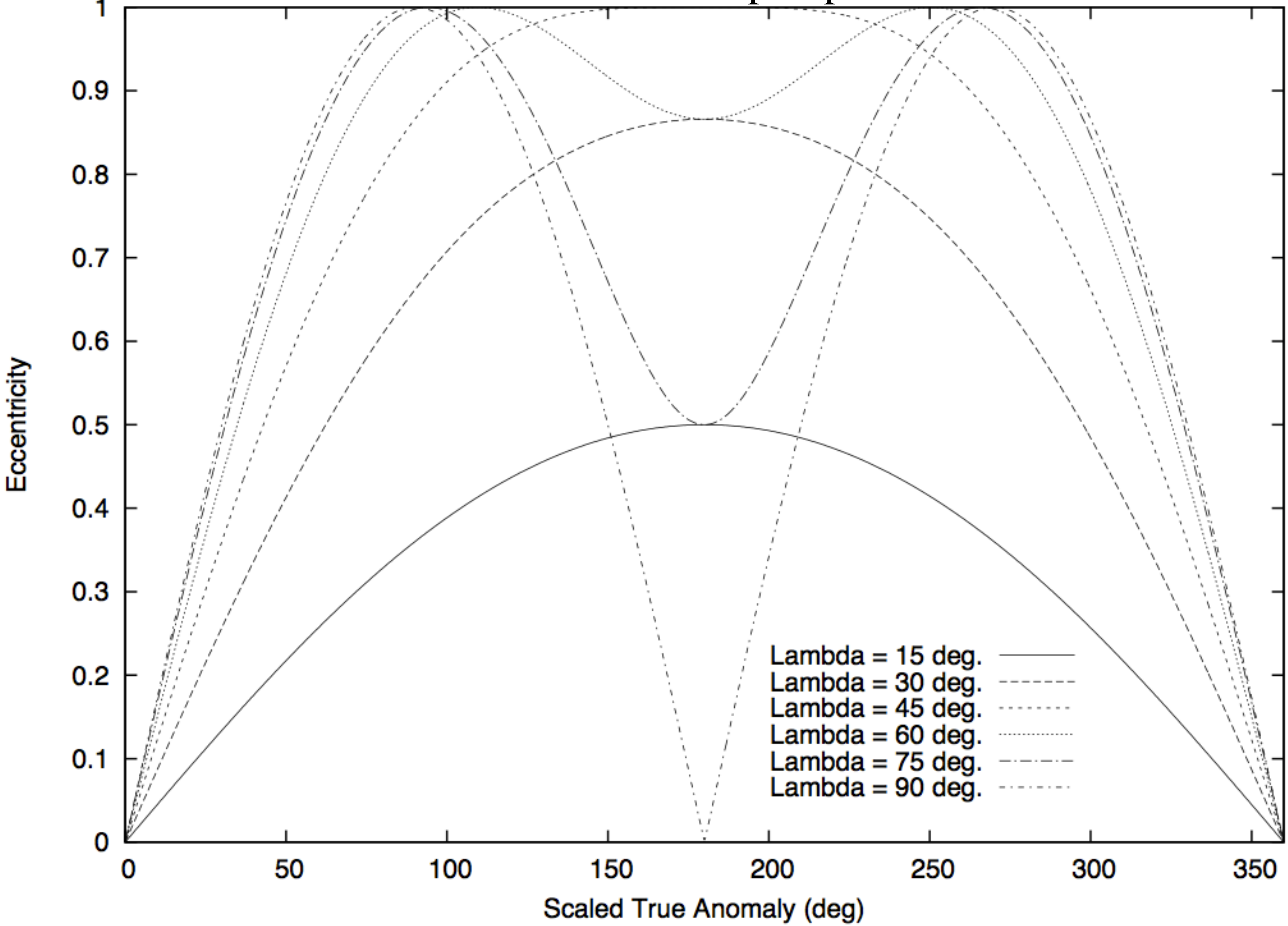
Orbit starts in the terminator plane



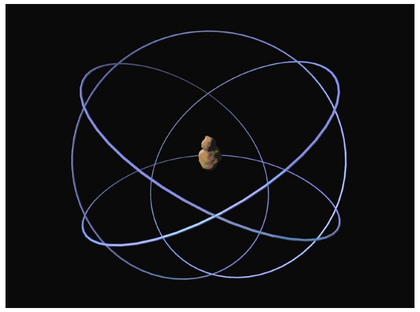
Orbit starts perpendicular to the terminator and ecliptic plane



Orbit starts in the ecliptic plane



SRP Frozen Orbits



- Two types of frozen orbits exist:

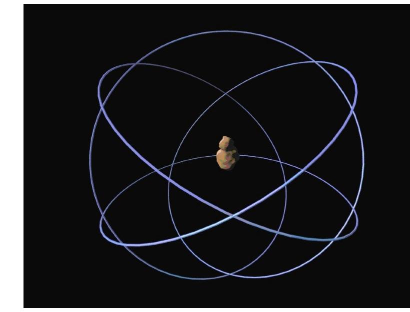
- ~~Frozen orbits in the asteroid orbit plane~~
  - ~~Periapsis vector parallel to sun-line~~

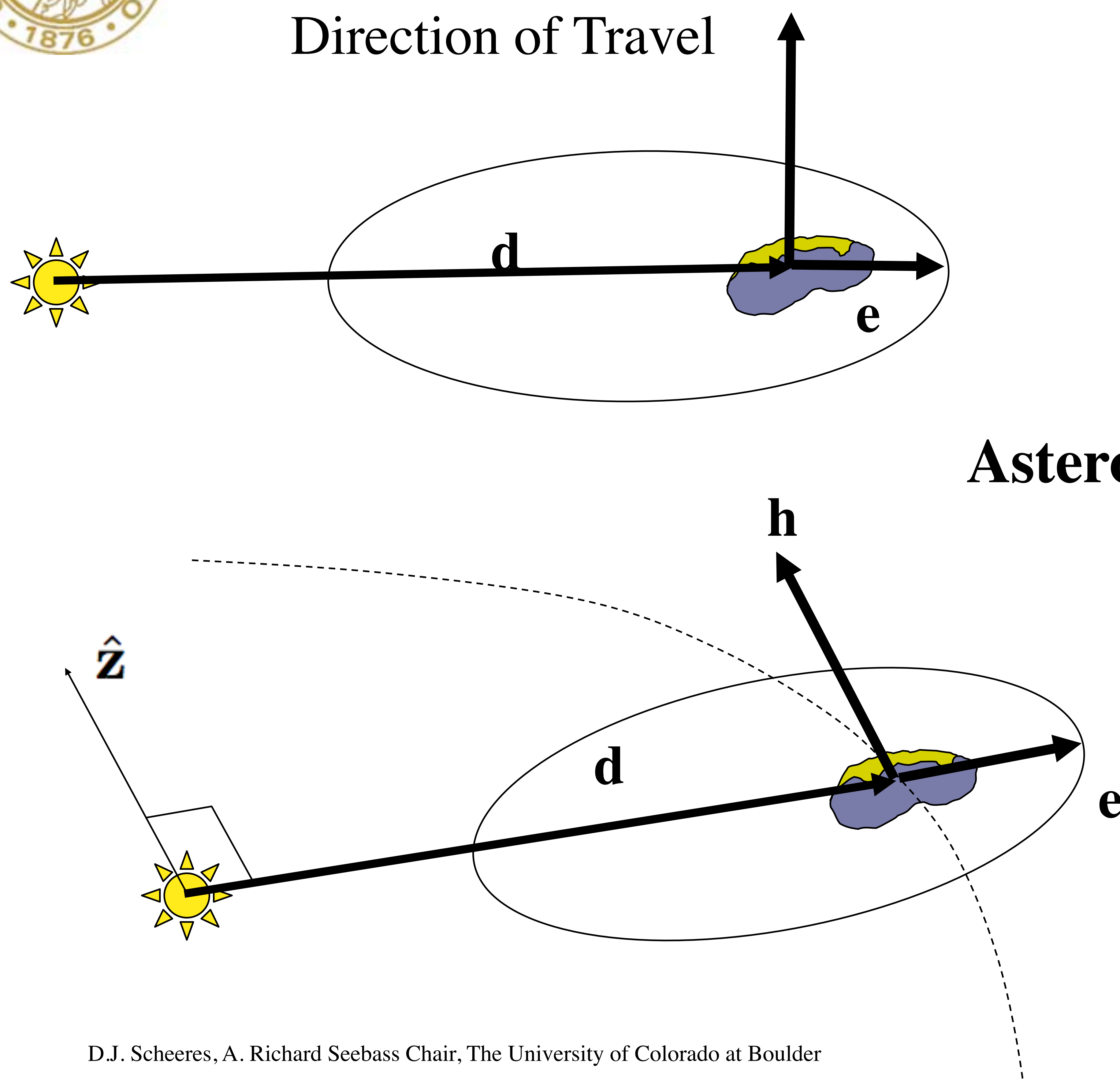
$$\cancel{\hat{\mathbf{d}} \cdot \mathbf{e} = \pm \sin \Lambda \qquad \hat{\mathbf{z}} \cdot \mathbf{h} = \mp \cos \Lambda}$$

- Solar Plane-of-Sky orbits in the terminator plane
  - Angular momentum vector parallel to sun-line

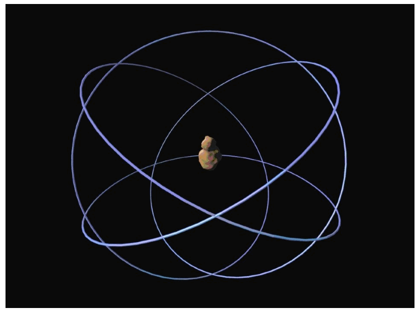
$$\hat{\mathbf{d}} \cdot \mathbf{h} = \mp \sin \Lambda \qquad \hat{\mathbf{z}} \cdot \mathbf{e} = \pm \cos \Lambda$$

  - These are preferred for mission applications



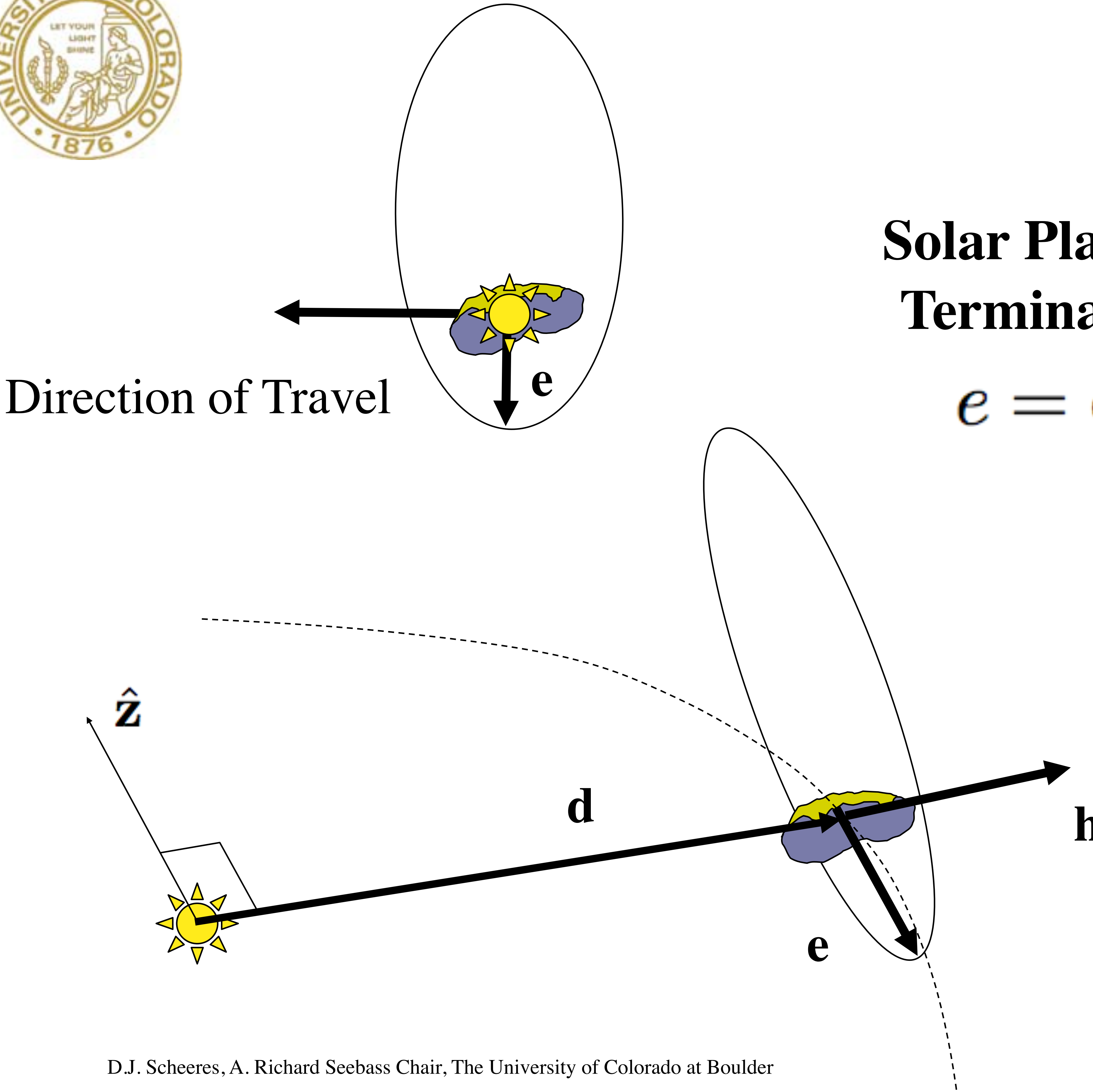

Direction of Travel
d
e
Asteroid Orbit Plane
h
$\hat{\mathbf{z}}$
d
e

$$e = \sin \Lambda$$

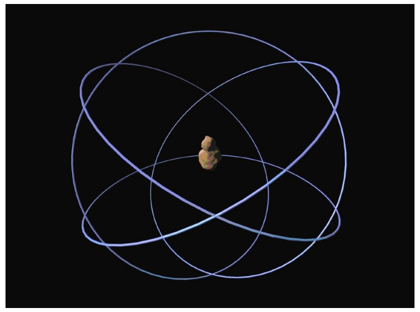


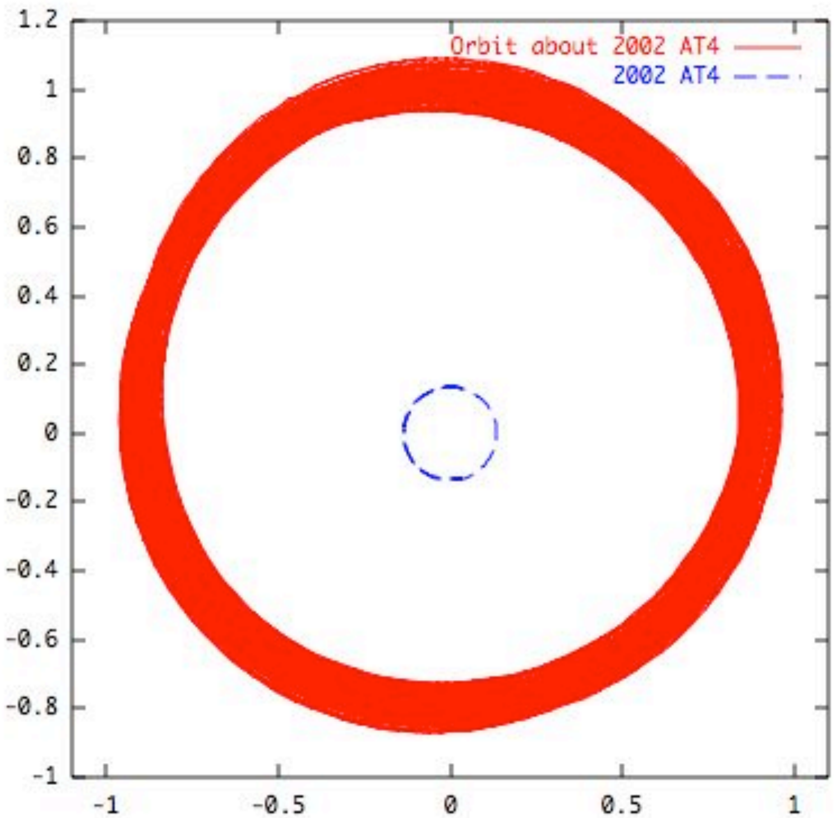
Solar Plane-of-Sky / Terminator Orbits

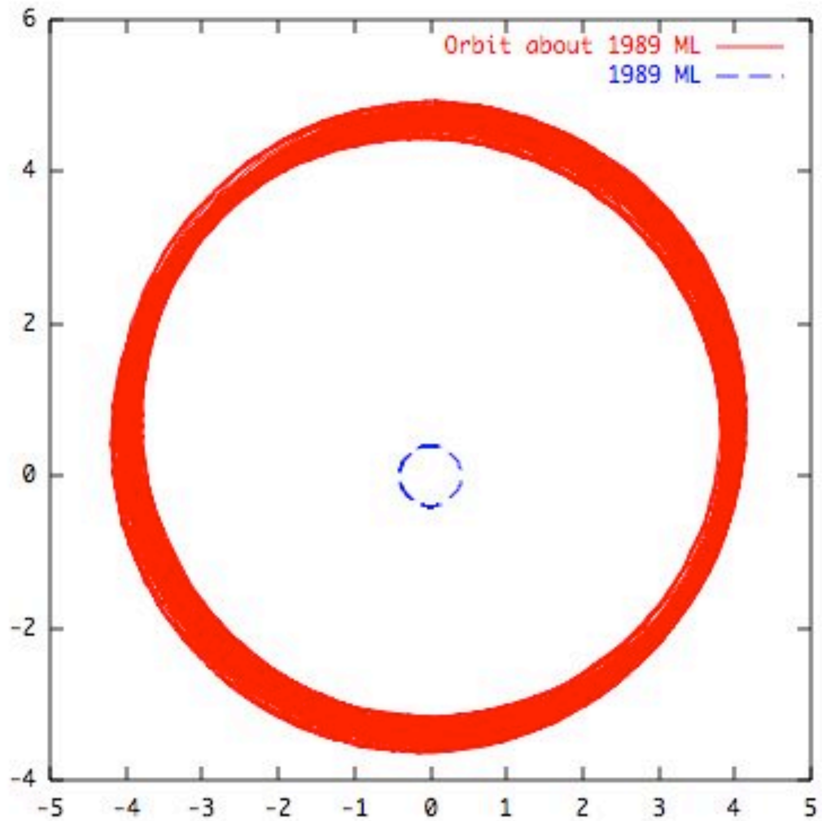
$$e = \cos \Lambda$$



Stable Frozen Terminator Orbits

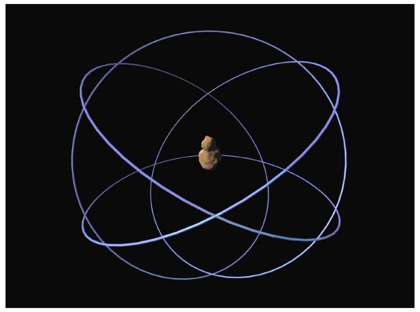


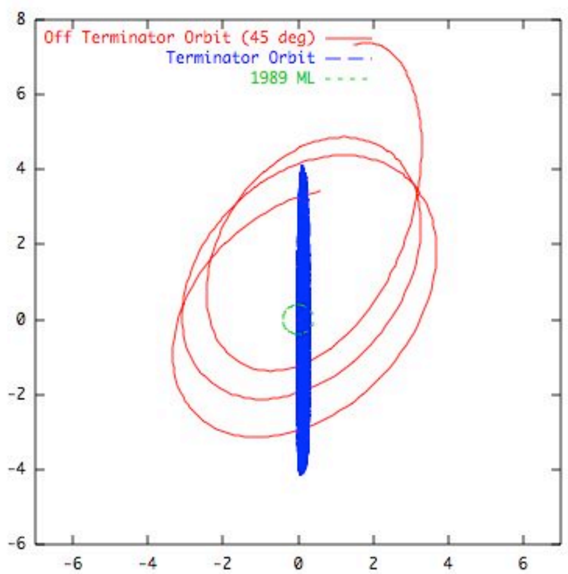




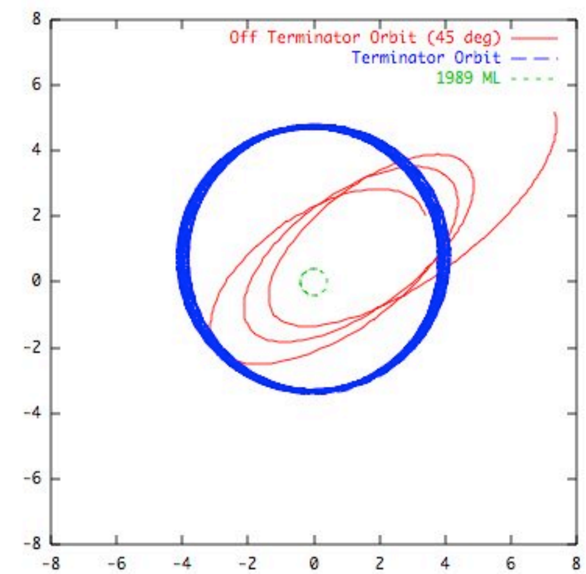
- Orbits lie in the sun-terminator plane
- Orbit radius must be small enough to not be stripped away
- SRP force makes them sun-synchronous
- Very robust and stable – above are integrated over an asteroid heliocentric period

Terminator vs. Non-Terminator Orbit

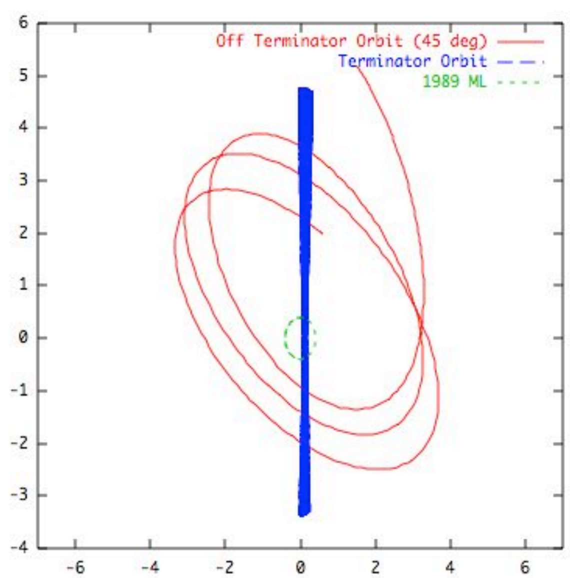




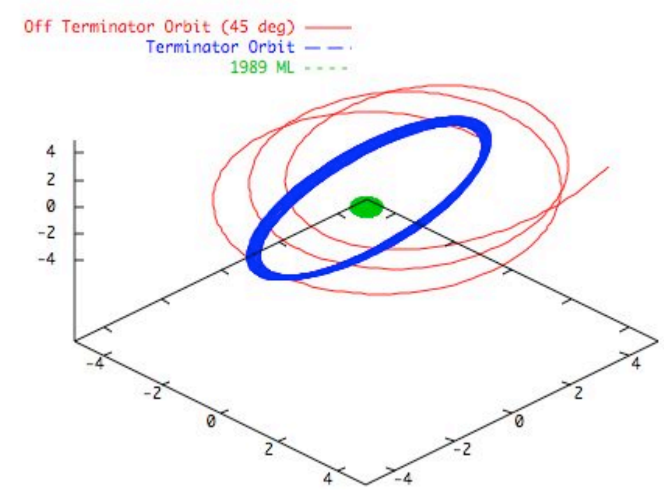
View in asteroid orbit plane



View from the sun

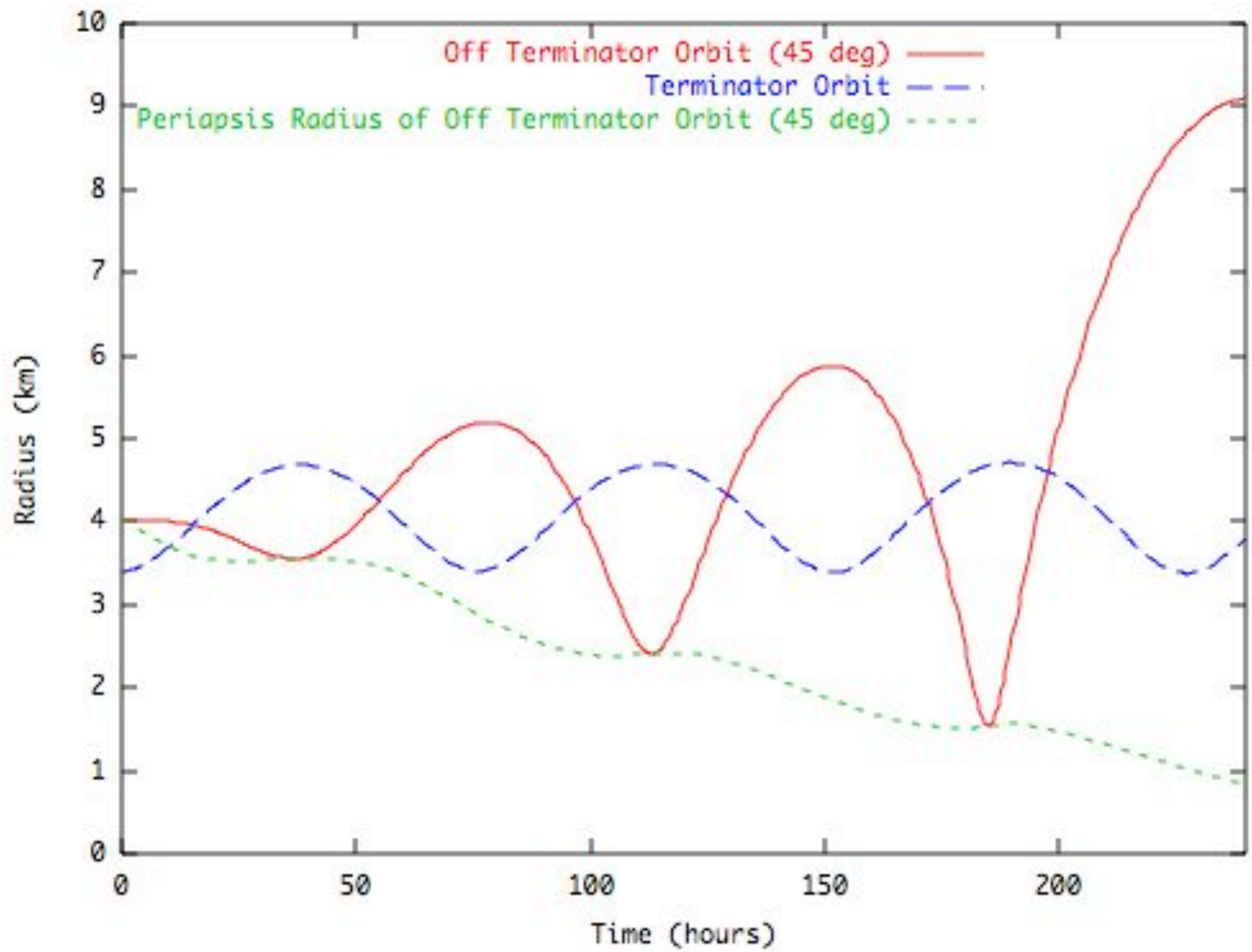


Looking down on asteroid orbit plane



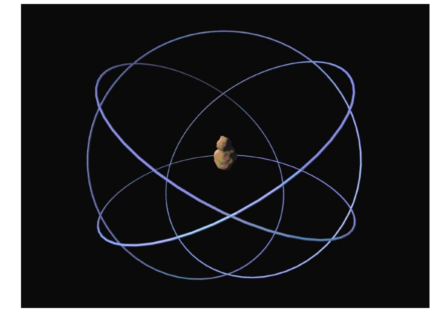
Terminator Orbit in above propagated over 100 days

Terminator vs. Non-Terminator Orbit



Mixed Regime: Interactions between SRP and Gravity Fields

- Orbits about real bodies will have a lower bound on radius for orbit stability due to mass distribution effects
  - Joint perturbations between SRP and non-spherical gravity terms are generally destabilizing
  - Limiting radius determined by size of frozen eccentricity and magnitude of inclination oscillations
- Destabilization due to two effects:
  - Oblateness causes precession of orbit plane out of terminator orbit
    - Causes oscillation in eccentricity that leads to stronger interactions with the gravity field
  - Ellipticity causes fluctuations in orbit semi-major axis, eccentricity and inclination
    - Causes orbit to migrate away from frozen orbit
    - Subsequent motion becomes chaotic and can lead to impact or escape

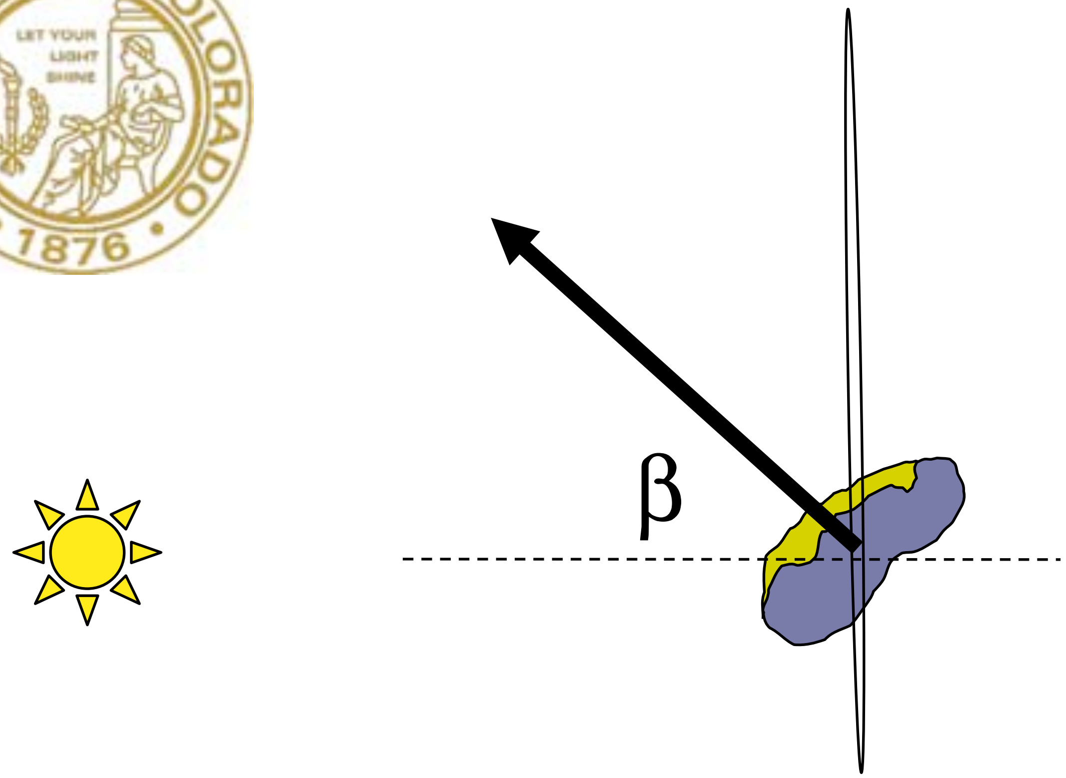


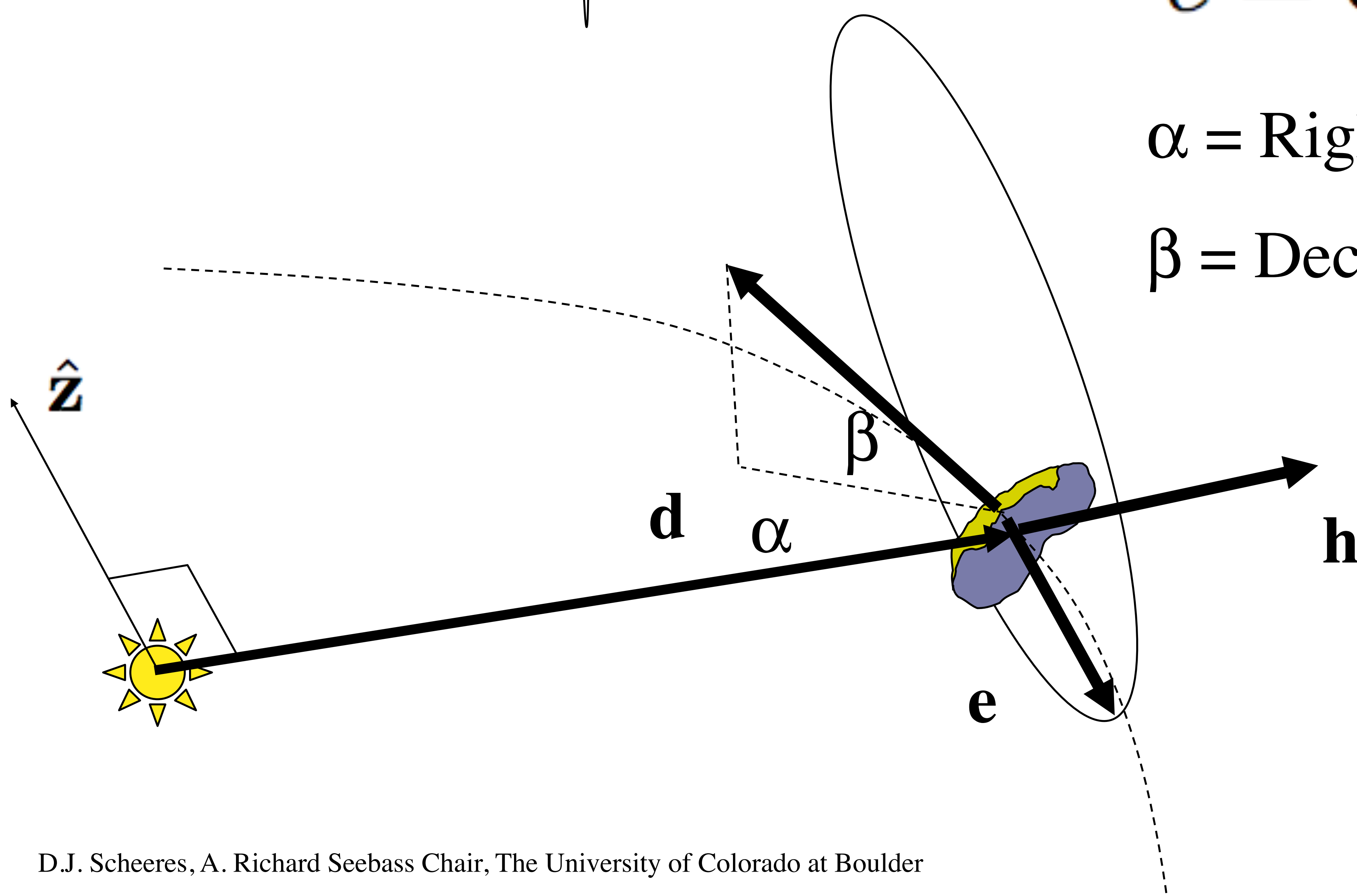
Solar Plane-of-Sky / Terminator Orbits

$$e = \cos \Lambda$$

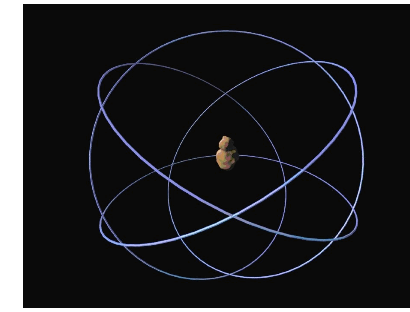
α = Right Ascension

β = Declination





Influence of Oblateness



- Perturbation equations for a terminator orbit about an asteroid with obliquity β and right ascension α:

$$e = \cos\Lambda + \delta e; \Omega = \pm\pi/2 + \delta\Omega \qquad \omega = \mp\pi/2 + \delta\omega; i = \pi/2 + \delta i$$

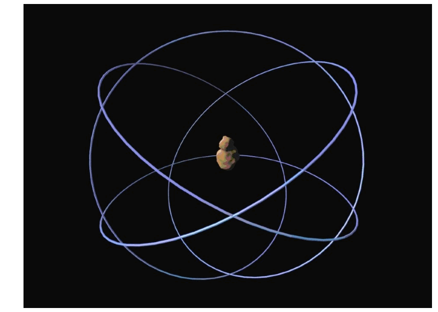
$$\frac{d\delta e}{d\nu} = -\frac{\sin^2\Lambda}{\cos\Lambda}\delta\Omega$$

$$\frac{d\delta\Omega}{d\nu} = \frac{1}{\sin^2\Lambda\cos\Lambda}\delta e \pm \frac{3}{4\dot{\nu}}\frac{n(I_a - I_t)}{a^2\sin^4\Lambda}\sin 2\beta\sin\alpha$$

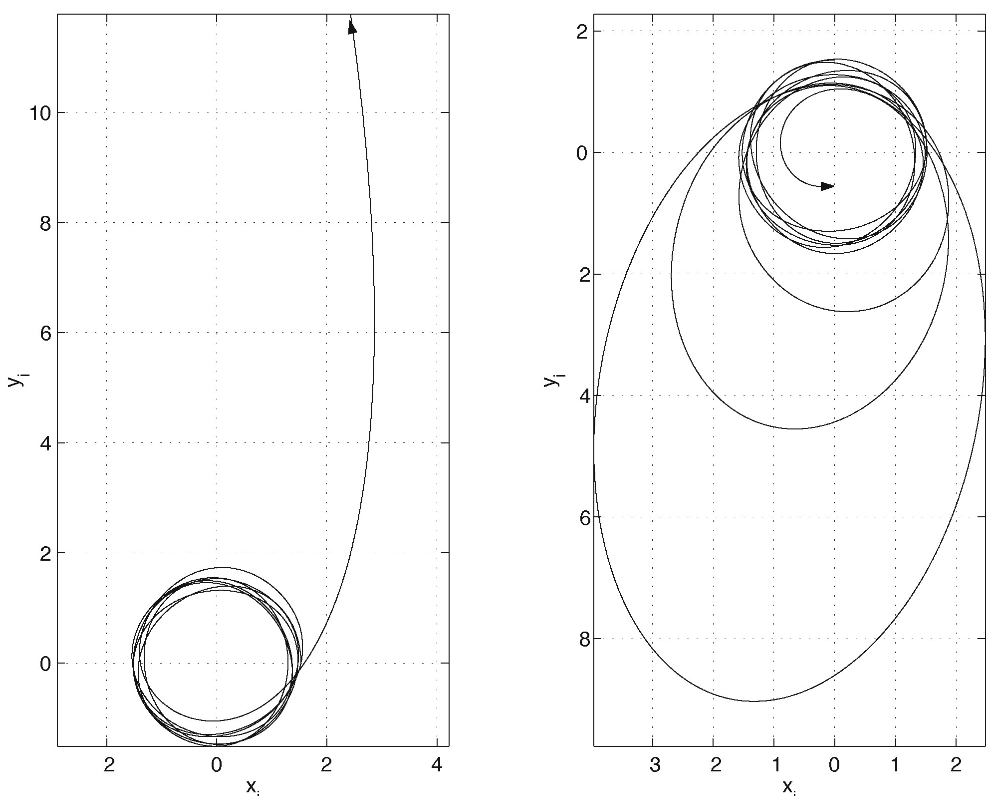
$$\frac{d\delta\omega}{d\nu} = \frac{1}{\cos^2\Lambda}\delta i + \frac{3}{4\dot{\nu}}\frac{n(I_a - I_t)}{a^2\sin^4\Lambda}\left[5 - 3\sin^2\beta\sin^2\alpha\right]$$

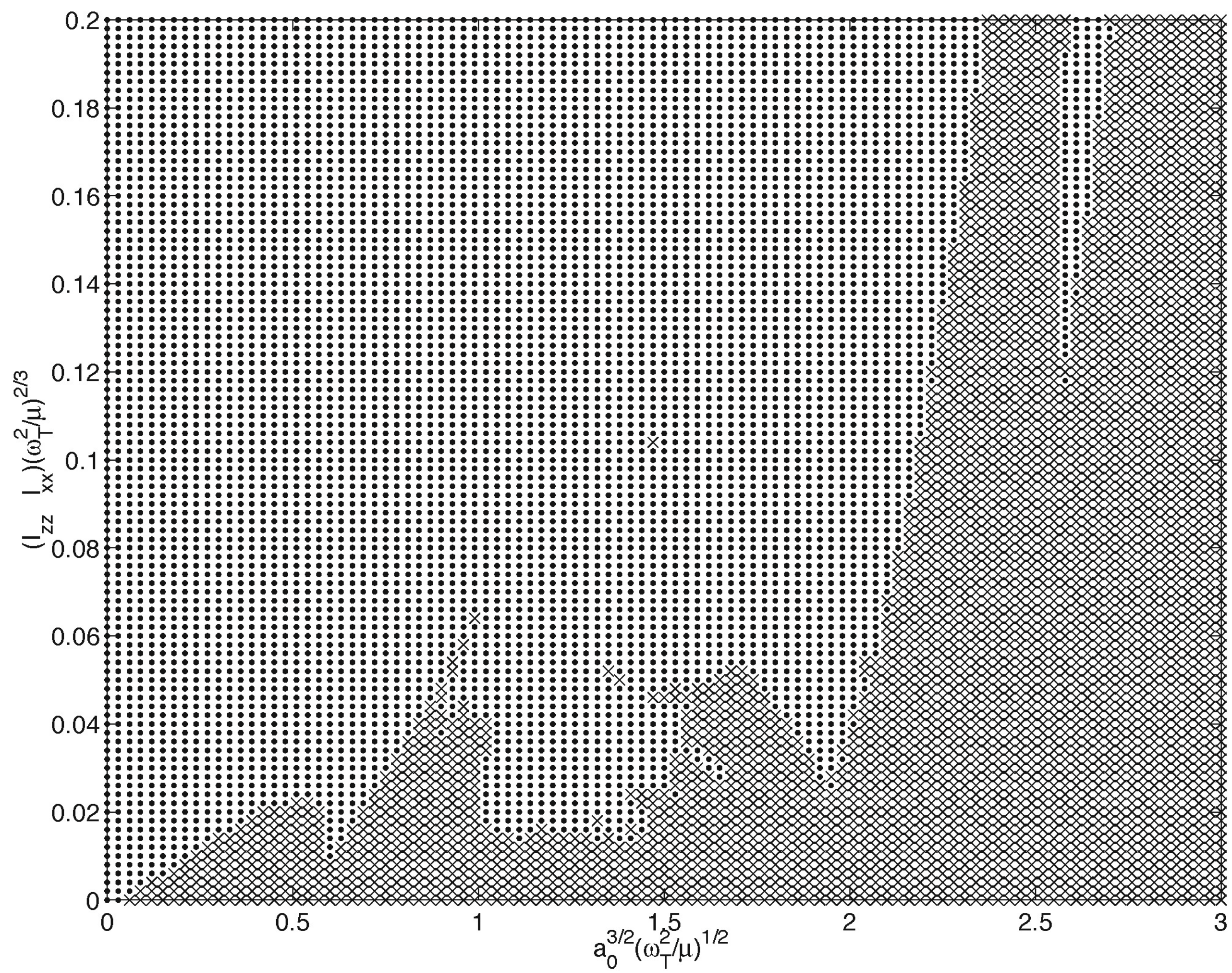
$$\frac{d\delta i}{d\nu} = -\delta\omega + \frac{3}{4\dot{\nu}}\frac{n(I_a - I_t)}{a^2\sin^4\Lambda}\sin^2\beta\sin 2\alpha$$

Influence of Ellipticity



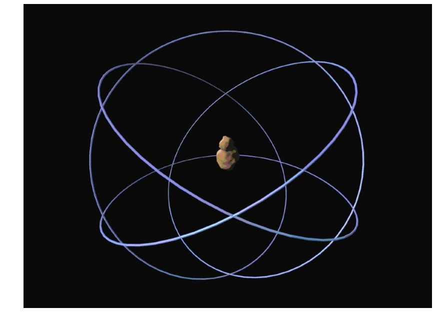
- Resonant interactions between the orbit and the rotating gravity field can induce chaotic dynamics





From: Weiduo Hu & D.J. Scheeres, Planetary and Space Science 52: 685-692, 2004

Bounds on effects



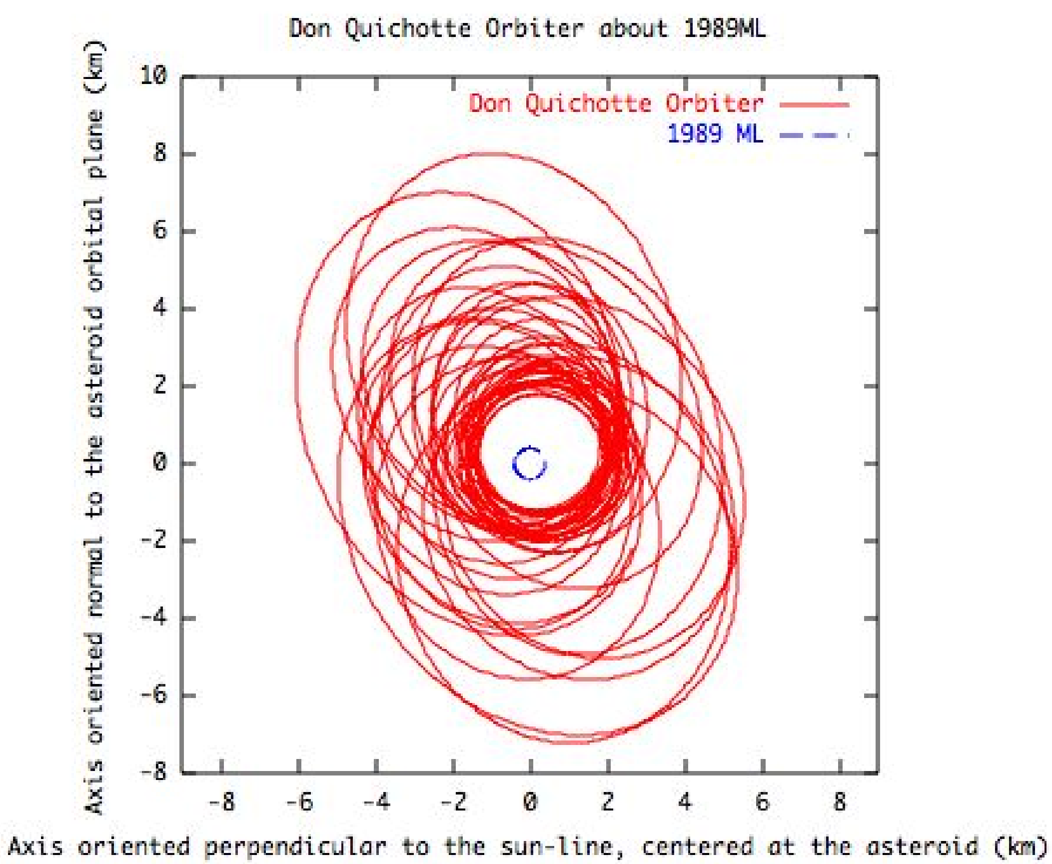
- Shape Oblateness causes fluctuations in eccentricity from the frozen orbit value:

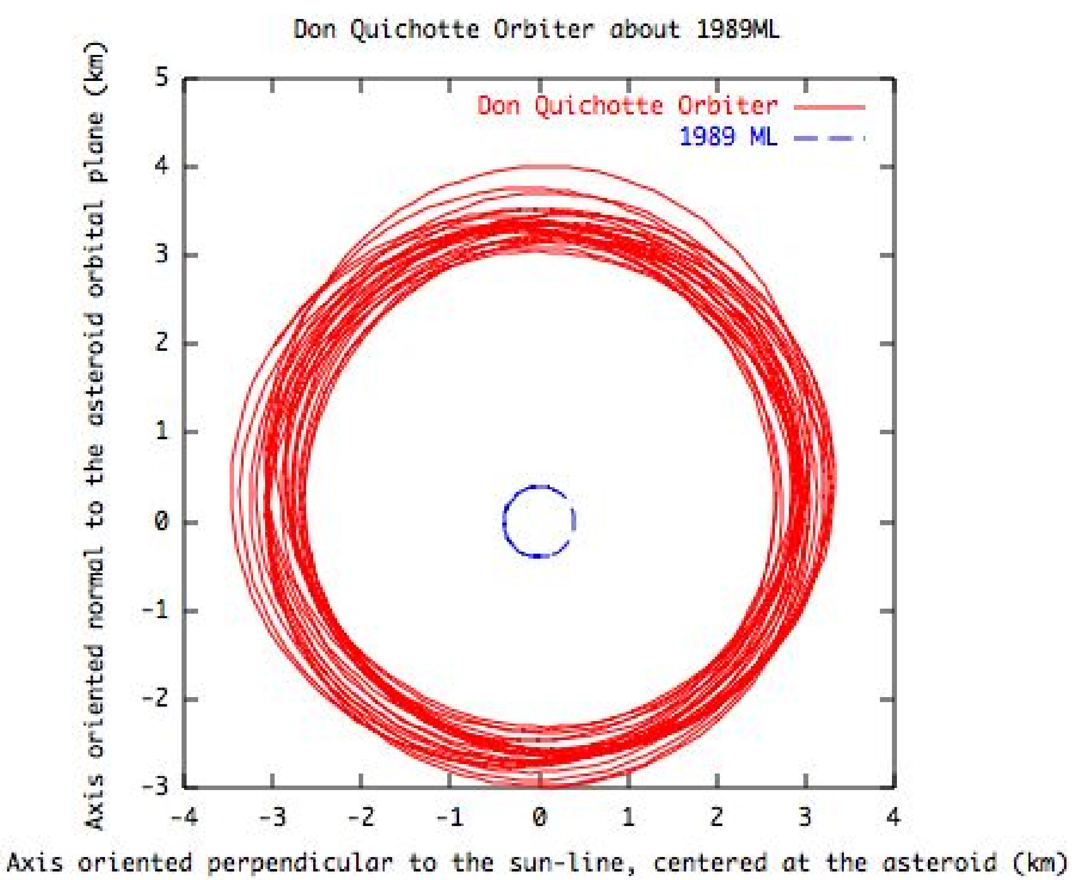
$$|\delta e| < \frac{3n}{2a^2}(I_a - I_t)\frac{\cos^2\Lambda}{\sin^4\Lambda}\frac{\sin 2\beta \sin\alpha}{\sqrt{\mu_{sun}P}}d^2$$

- Shape Ellipticity causes chaotic variations in orbit when the orbit period is within a 1.5 resonance radii:

$$a < \frac{3}{2}\left(\frac{T^2\mu}{4\pi^2}\right)^{1/3}$$

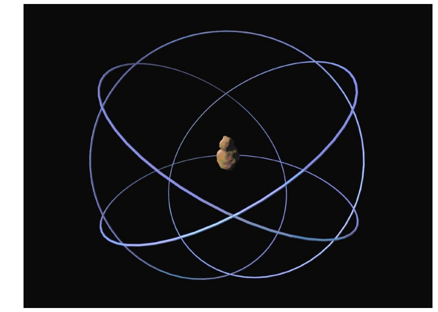
Mixed Perturbations





- Smaller orbit sizes can lead to destabilizing interactions between SRP and gravity field perturbations
- RHS, larger initial semi-major axis
  - stable against gravity and SRP
- LHS, smaller semi-major axis
  - orbit precession induces unstable motion

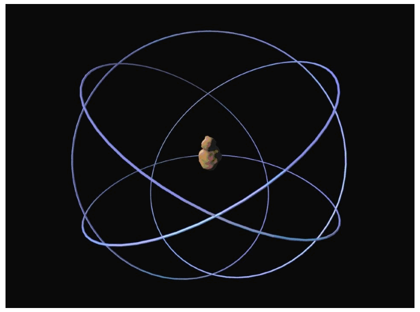
Case Study: Hayabusa at Itokawa

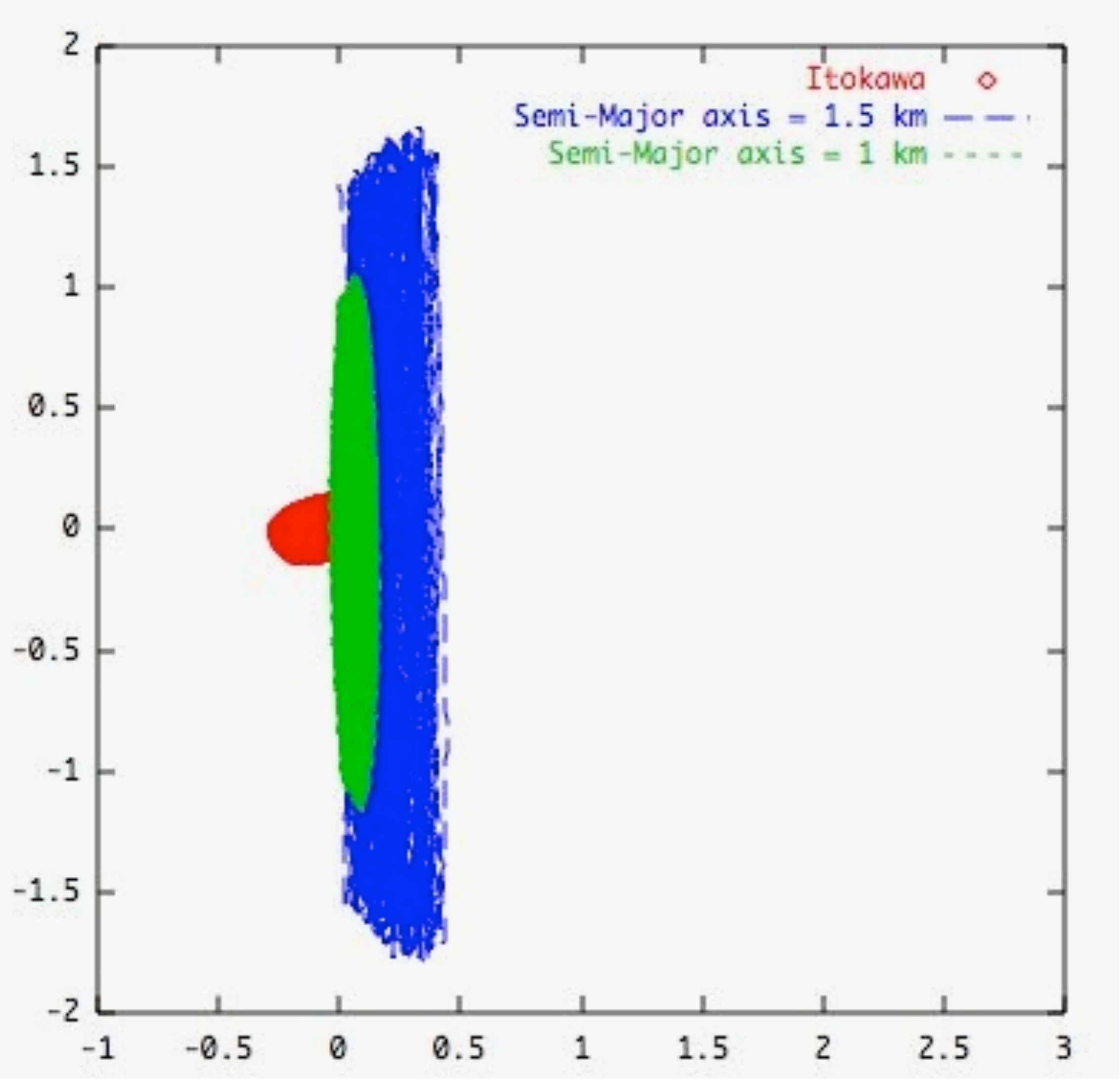


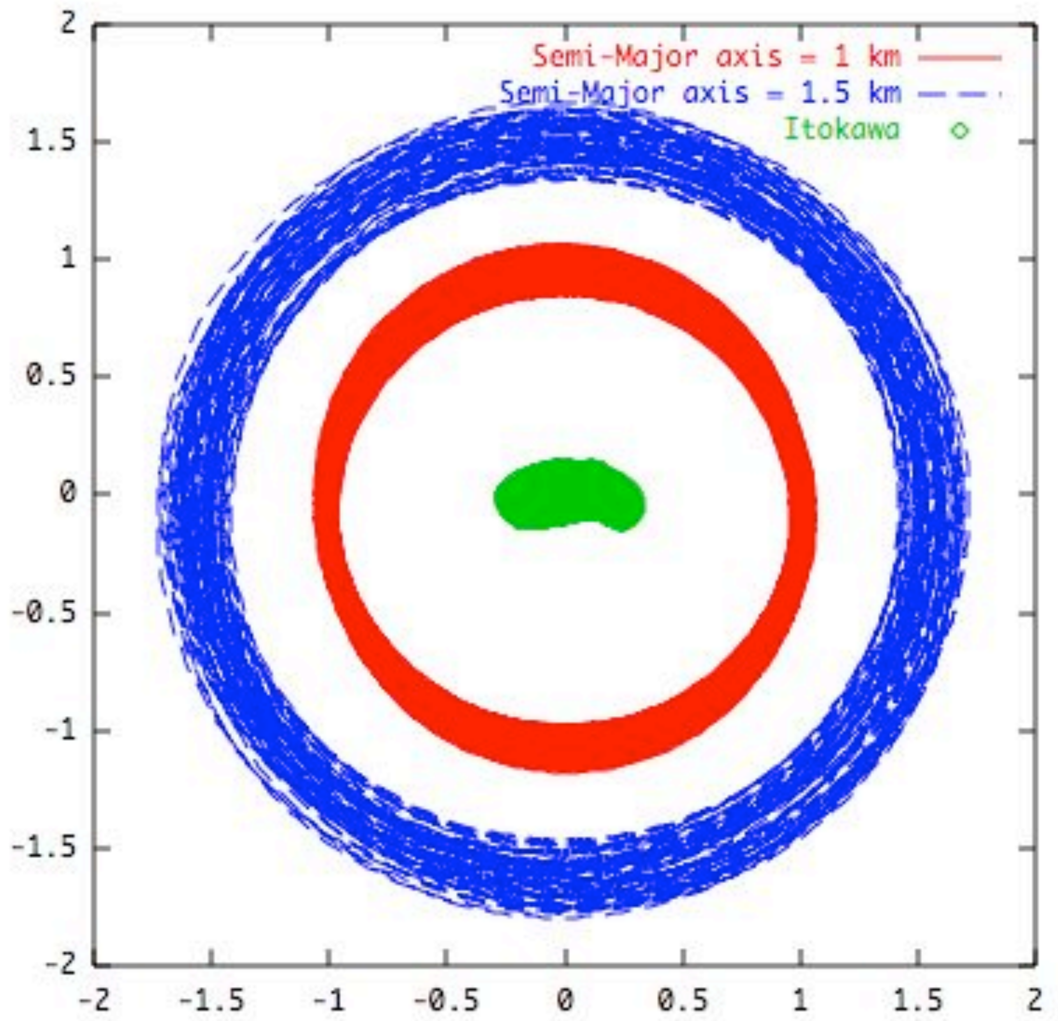
- Could the Hayabusa S/C have orbited the asteroid Itokawa?
  - Mission took a hovering approach, due to sampling technique and uncertainty of true asteroid mass
  - An orbital analysis was performed with the estimated asteroid mass and shape, and the S/C mass and projected area
  - An orbital mission was possible, but with tight constraints on semi-major axis:

$$\mathbf{\mathit{1.0\ km < a < 1.5\ km}}$$

Possible Hayabusa Orbits at Itokawa

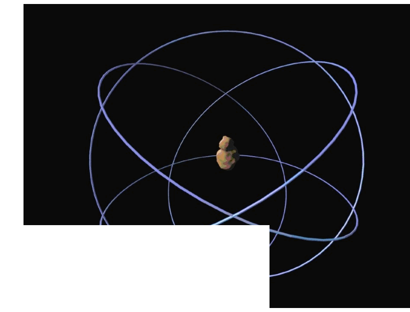


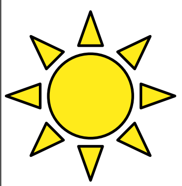


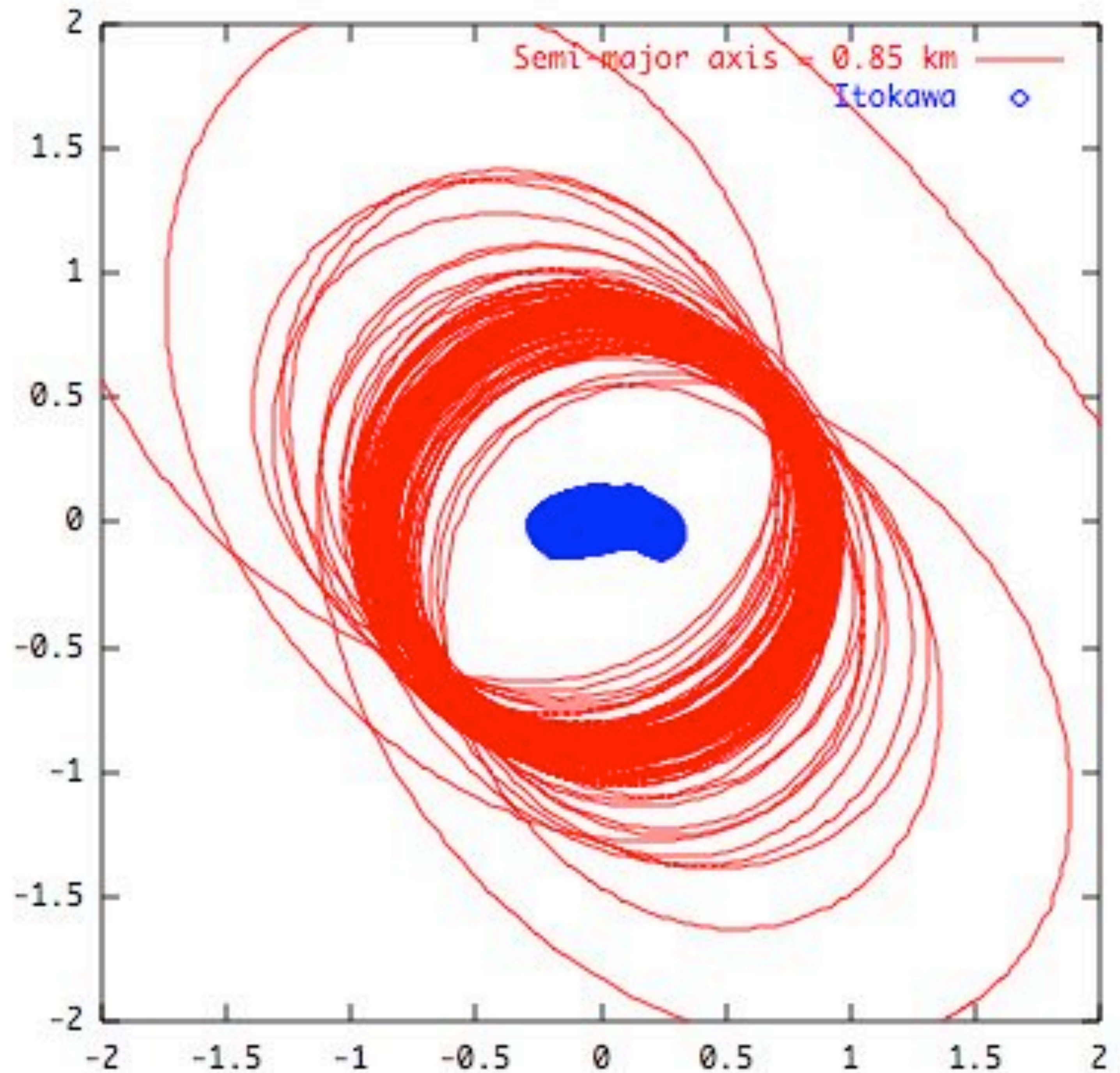


Larger or smaller orbits are unstable and escape

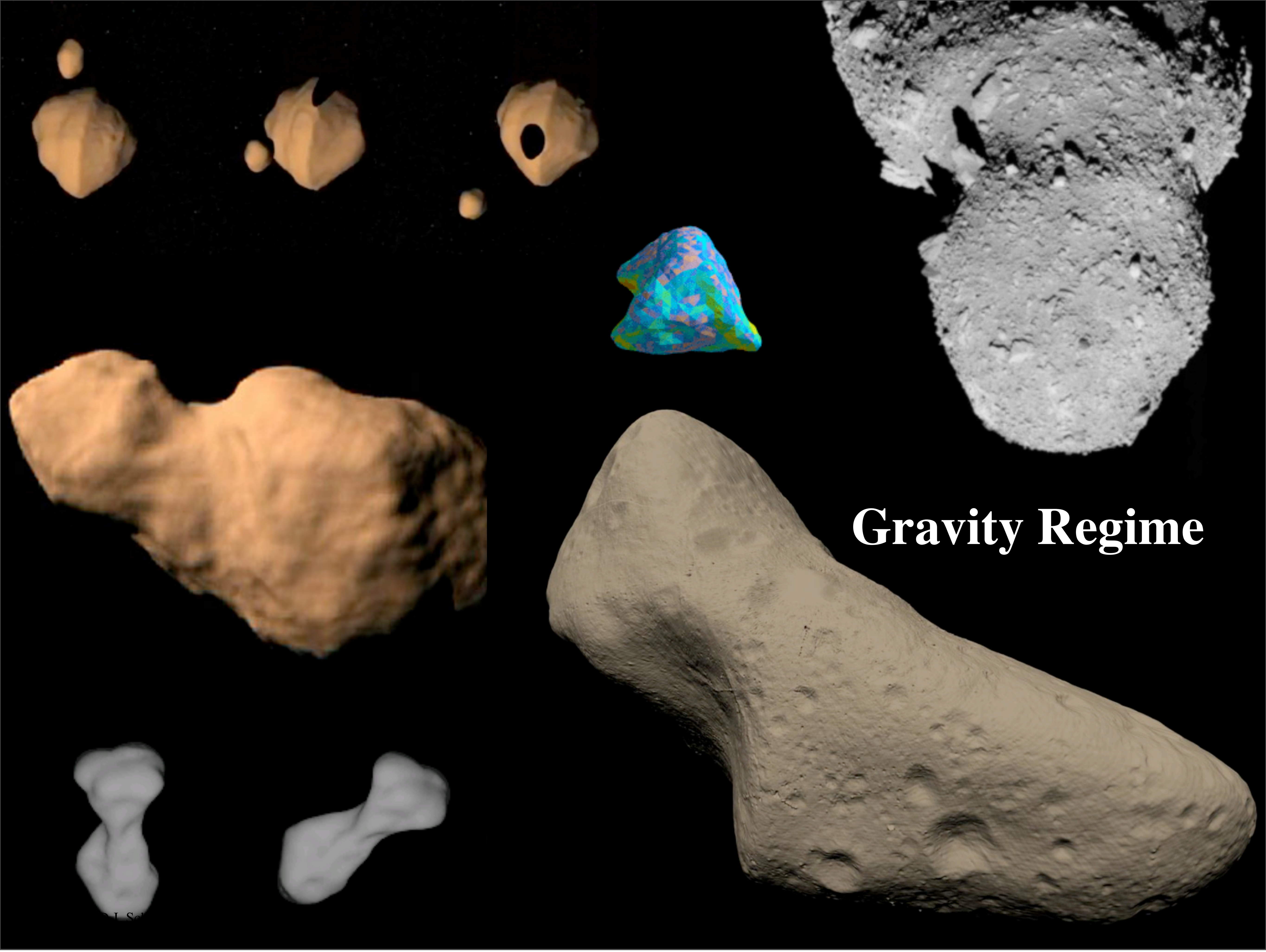
Possible Hayabusa Orbits at Itokawa



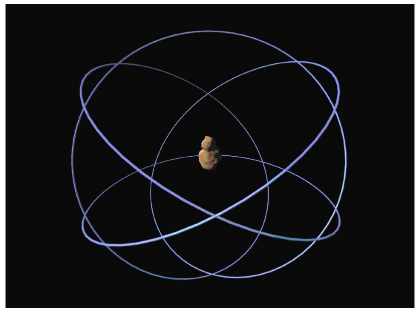




Large


Gravity Regime

Motion “Close” to the Asteroid



- If the asteroid is large or the particle is relatively massive, and we orbit close to the asteroid we can further restrict our equations of motion:

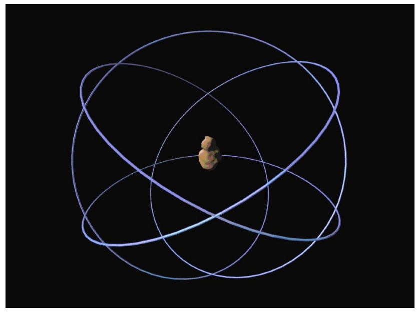
$$\ddot{\boldsymbol{r}} + 2\tilde{\boldsymbol{\omega}} \cdot \dot{\boldsymbol{r}} + \dot{\tilde{\boldsymbol{\omega}}} \cdot \boldsymbol{r} + \tilde{\boldsymbol{\omega}} \cdot \tilde{\boldsymbol{\omega}} \cdot \boldsymbol{r} \quad = \quad \frac{\partial U}{\partial \boldsymbol{r}} + \cancel{\frac{g_s}{d^2 B} \boldsymbol{T}^T \cdot \hat{\boldsymbol{d}}} + \cancel{\frac{\mu_s}{d^3} \left[3(\hat{\boldsymbol{d}} \cdot \boldsymbol{r}) \boldsymbol{T}^T \cdot \hat{\boldsymbol{d}} - \boldsymbol{r} \right]}$$

- If not uniformly rotating, EOM are time periodic
- If uniformly rotating, a Jacobi integral exists:

$$J \quad = \quad \frac{1}{2} \dot{\boldsymbol{r}} \cdot \dot{\boldsymbol{r}} - \frac{1}{2} \omega^2 \left(x^2 + y^2 \right) - U(\boldsymbol{r})$$

  – This is a non-integrable problem distinct from, and more difficult than, the R3BP

Motion "Close" to the Asteroid



- If the asteroid is large or the particle is relatively massive, and we orbit close to the asteroid we can further restrict our equations of motion:

$$\ddot{\boldsymbol{r}} + 2\tilde{\boldsymbol{\omega}}\cdot\dot{\boldsymbol{r}} + \cancel{\dot{\tilde{\boldsymbol{\omega}}}}\cdot\boldsymbol{r} + \tilde{\boldsymbol{\omega}}\cdot\tilde{\boldsymbol{\omega}}\cdot\boldsymbol{r} \quad = \quad \frac{\partial U}{\partial \boldsymbol{r}} + \cancel{\frac{\mathcal{G}_s}{d^2 B}\boldsymbol{T}^T\cdot\hat{\boldsymbol{d}}} + \cancel{\frac{\mu_s}{d^3}\left[3(\hat{\boldsymbol{d}}\cdot\boldsymbol{r})\boldsymbol{T}^T\cdot\hat{\boldsymbol{d}} - \boldsymbol{r}\right]}$$

If uniformly rotating

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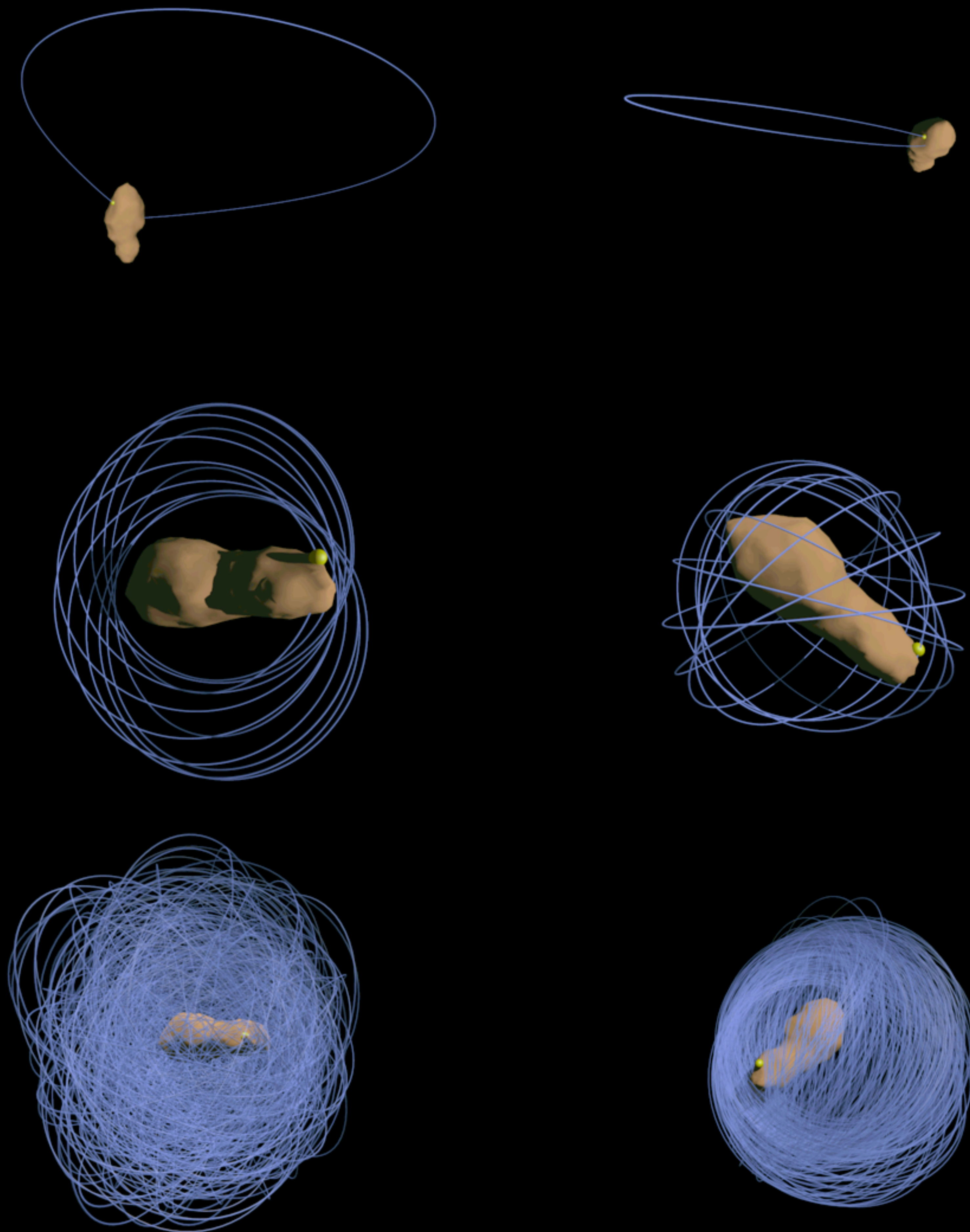
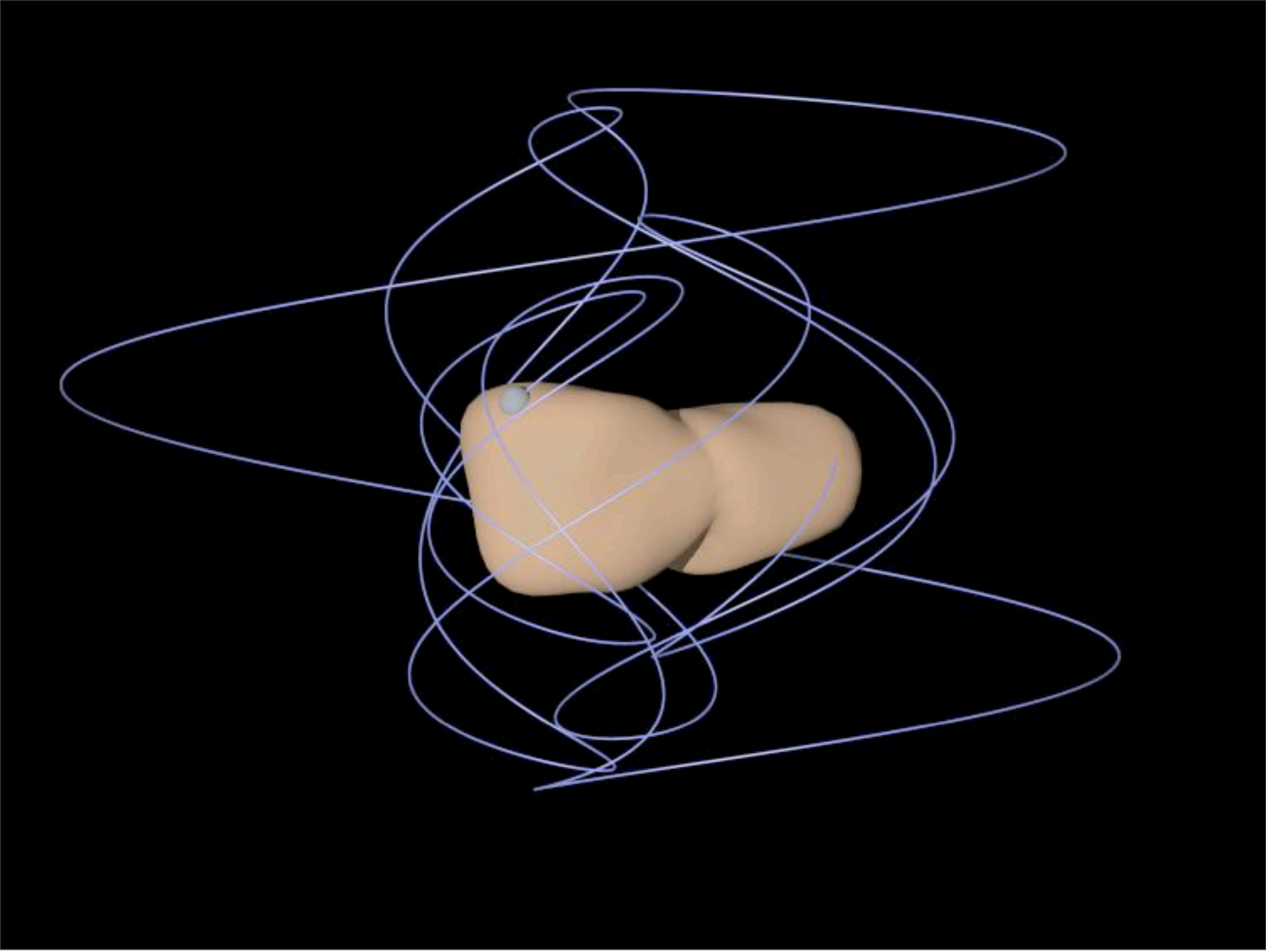
$$J \quad = \quad \frac{1}{2}\dot{\boldsymbol{r}}\cdot\dot{\boldsymbol{r}} - \frac{1}{2}\omega^2\left(x^2+y^2\right) - U(\boldsymbol{r})$$

  – This is a non-integrable problem distinct from, and more difficult than, the R3BP

Gravity Regime

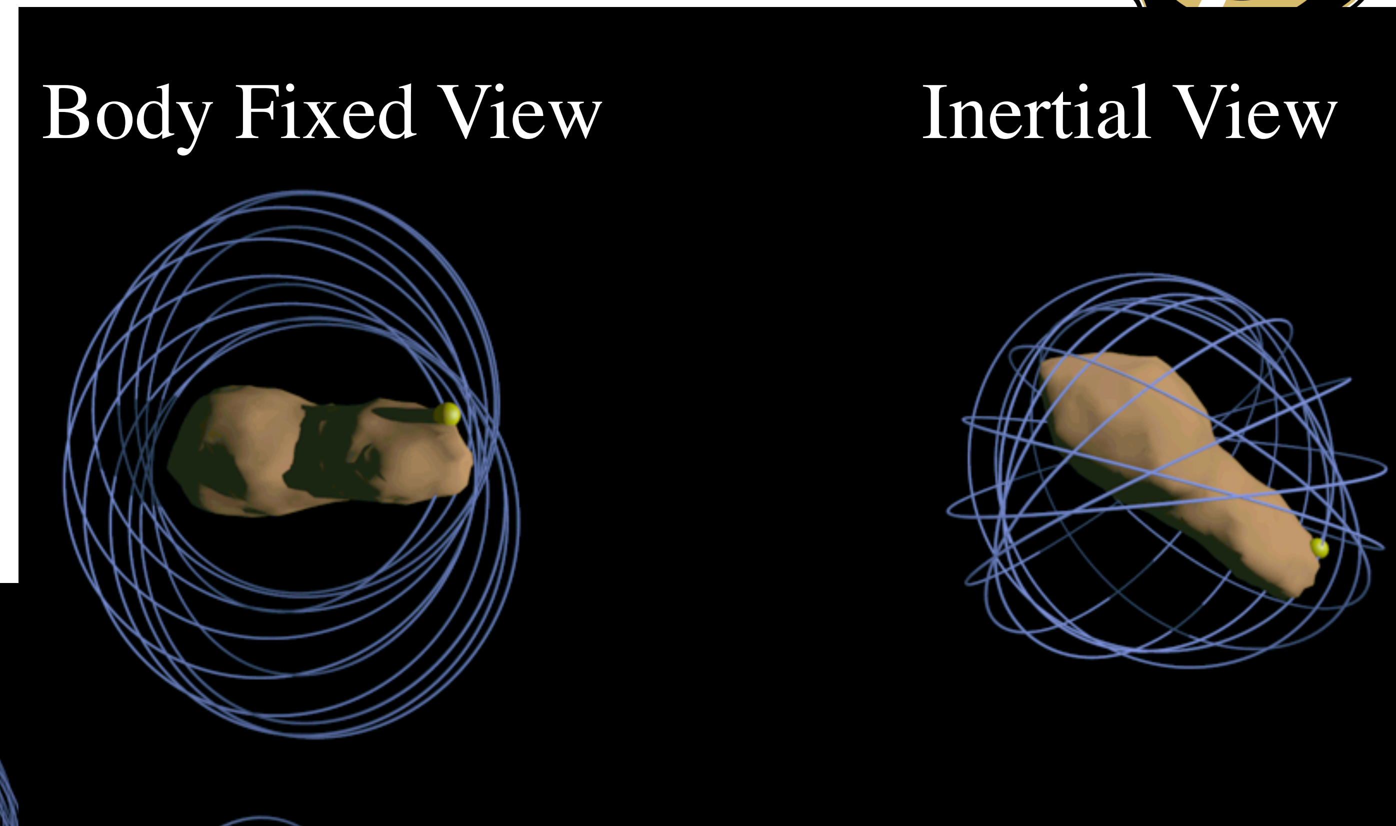
- Mass distribution and rotation state dominates motion.
- Use of classical analytical theories is challenging:
  - At Eros, the secular effect of J_2 is ***200*** times stronger than at Earth, high order zonal and tesseral coefficients are relatively even larger.
  - Convergent series for analytical descriptions must extend to much higher orders, incorporate many more effects.
  - Resonant interactions with the rotating gravity field causes orbital motion to become chaotic – cannot be described by analytical theories.
- Alternate tools for stable orbit design are needed and include:
  - Averaging to identify first-order effects
  - Periodic orbits to delineate regions of stability
  - Hill stability to guarantee no-impact with the body (Lagrange stability)
  - Semi-analytic evaluations to identify conditions for instability



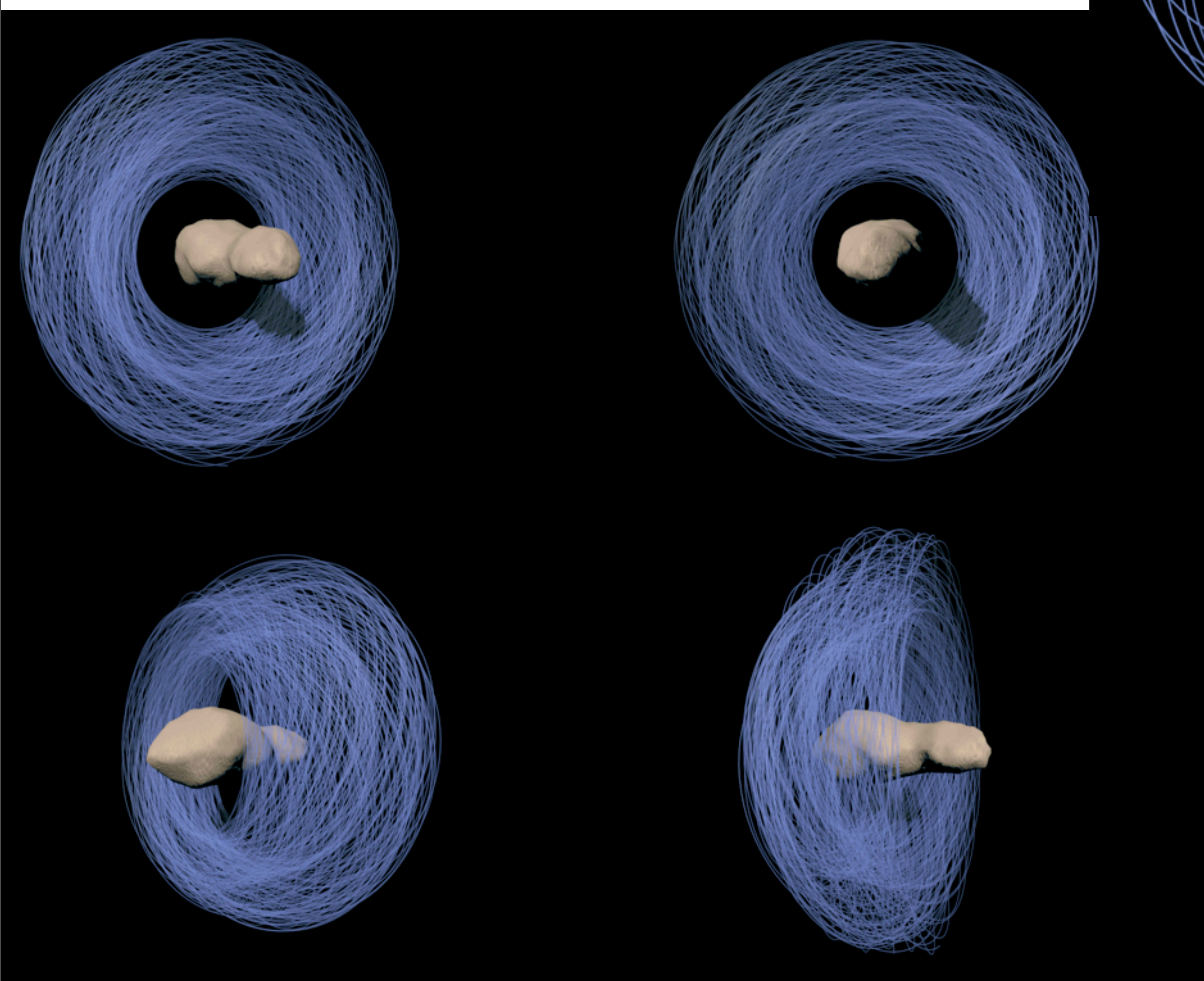


Averaging for understanding

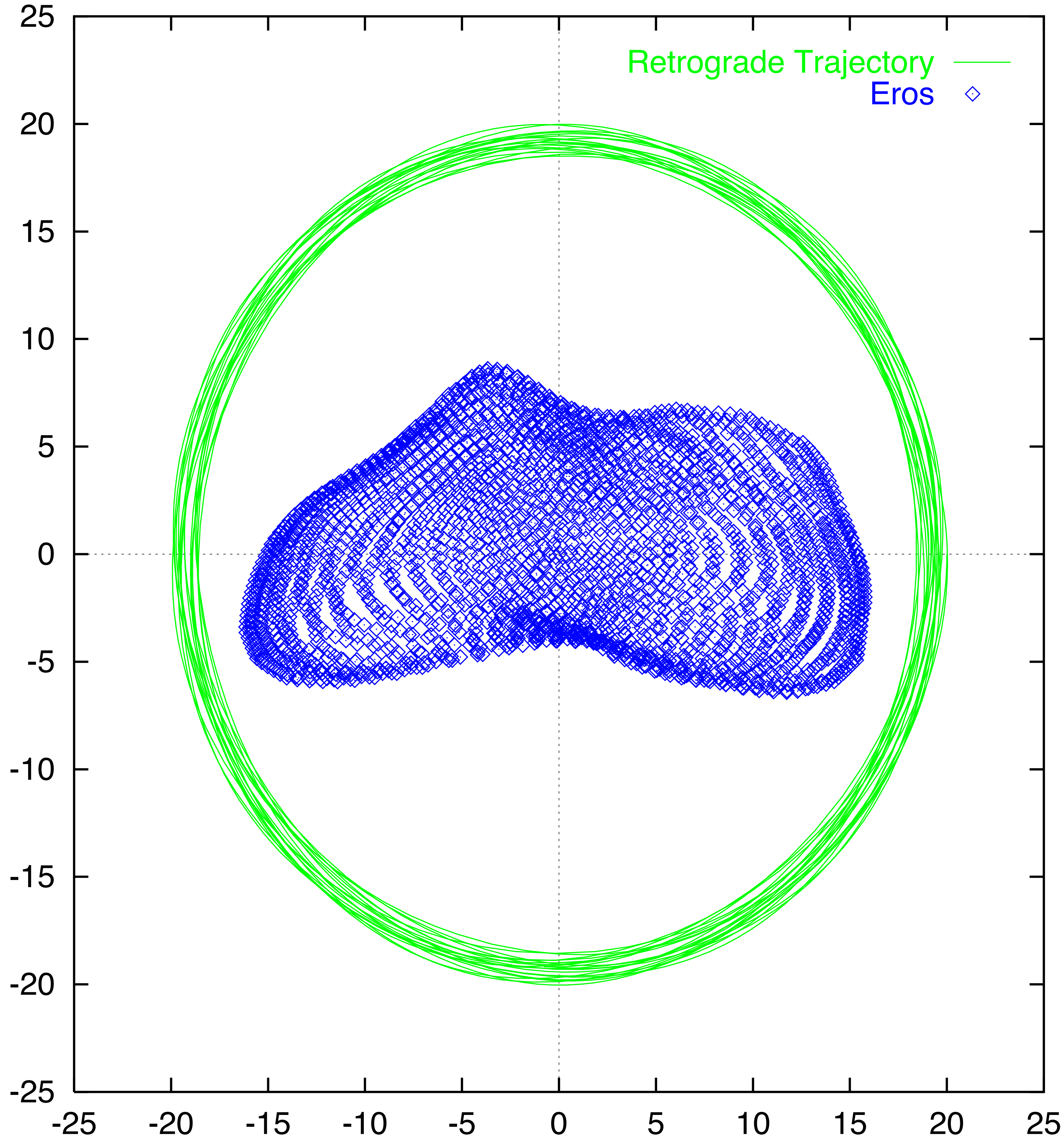
First-order averaging analysis suggests stable orbit designs and identifies the controlling, fundamental dynamical effects.



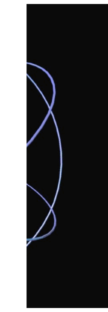
Orbit plane “dragging” by mass distribution, predicted by averaging theory.

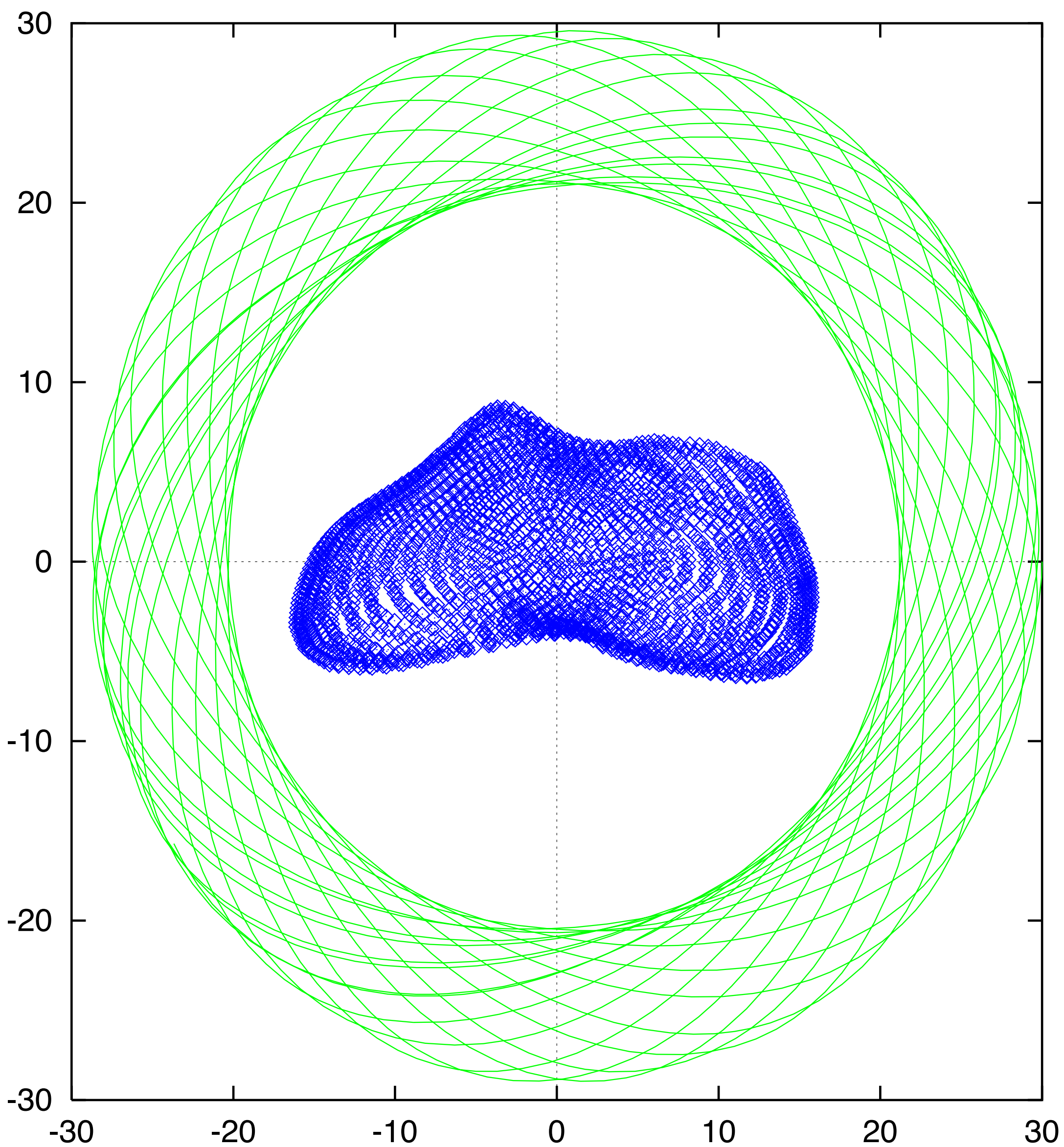


Stable orbit viewed in asteroid-fixed frame, identified using averaging analysis for motion about a non-uniform rotator.

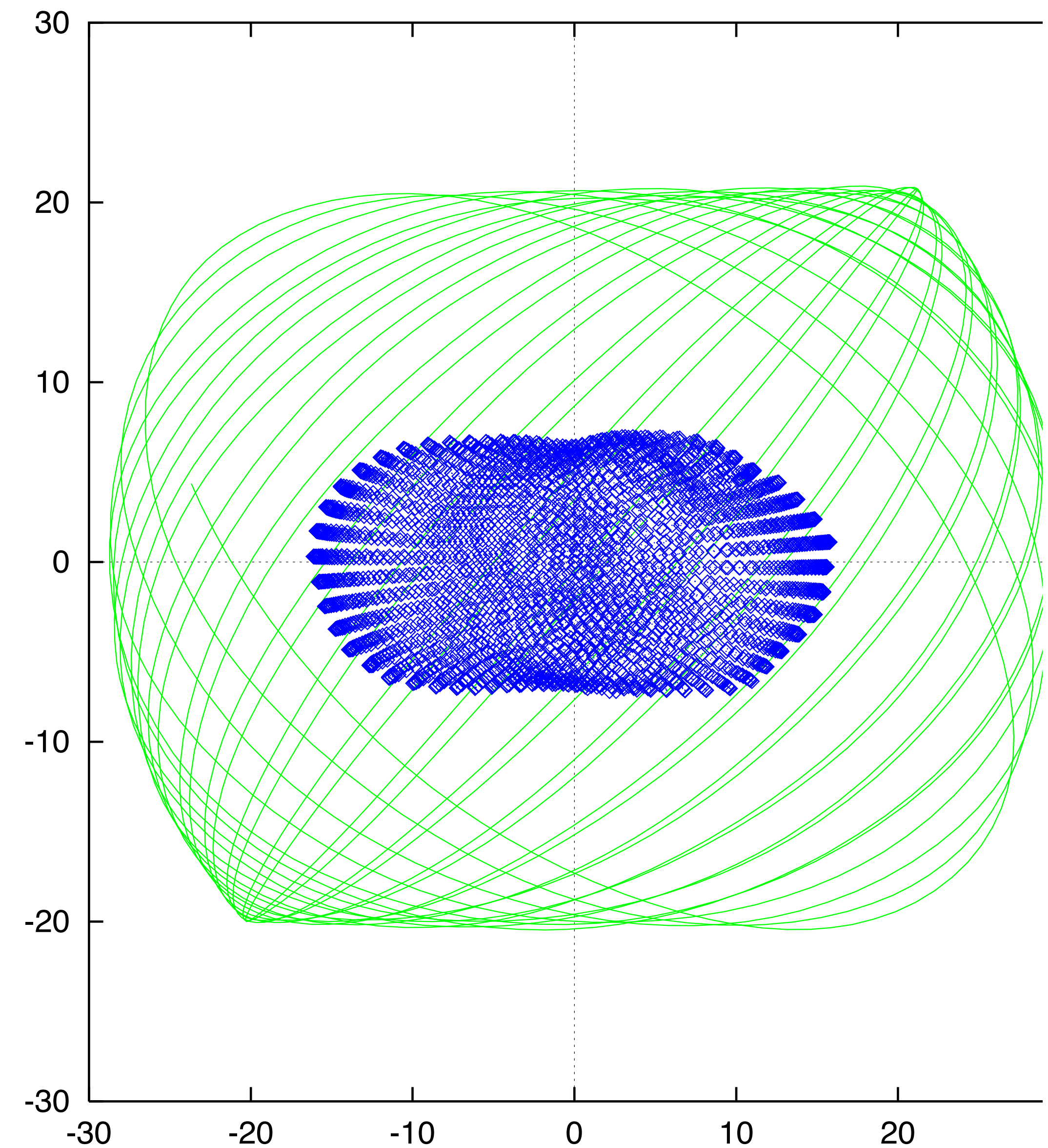

Retrograde Trajectory
Eros
25
20
15
10
5
0
-5
-10
-15
-20
-25
-25 -20 -15 -10 -5 0 5 10 15 20 25

30 km orbit with 135° inclination over 10 days



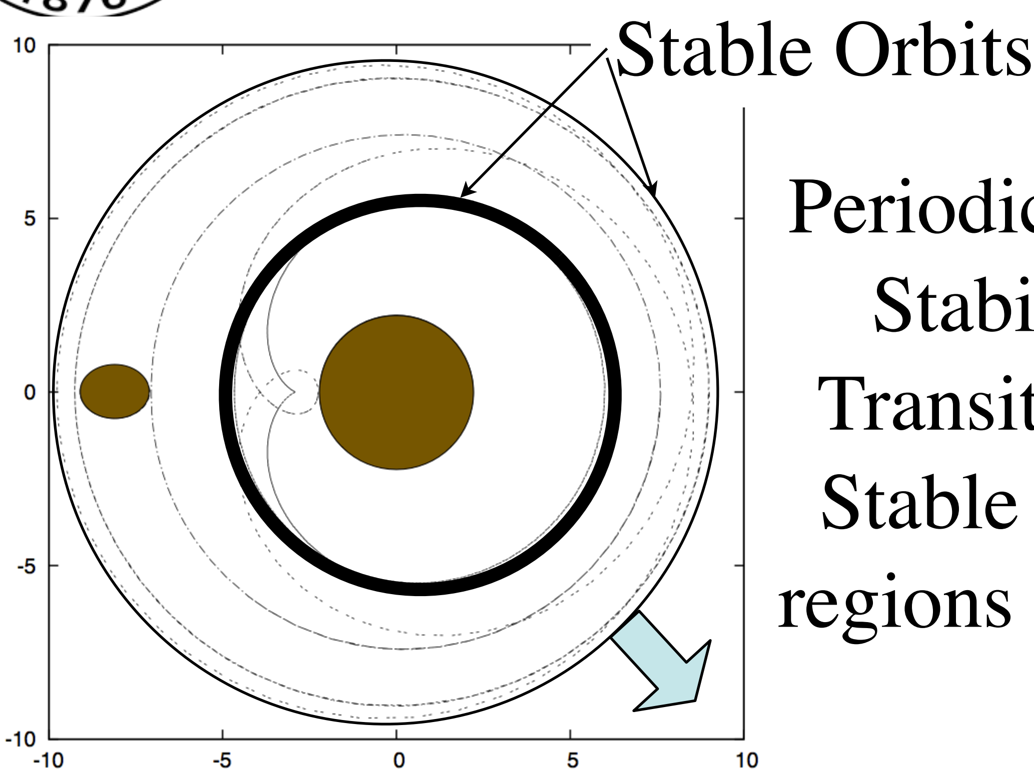


Projection into Eros Equatorial Plane

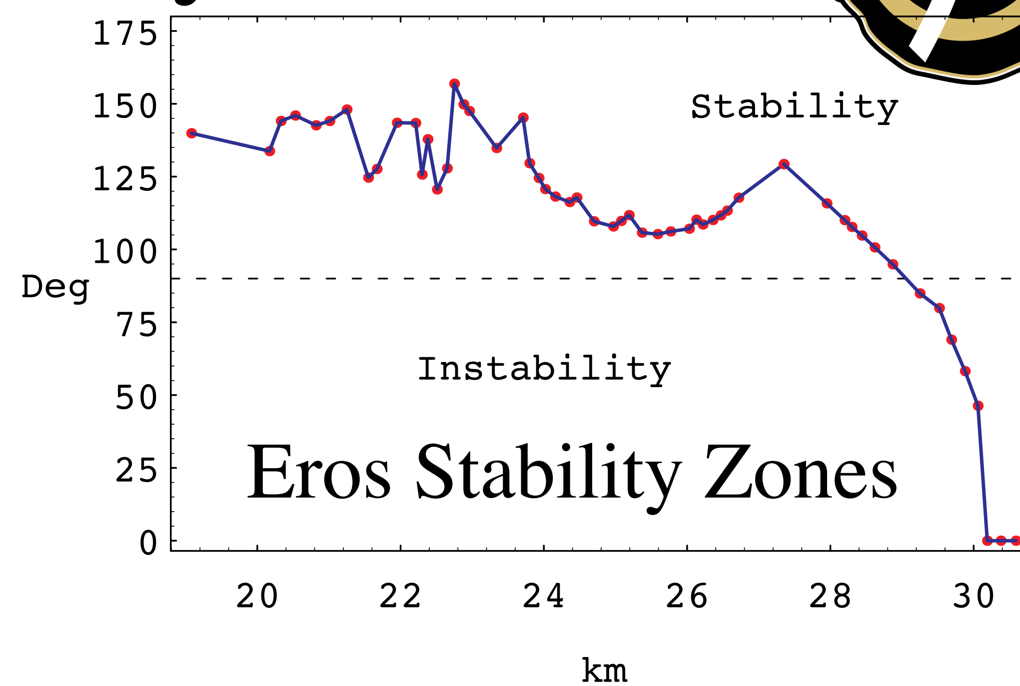


Projection Normal to Eros Equatorial Plane

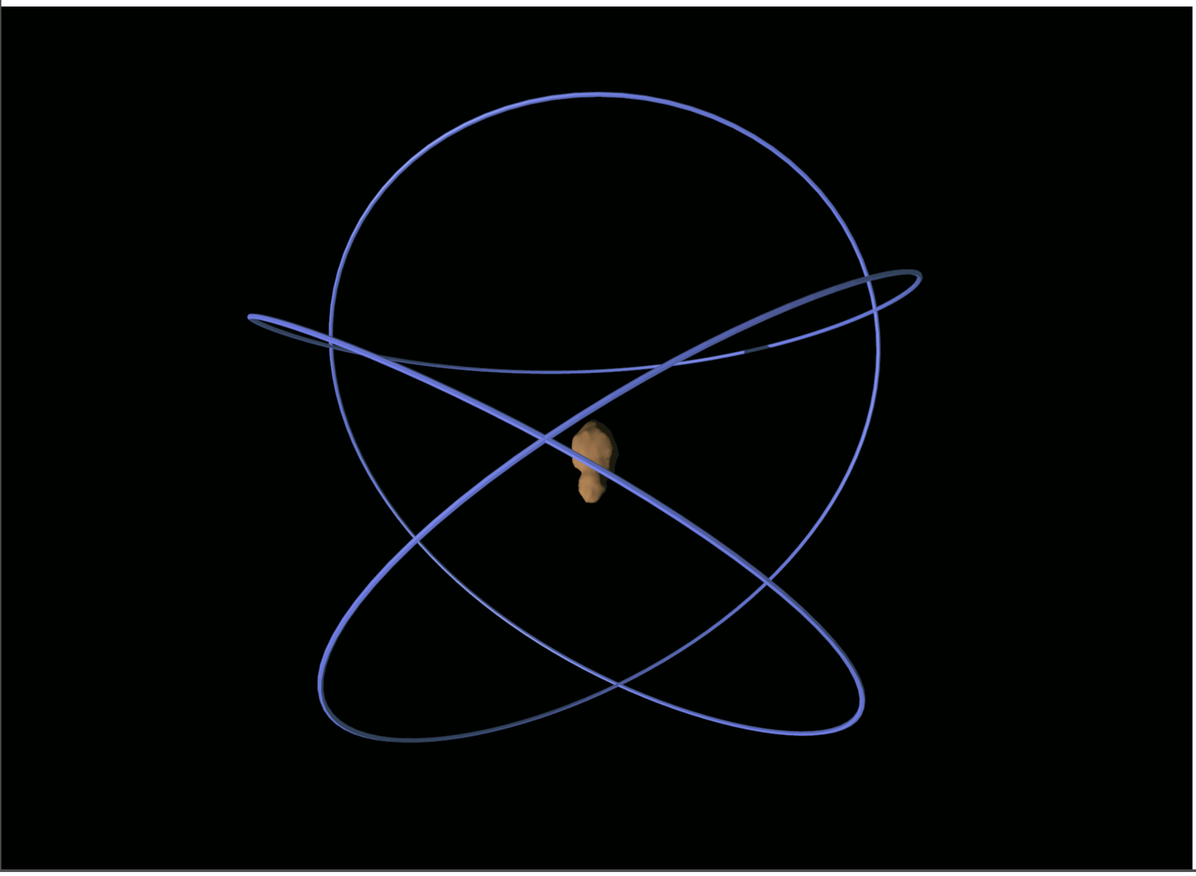
Periodic Orbits as Stability Probes



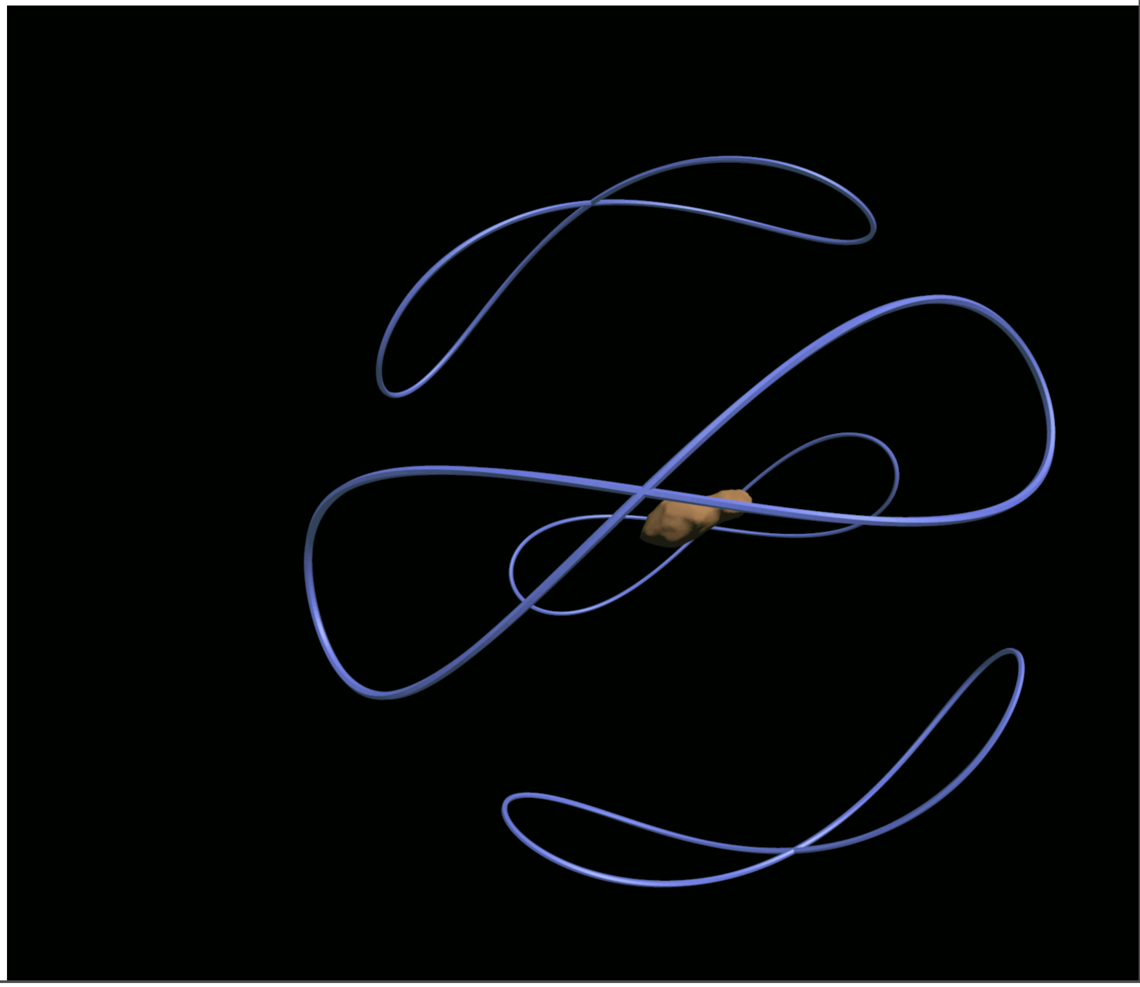
Periodic Orbit Family Stability Defines Transitions between Stable and Unstable regions of phase space



Lara & Scheeres (2002)

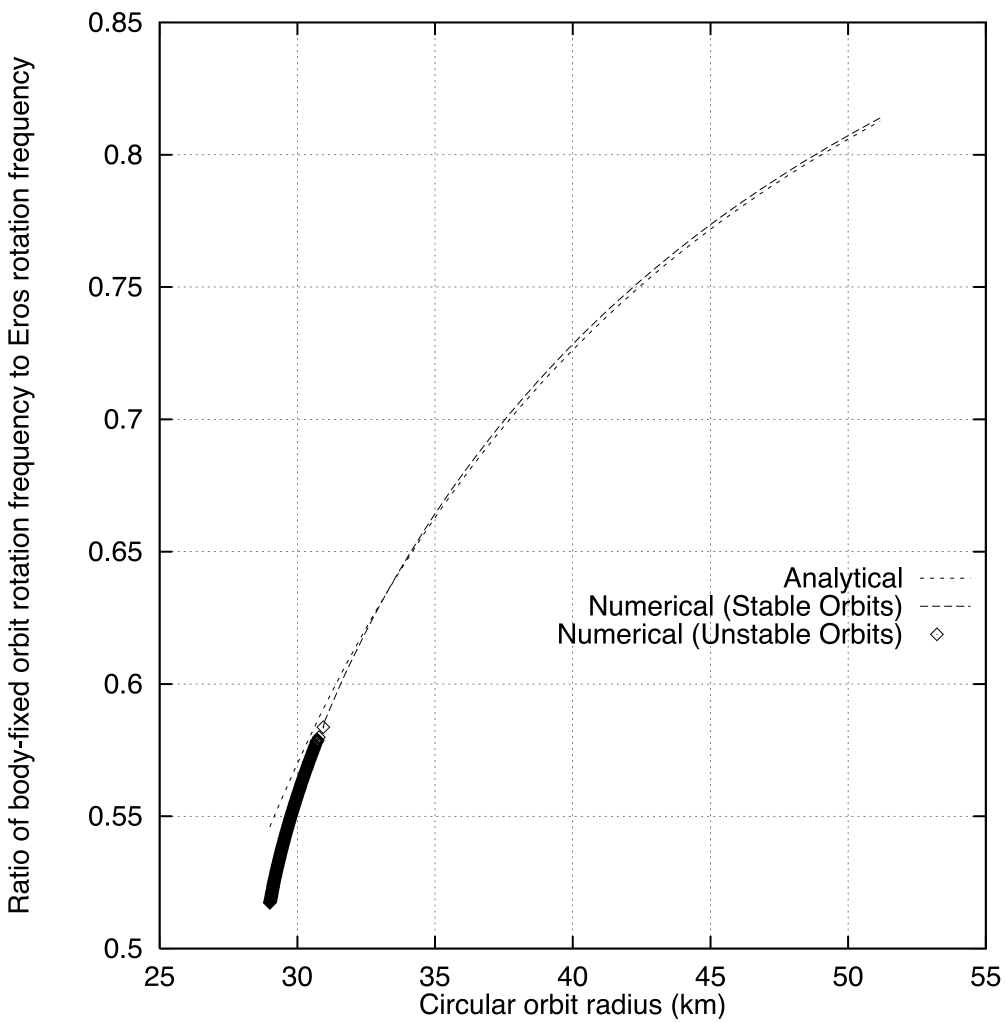


Stable Periodic Orbits at Toutatis

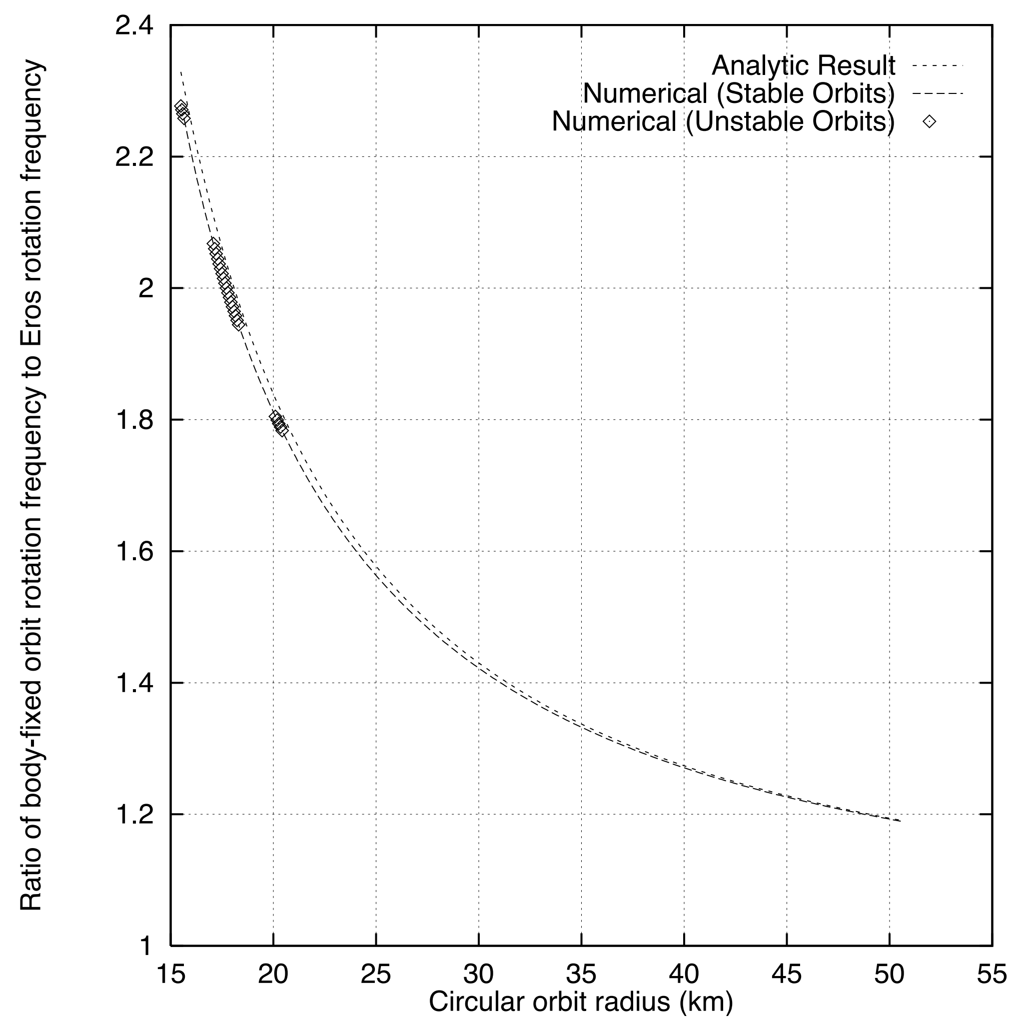


Periodic Orbit Stability at Eros

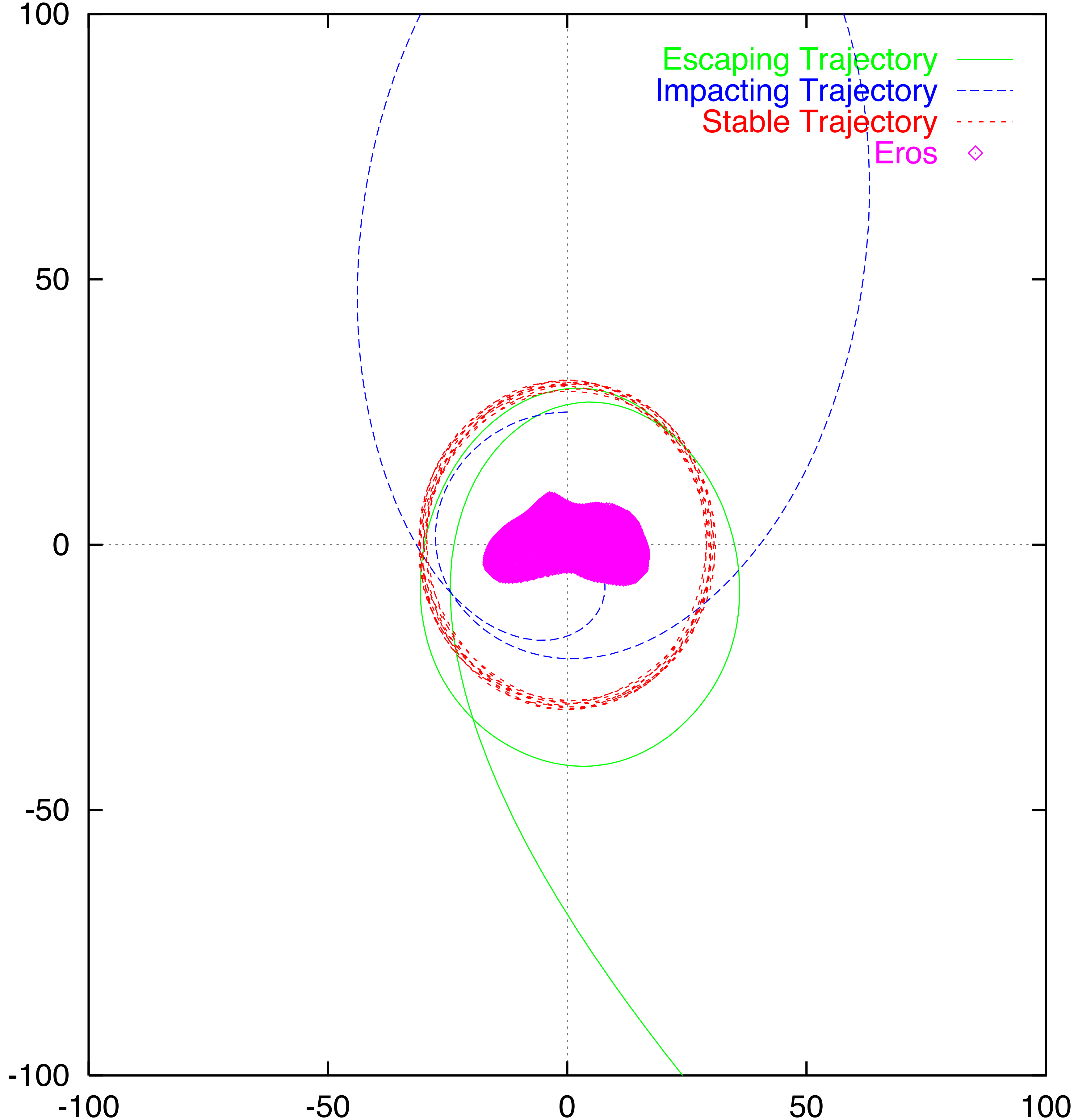
- Stability limits can be determined using periodic orbits and noting stability transitions



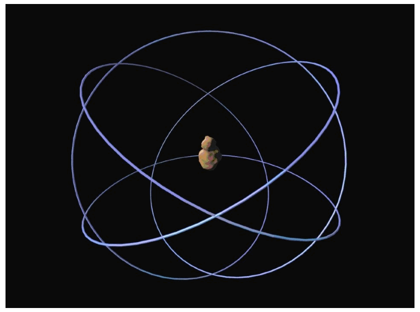
Direct Orbits



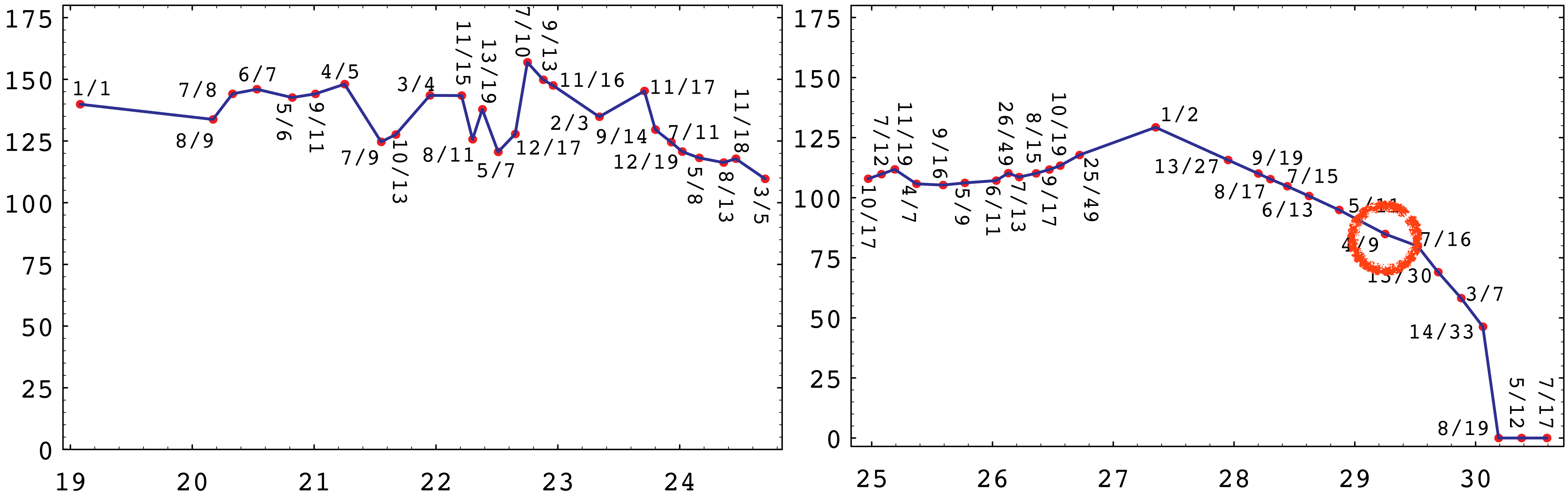
Retrograde Orbits

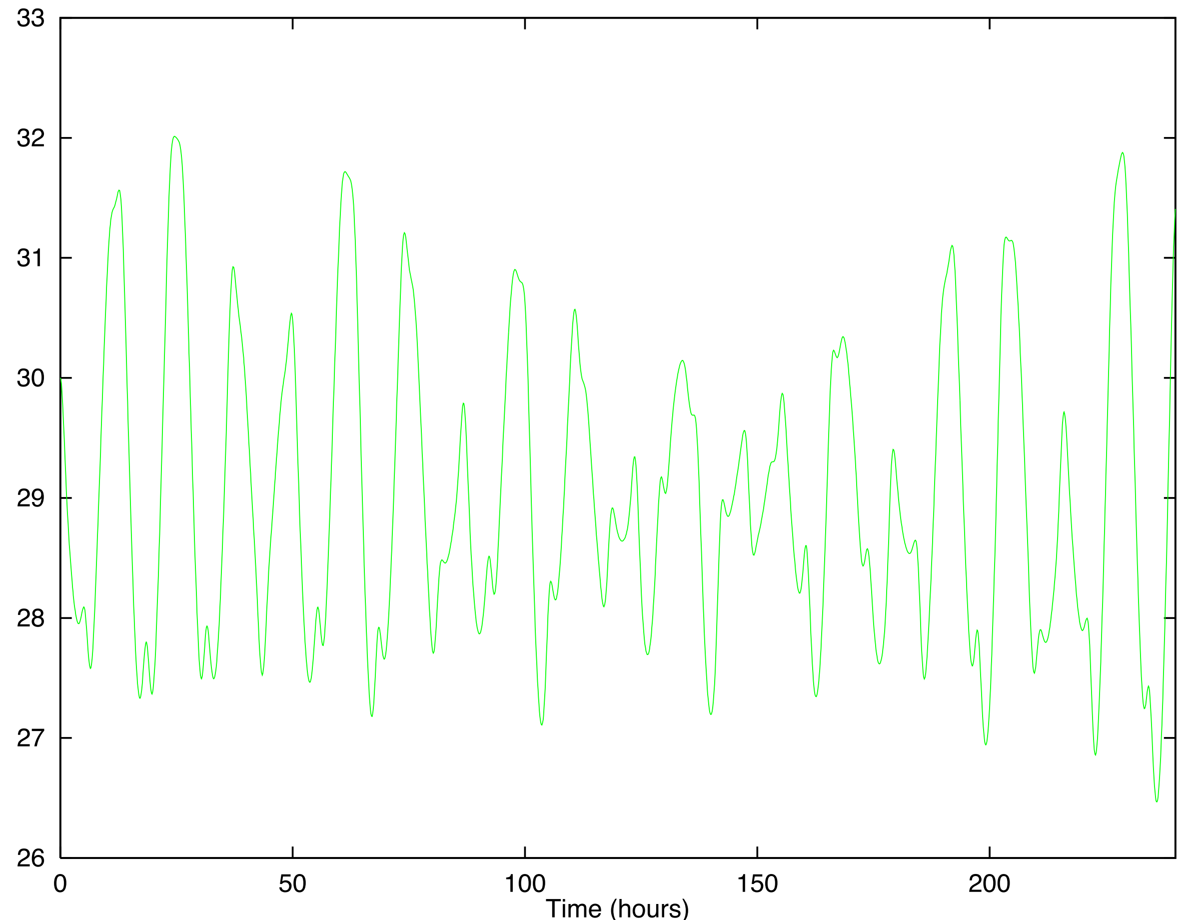

Escaping Trajectory
Impacting Trajectory
Stable Trajectory
Eros
100
50
0
-50
-100
-100
-50
0
50
100

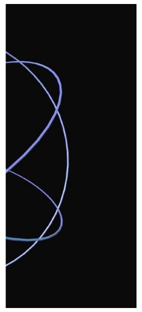
Non-Planar Orbit Stability



- Analysis can be extended to orbital stability vs inclination via resonant periodic orbits (*M. Lara & D.J. Scheeres, JAS 2002*):



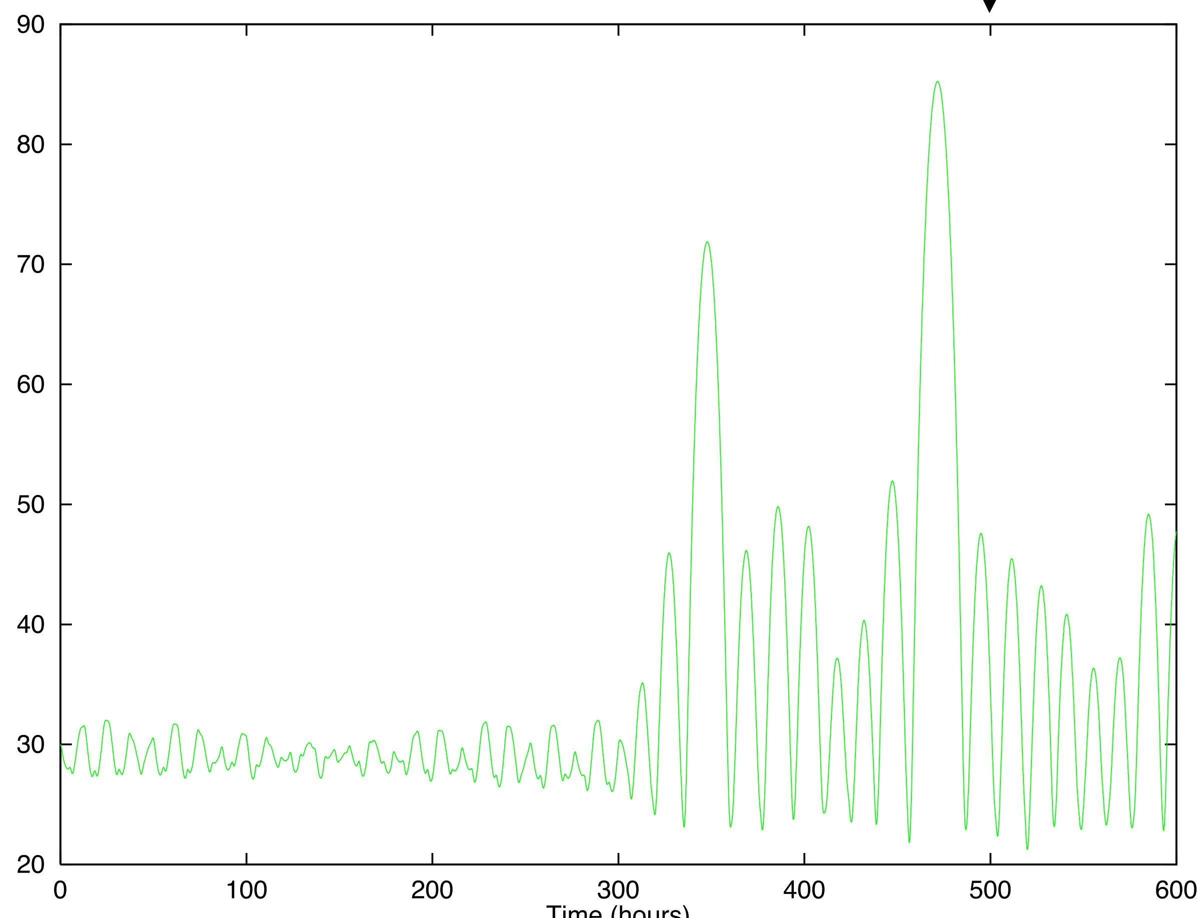




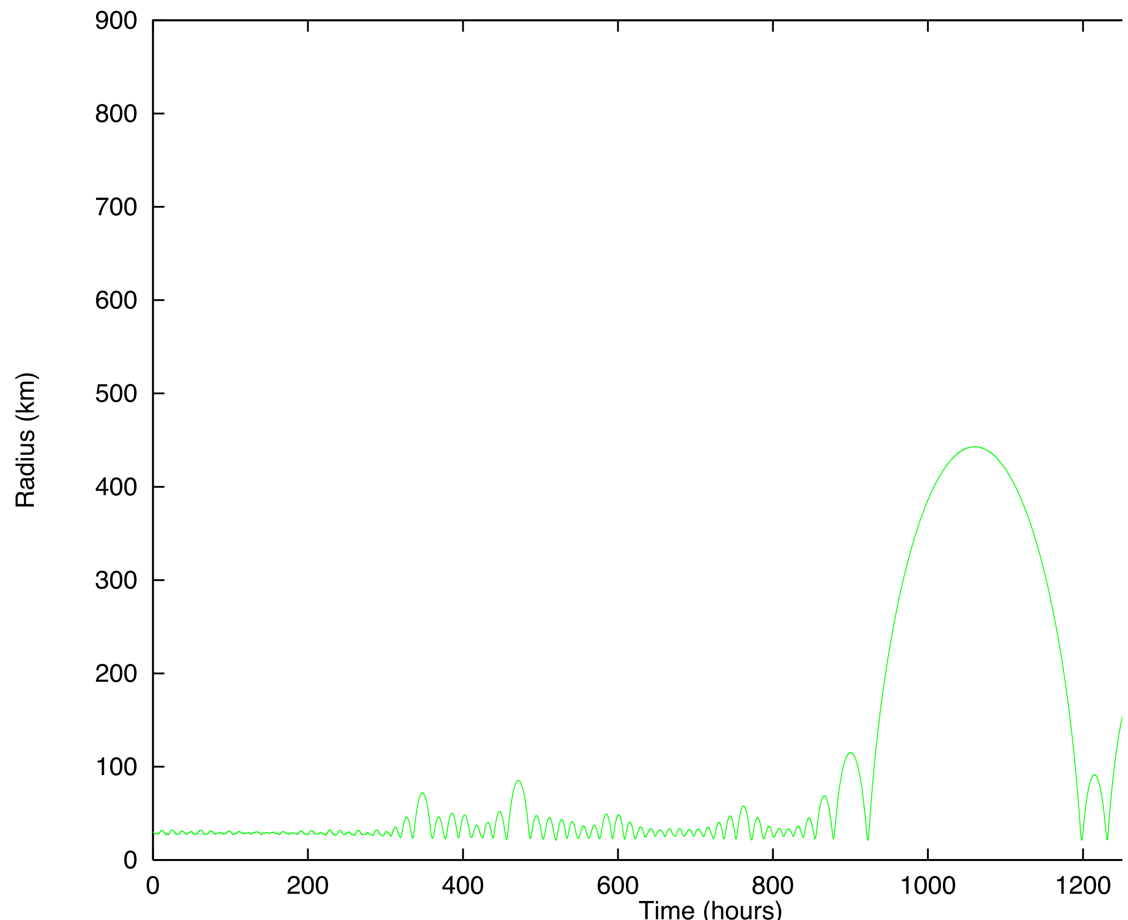
Initially Polar ~29 km Orbit
Radius is plotted after 10, 25, and 60 days

days

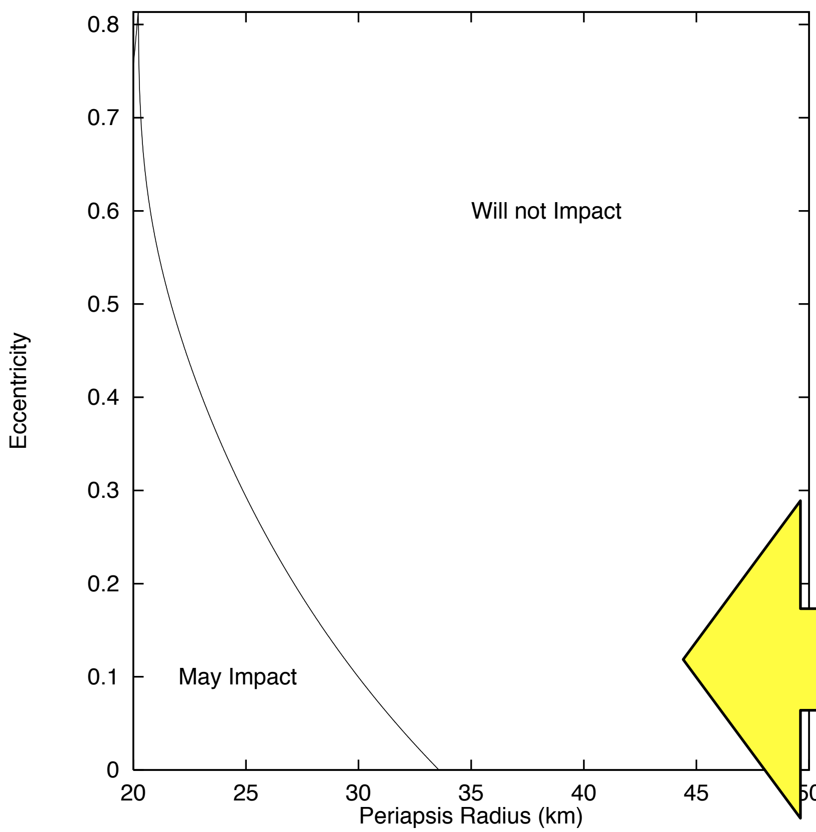
25 days



60 days

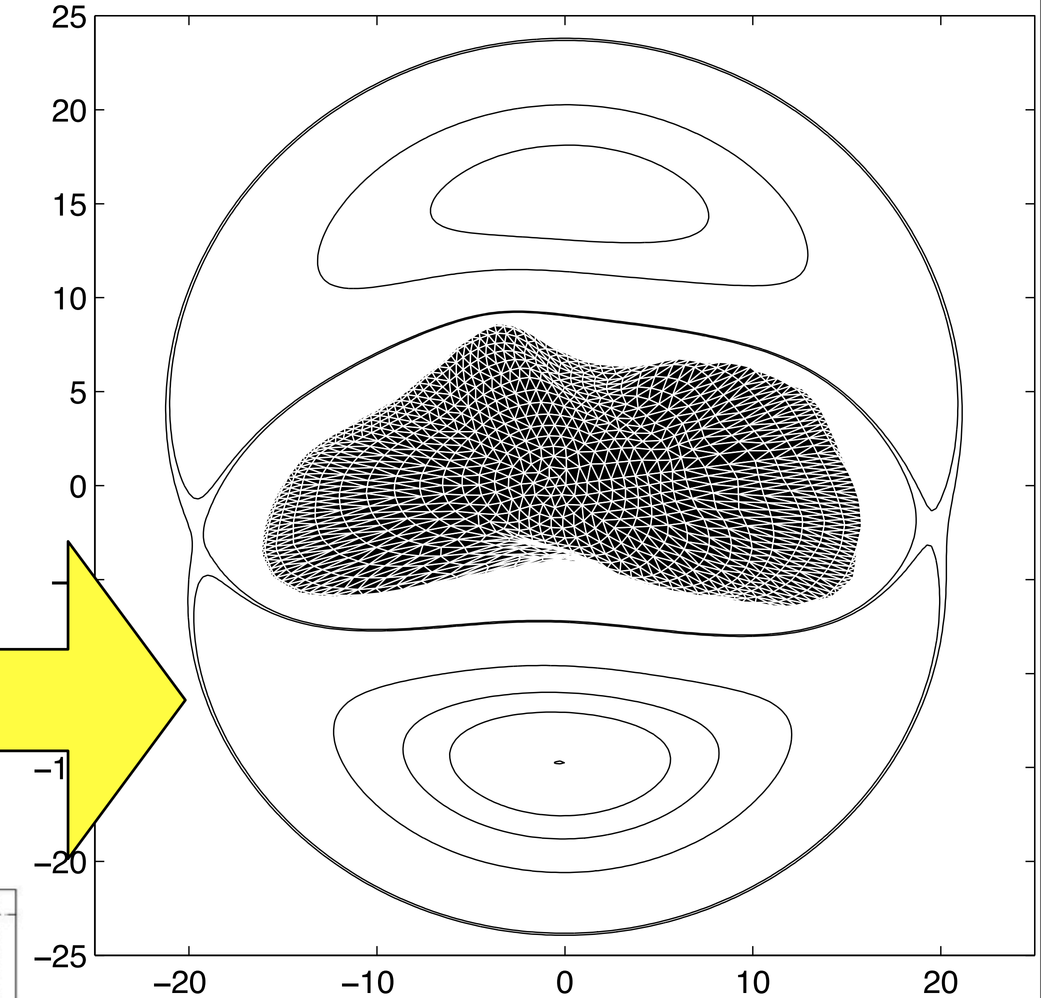


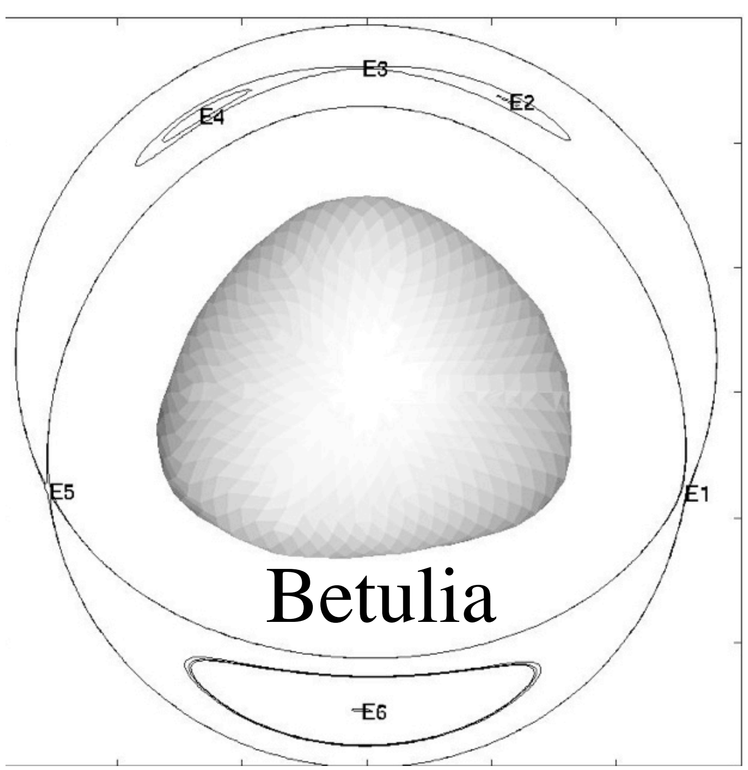
Zero-Velocity Curves to Define Impact-Free Orbits

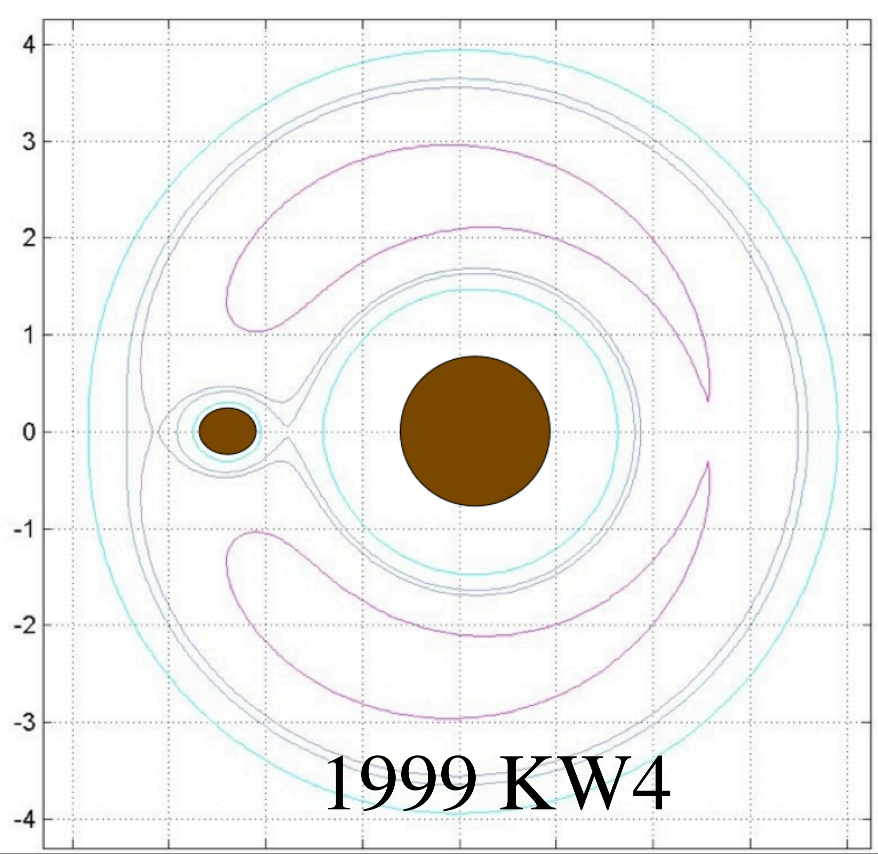


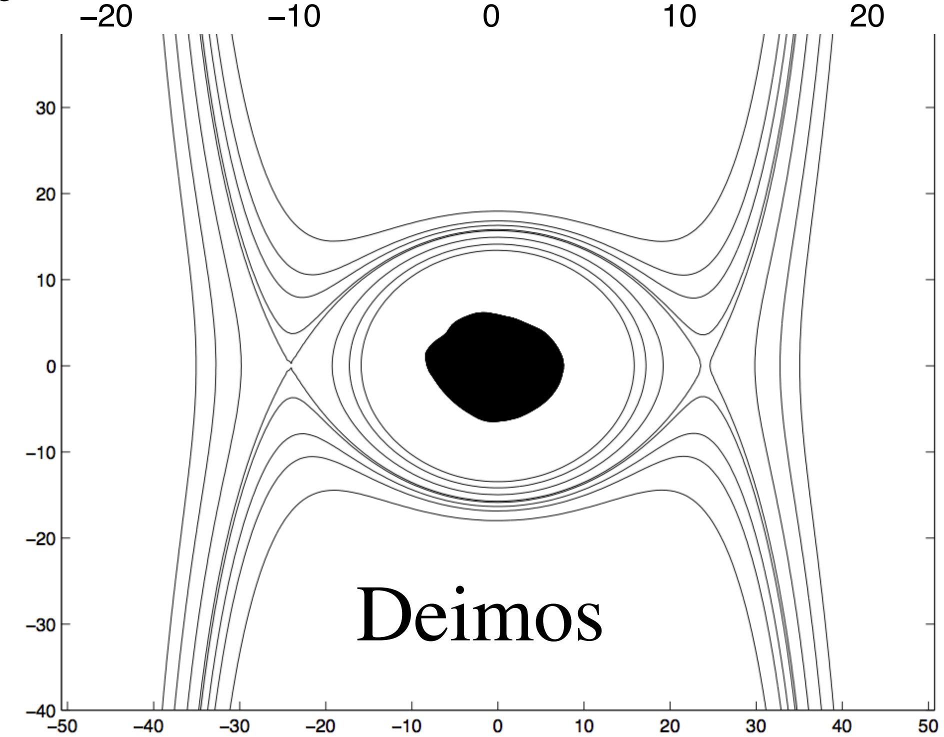
Eros Zero-Velocity Curves and resultant sharp limits on non-impacting orbits for uniformly rotating bodies

$$J = \frac{1}{2}\dot{\boldsymbol{r}} \cdot \dot{\boldsymbol{r}} - \frac{1}{2}\omega^2 \left(x^2 + y^2\right) - U(\boldsymbol{r})$$



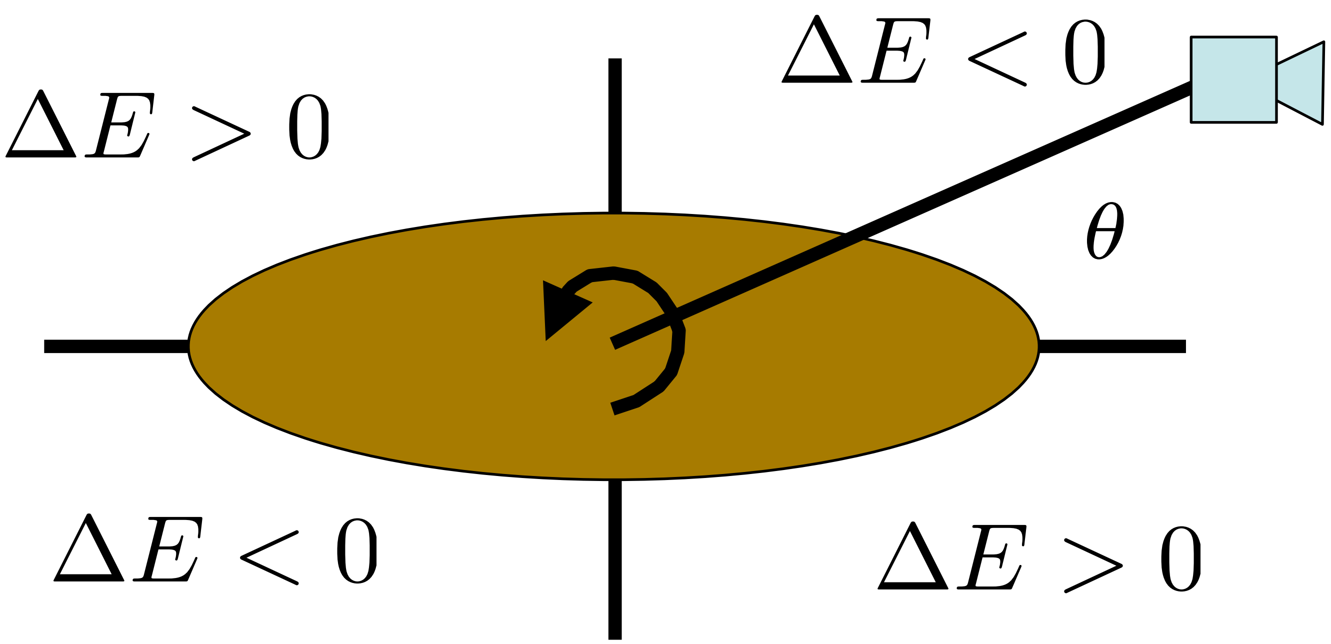


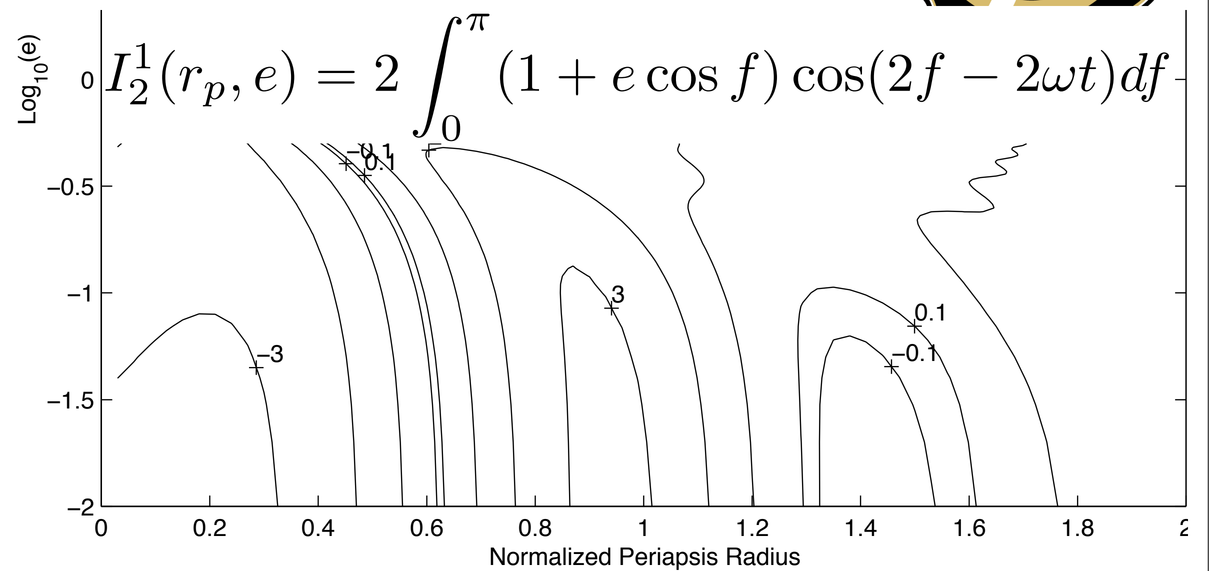


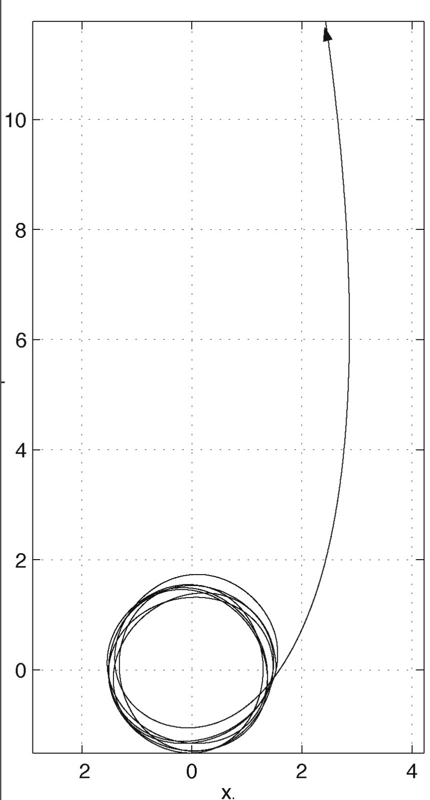


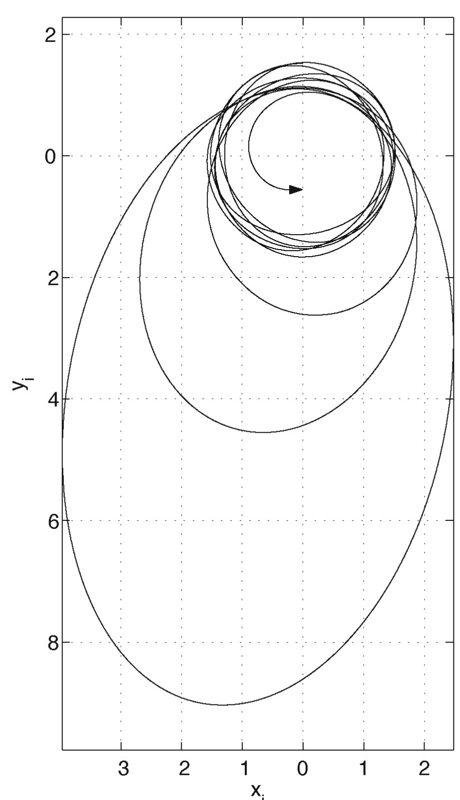
Semi-Analytical Results

$$\Delta E = -6\omega C_{22}R^2\sqrt{\frac{\mu}{p^3}}\cos^4(i/2)\sin 2(\theta) I_2^1(r_p, e)$$



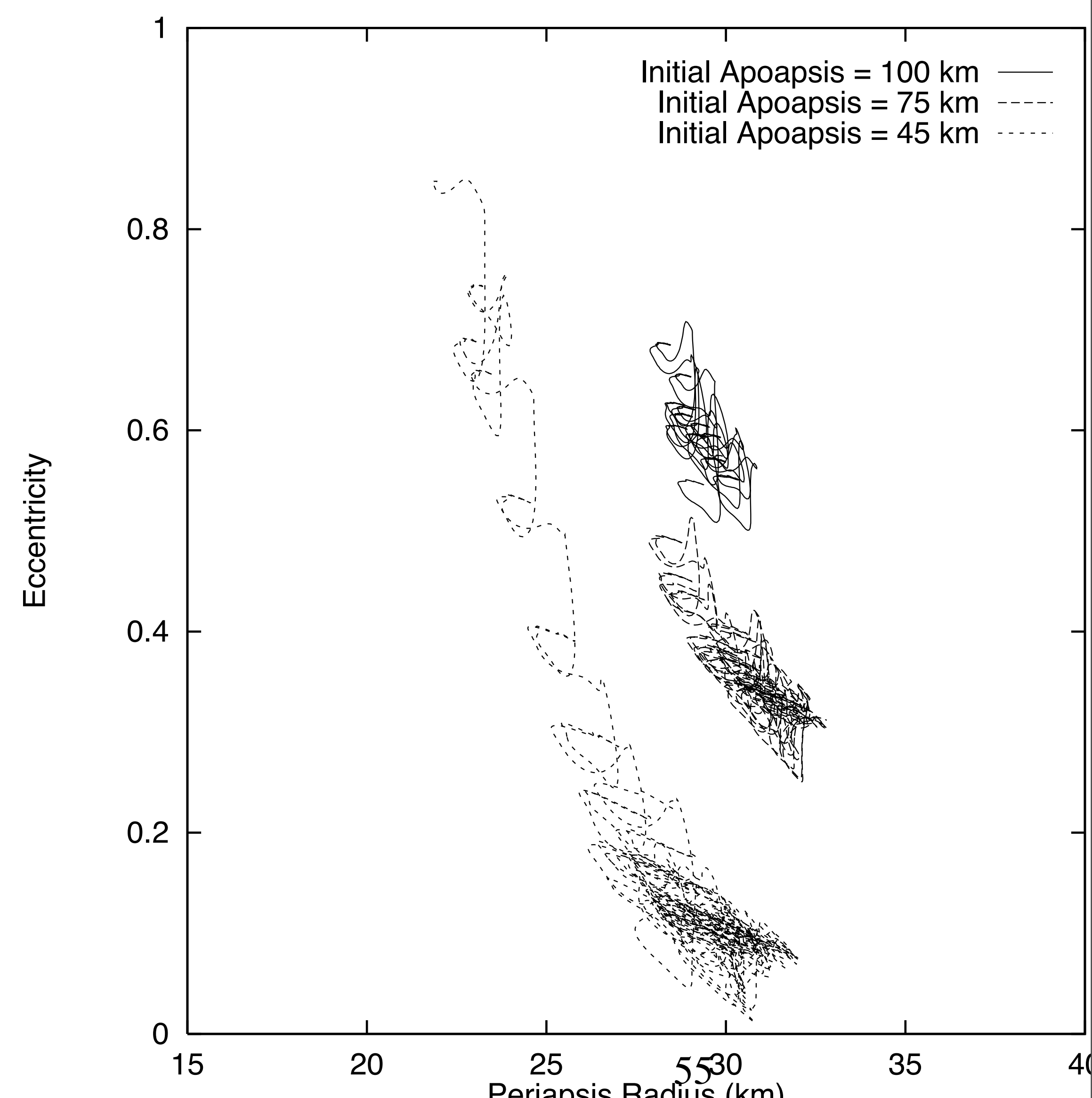




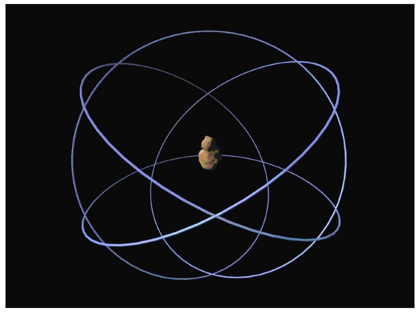


Placement of periapsis controls energy gain/loss. High inclination orbits suffer less perturbation

Boulder



Orbit Fluctuations

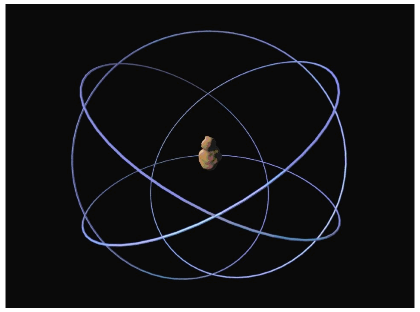


- Such strongly varying dynamics cannot be easily understood using classical perturbation theory
  - Example: we have developed a "discrete" perturbation theory that can compute changes in significant quantities each orbit:
    - G is angular momentum
    - C is Keplerian energy
    - $H = G\cos(i)$

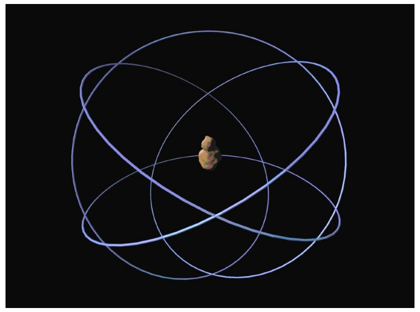
$$
\begin{aligned}
\Delta G &= -6C_{22}\sqrt{\frac{\mu}{p^3}} \\
&\quad \left[\cos^4(i/2)\sin 2(\omega+\Omega)I_2^1 + \sin^4(i/2)\sin 2(\omega-\Omega)I_{-2}^1\right] \\
\Delta H &= -6C_{22}\sqrt{\frac{\mu}{p^3}}\left[\frac{1}{2}\sin^2 i\sin 2\Omega I_0^1\right. \\
&\quad \left.+\cos^4(i/2)\sin 2(\omega+\Omega)I_2^1 - \sin^4(i/2)\sin 2(\omega-\Omega)I_{-2}^1\right] \\
\Delta C &= -6C_{22}\omega_E\sqrt{\frac{\mu}{p^3}}\left[\frac{1}{2}\sin^2 i\sin 2\Omega\left\{I_0^1-(1-e)^3I_0^{-2}\right\}\right. \\
&\quad +\cos^4(i/2)\sin 2(\omega+\Omega)\left\{I_2^1-(1-e)^3I_0^{-2}\right\} \\
&\quad \left.-\sin^4(i/2)\sin 2(\omega-\Omega)\left\{I_{-2}^1-(1-e)^3I_0^{-2}\right\}\right]
\end{aligned}
$$

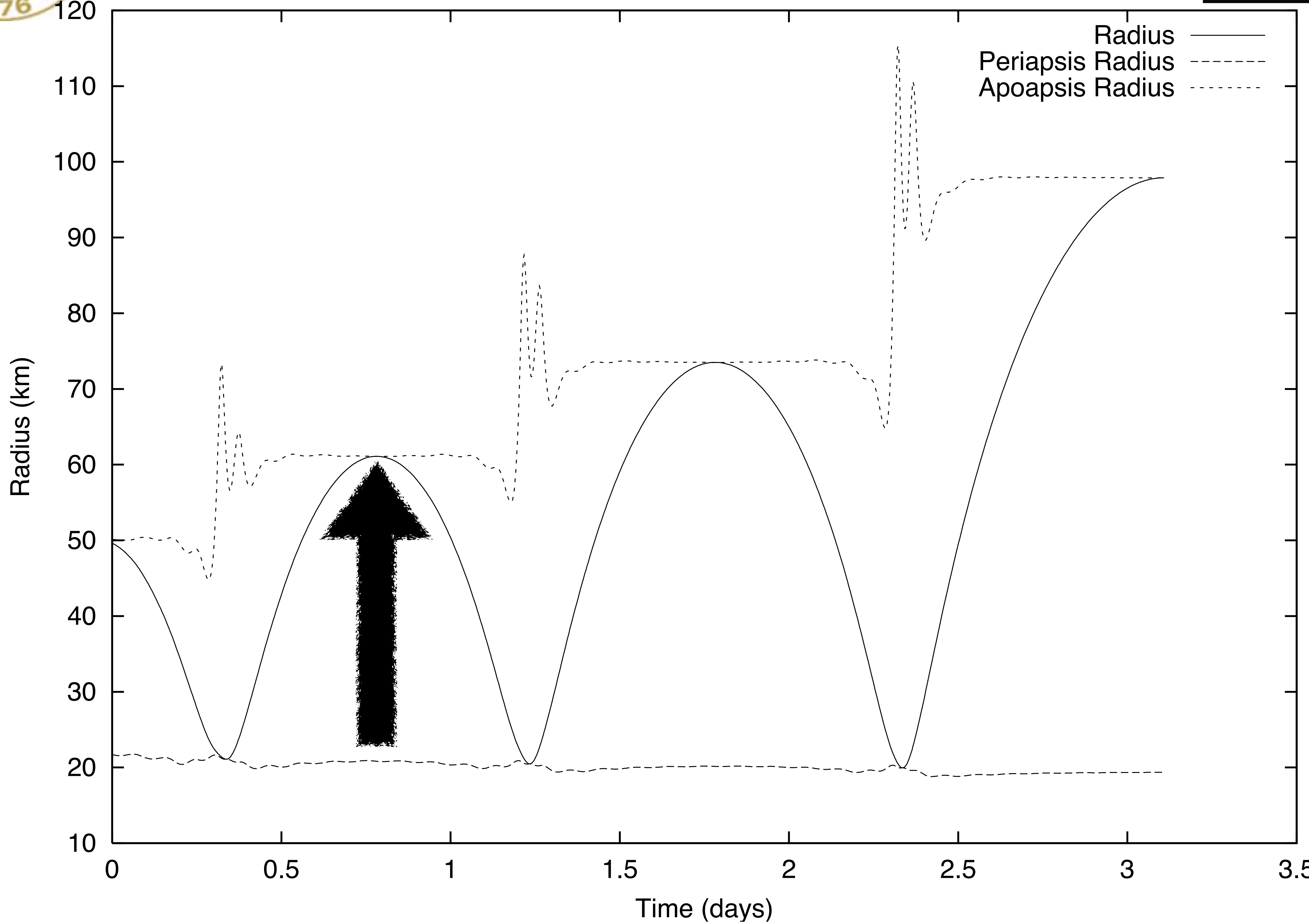
- The I_i^j are Hansen Coefficients and are only a function of a and e

Transient Motion and Chaoticity

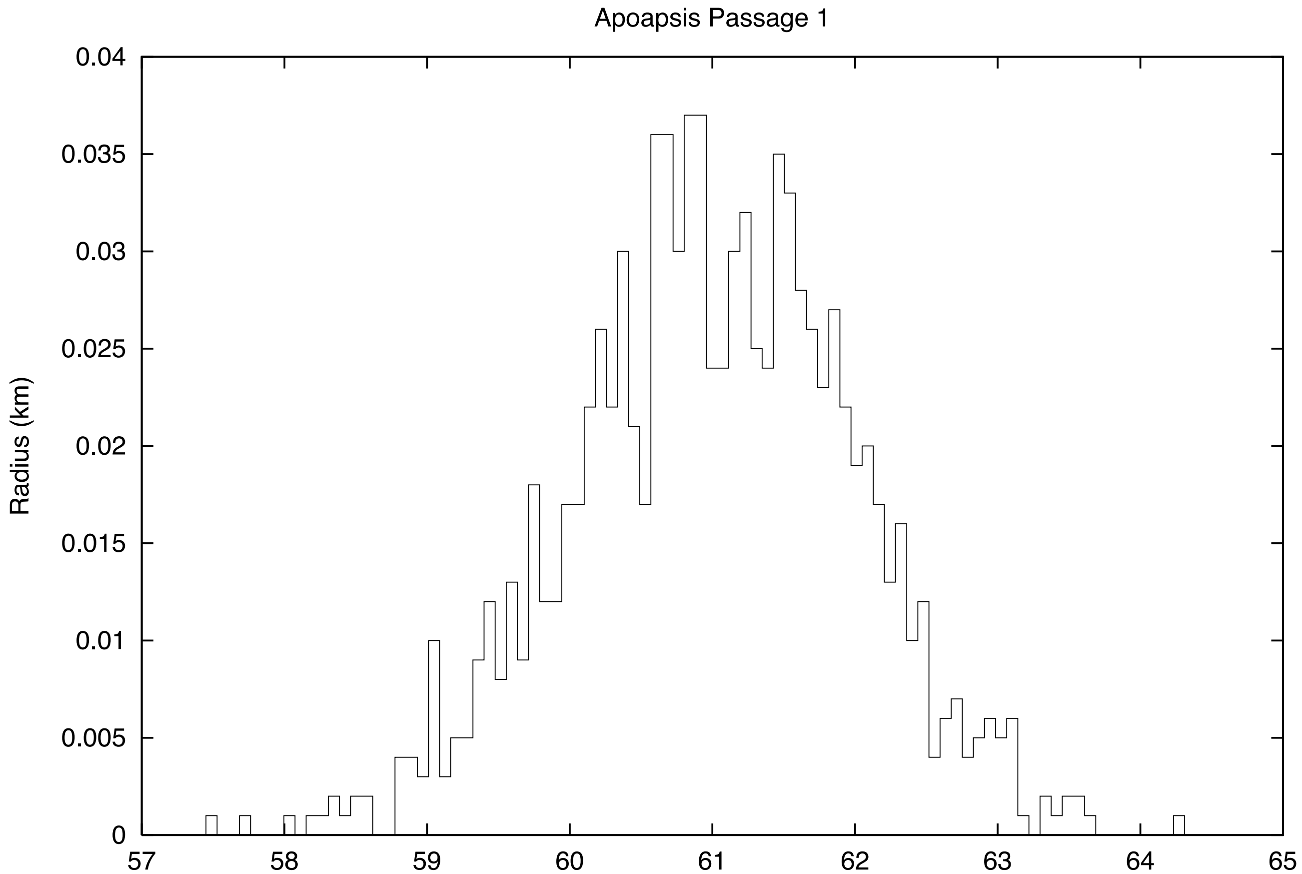


- The clearest example of the strength of orbital perturbations can be shown using orbital uncertainties
  - In the following we initiate an orbit with uncertainties of 10 meters in position and 1 cm/s in velocity about the asteroid **Eros**
  - We use 1000 points randomly drawn from a Gaussian distribution
  - The orbit and distributions are propagated for three full orbits about the asteroid passages
- The strongly non-Gaussian distribution of final orbit states indicates the difficulties associated with S/C navigation at small bodies

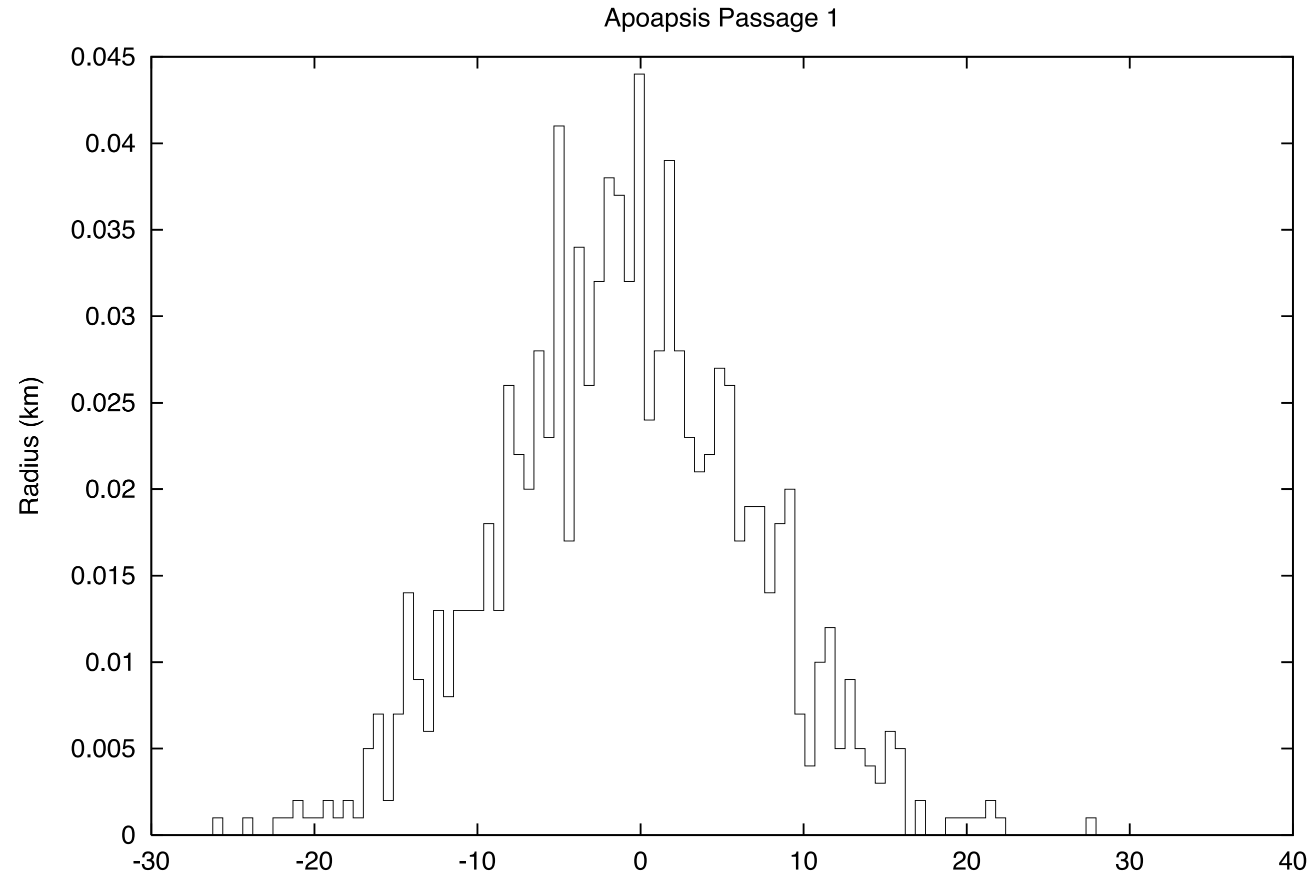




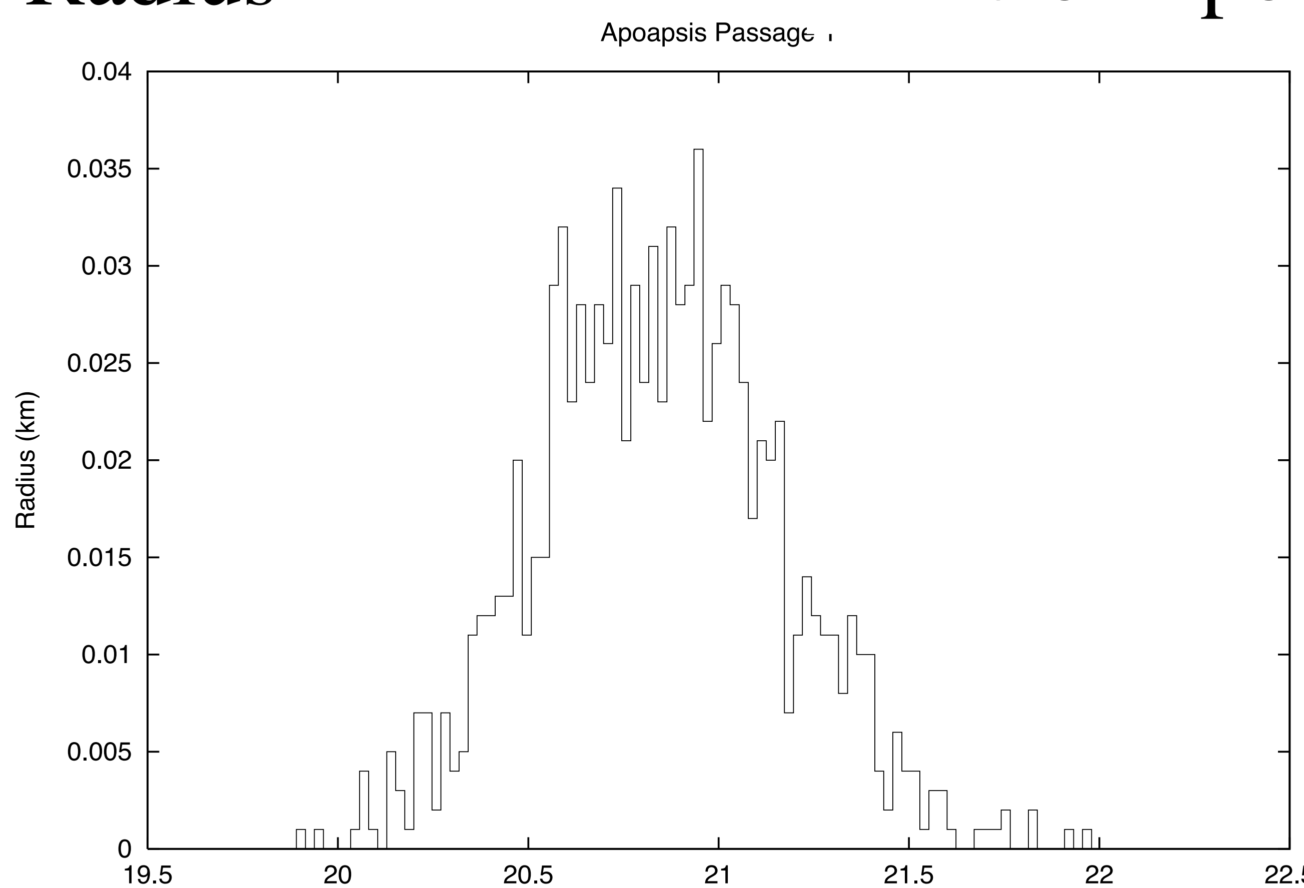
Radius
Periapsis Radius
Apoapsis Radius
Radius (km)
Time (days)
120
110
100
90
80
70
60
50
40
30
20
10
0
0.5
1
1.5
2
2.5
3
3.5



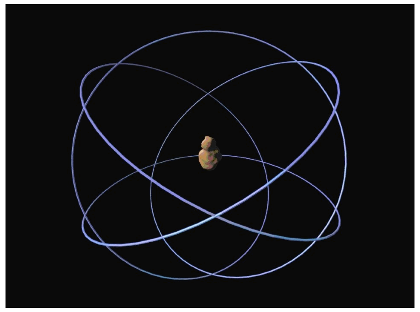
Apoapsis Radius

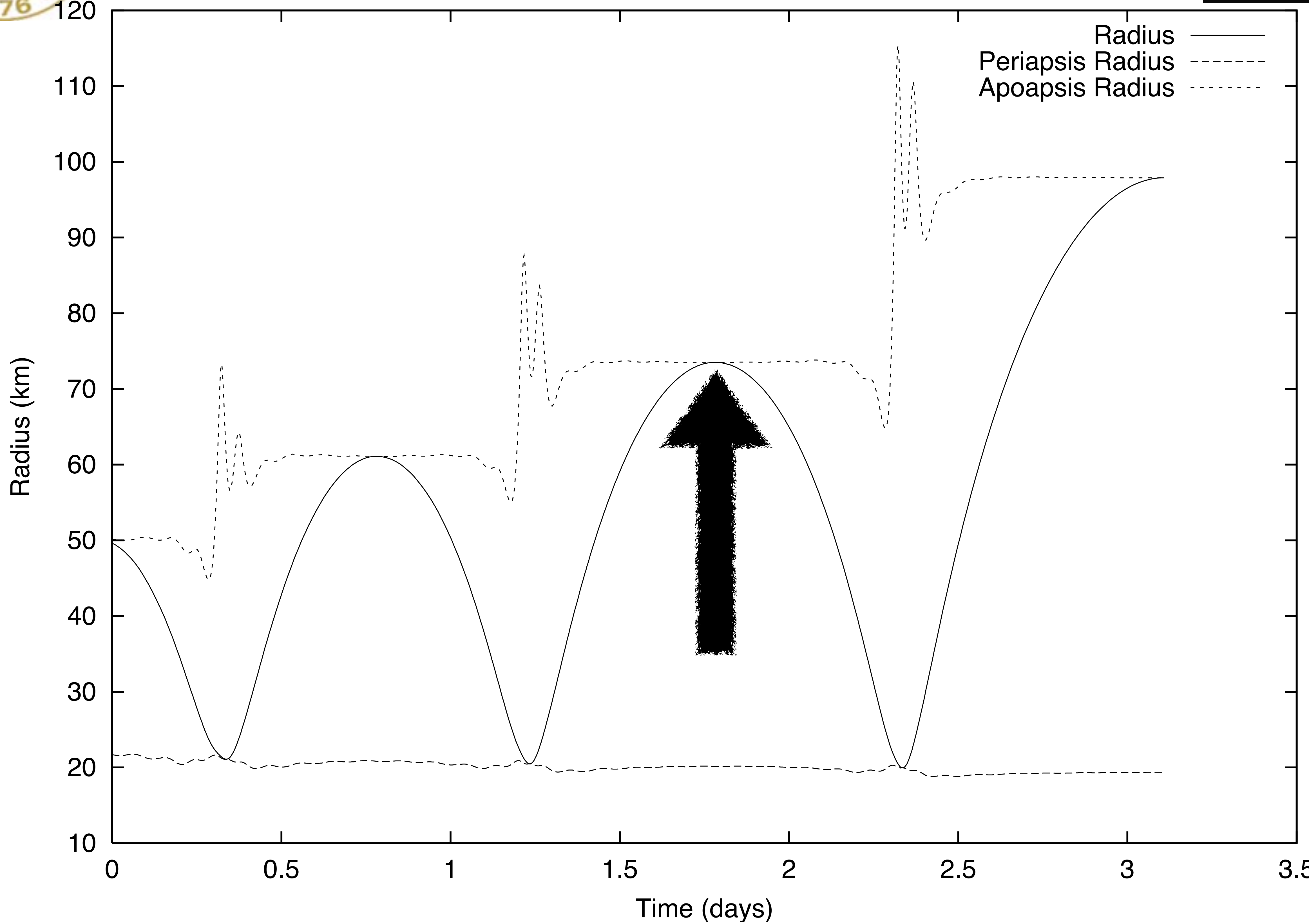


Time of Apoapsis Passage



Periapsis Radius





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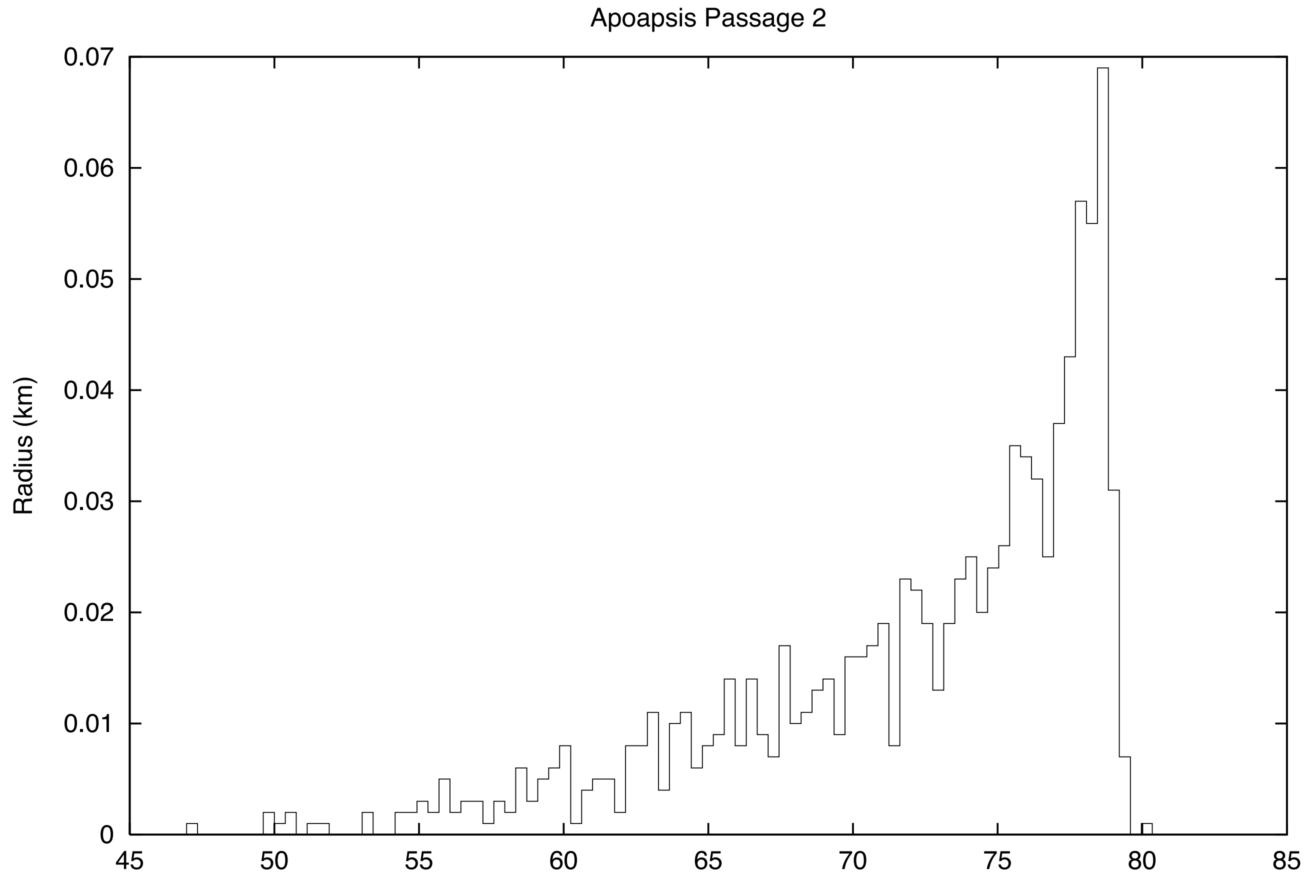
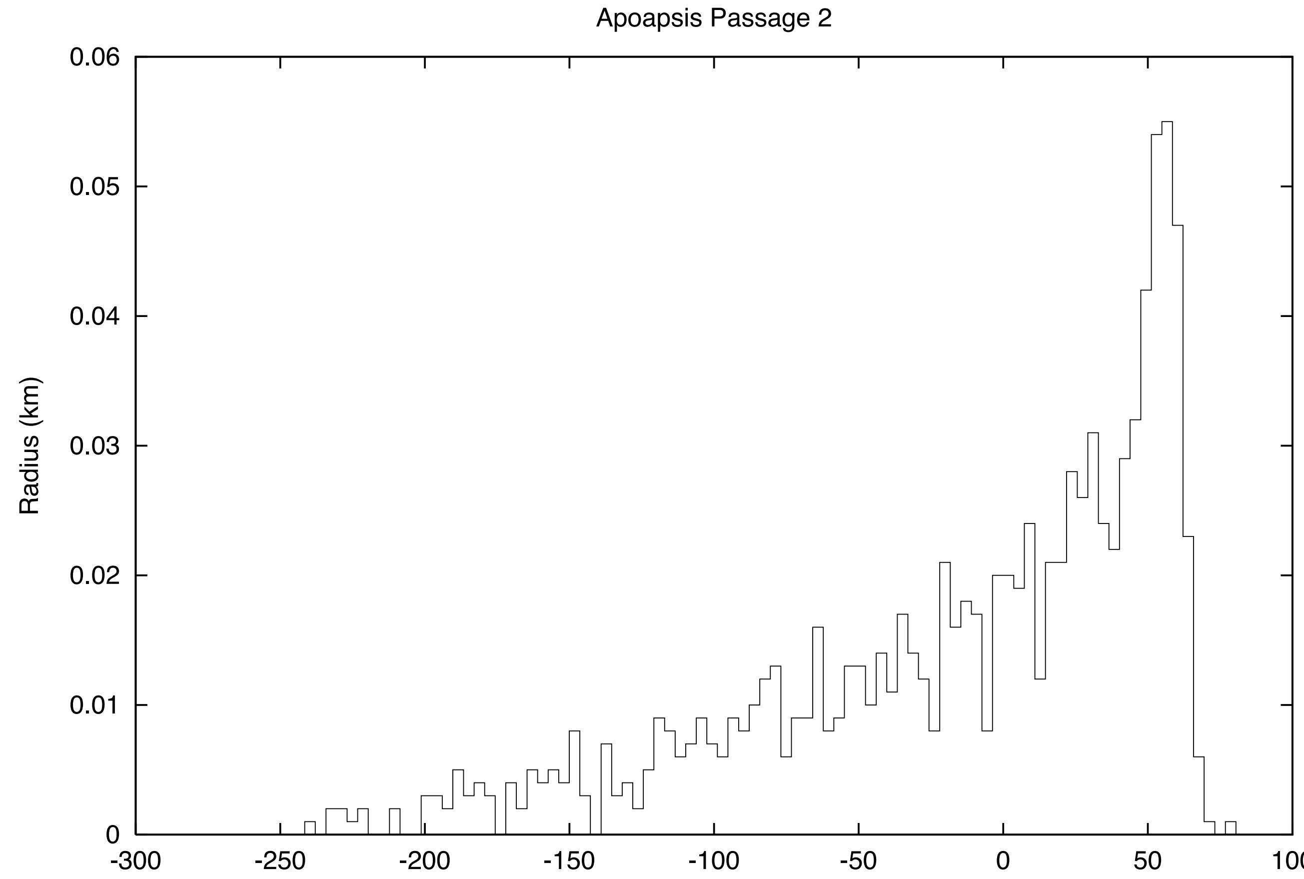
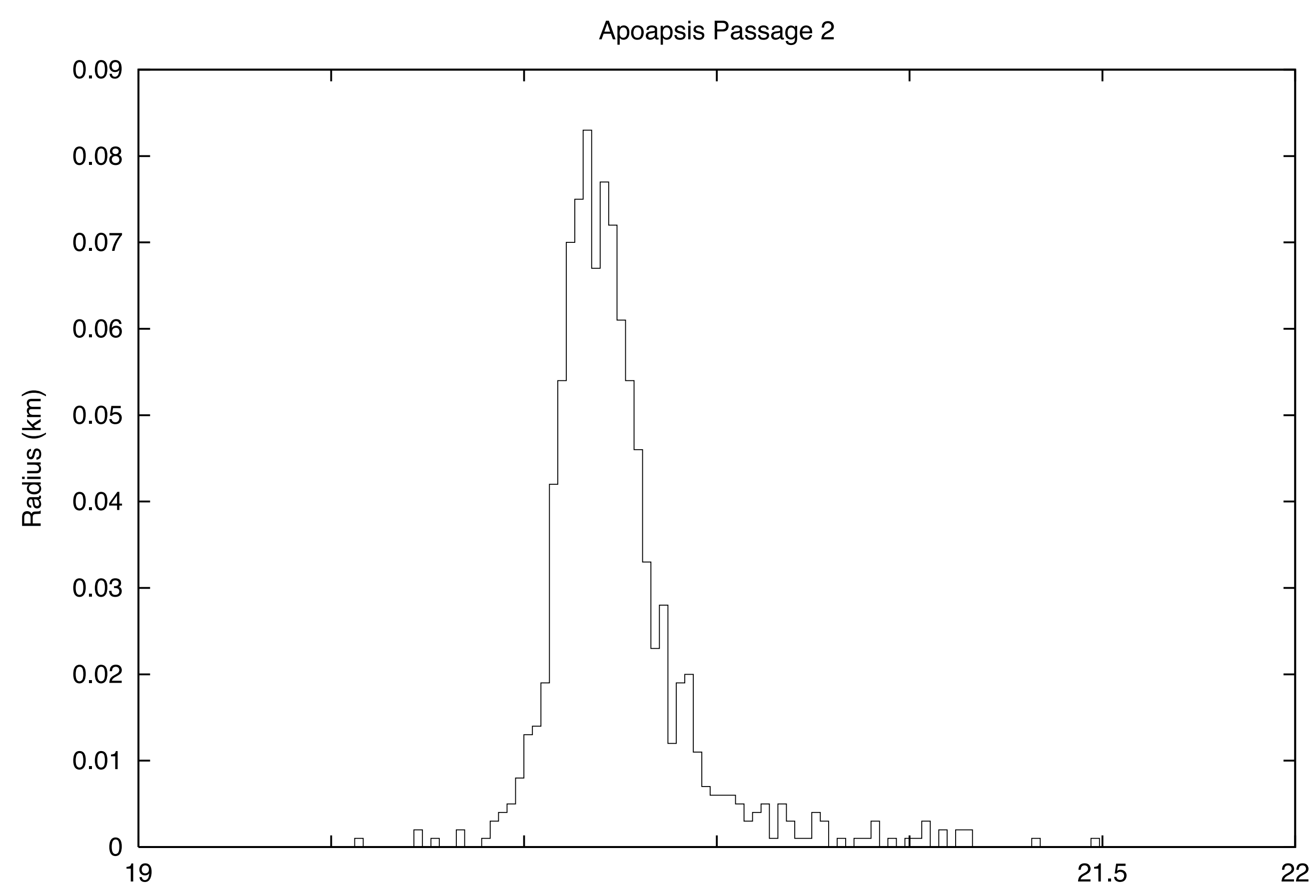


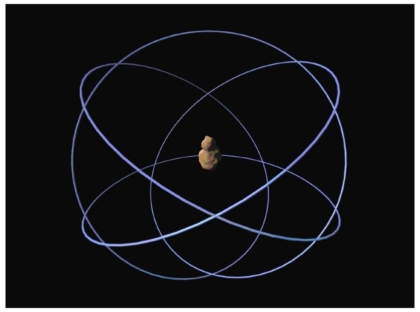
Apoapsis Radius

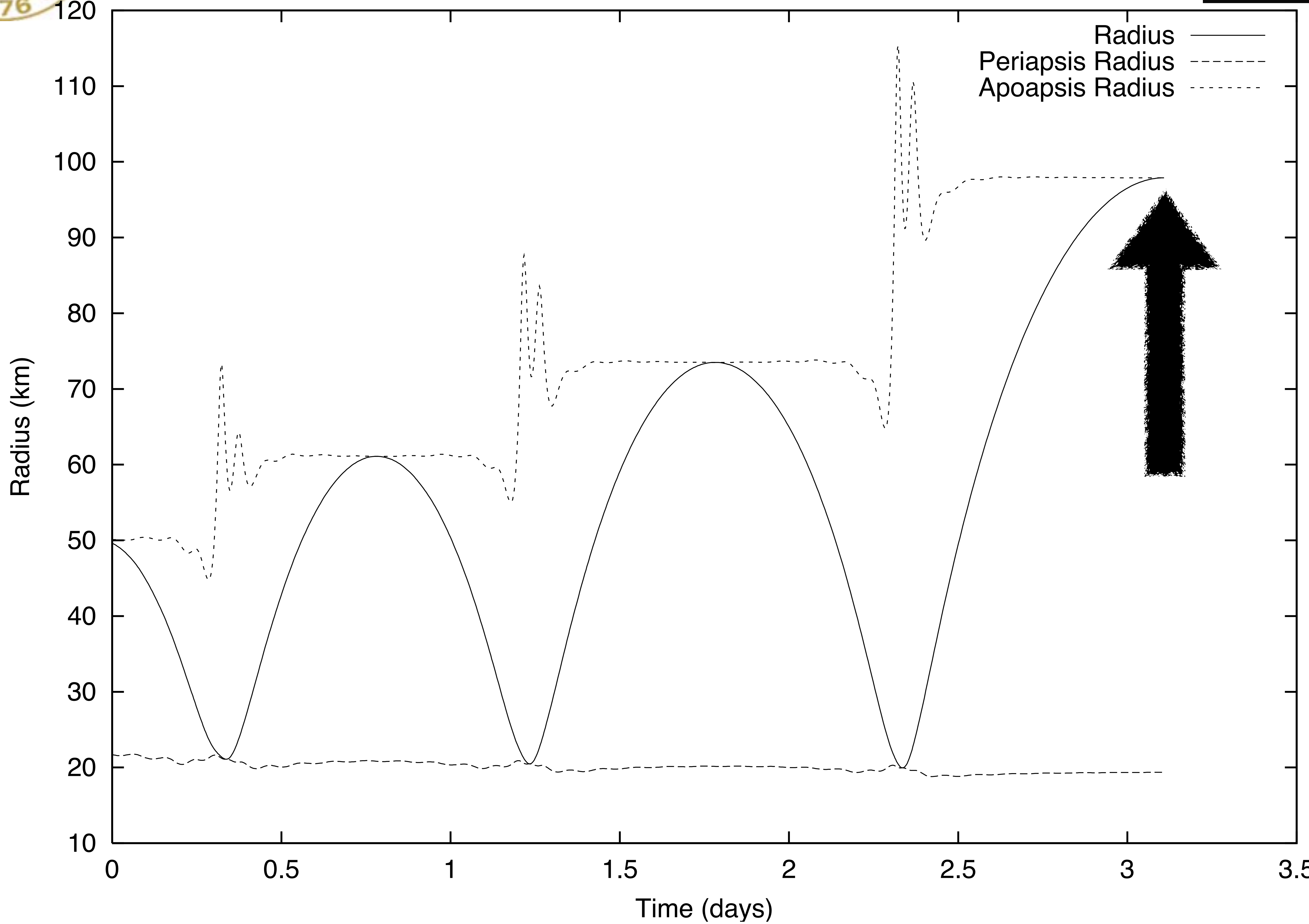


Time of Apoapsis Passage



Periapsis Radius





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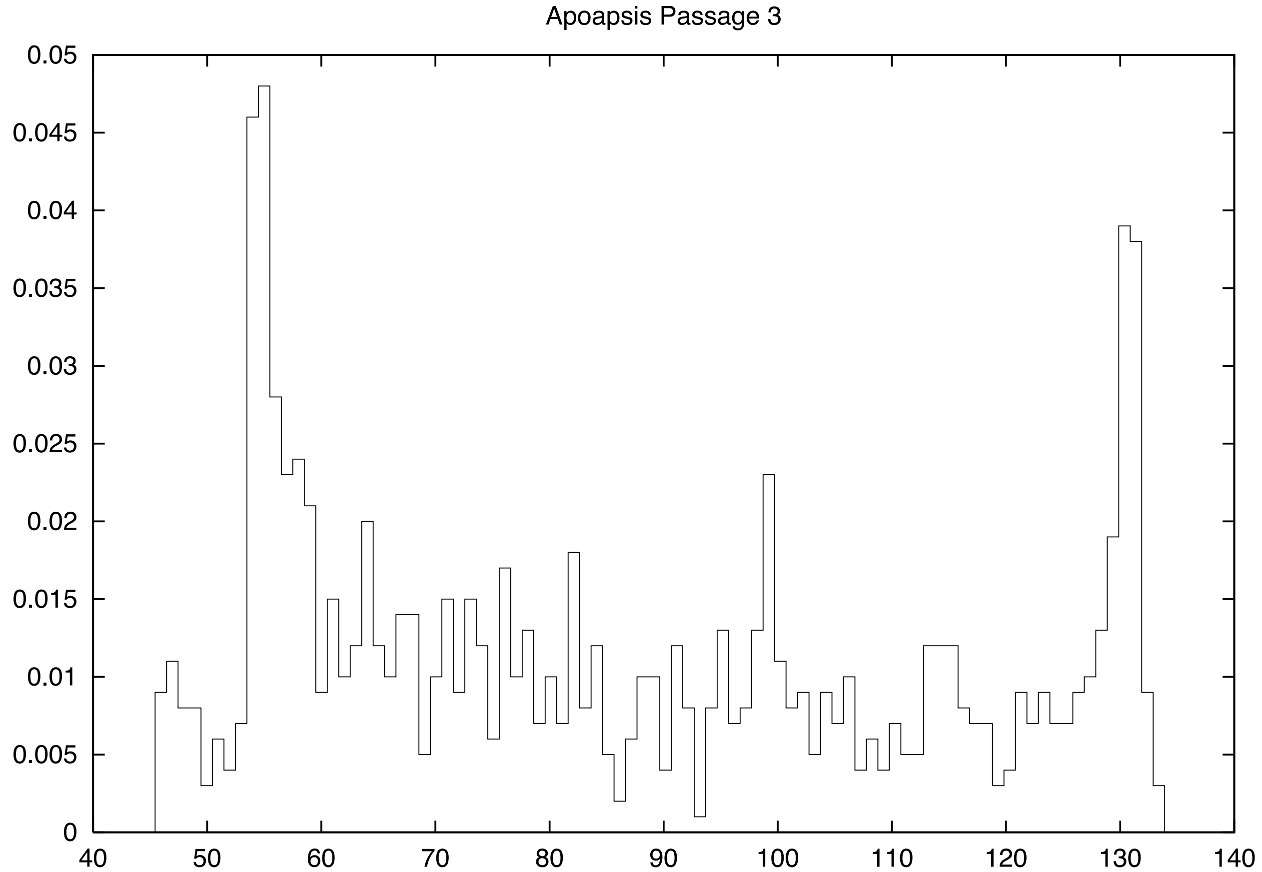
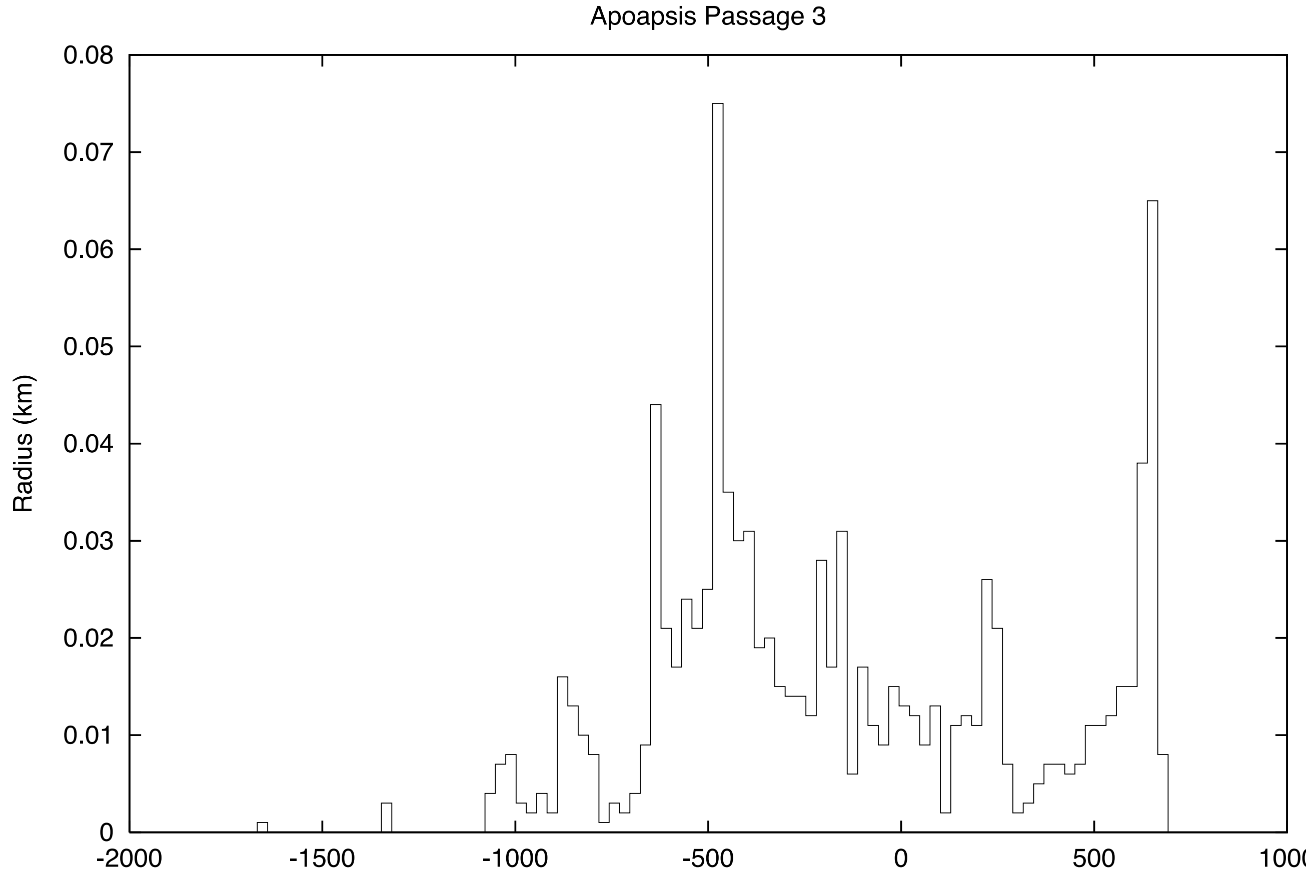
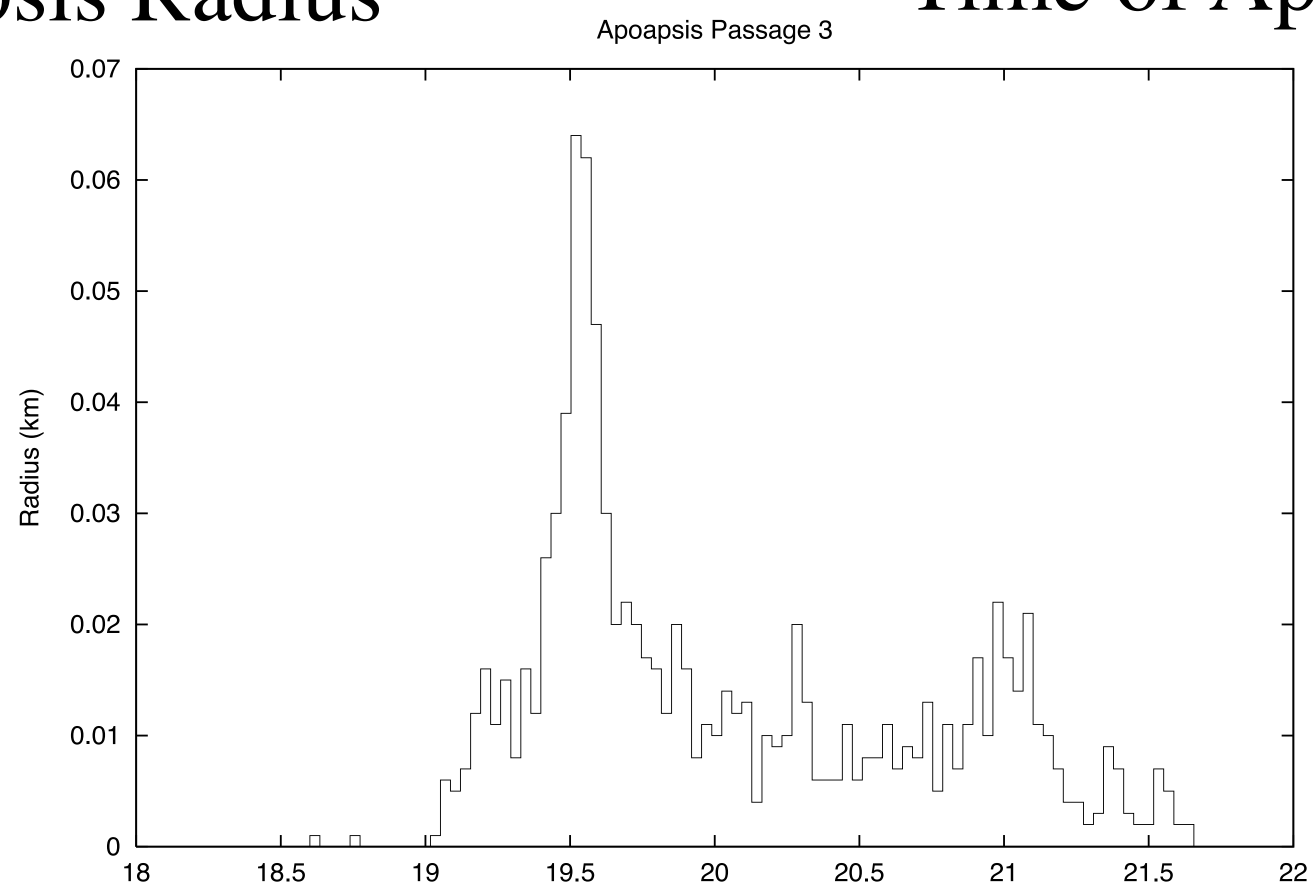


Apoapsis Radius



Time of Apoapsis Passage

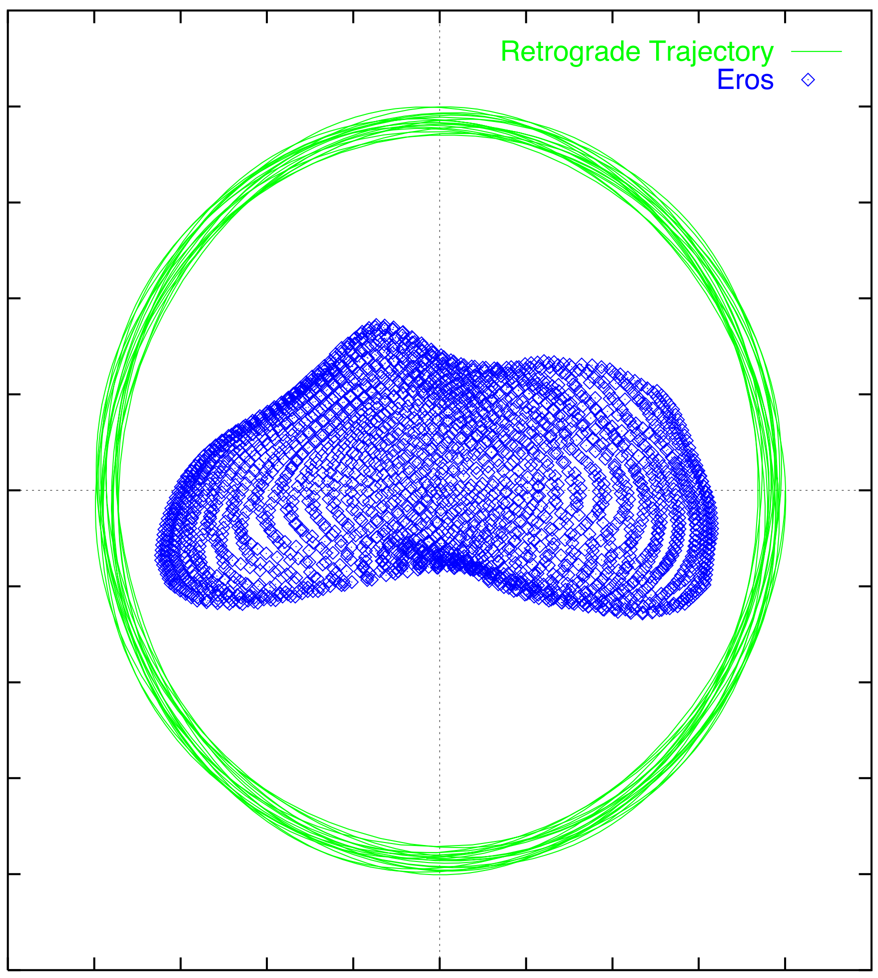


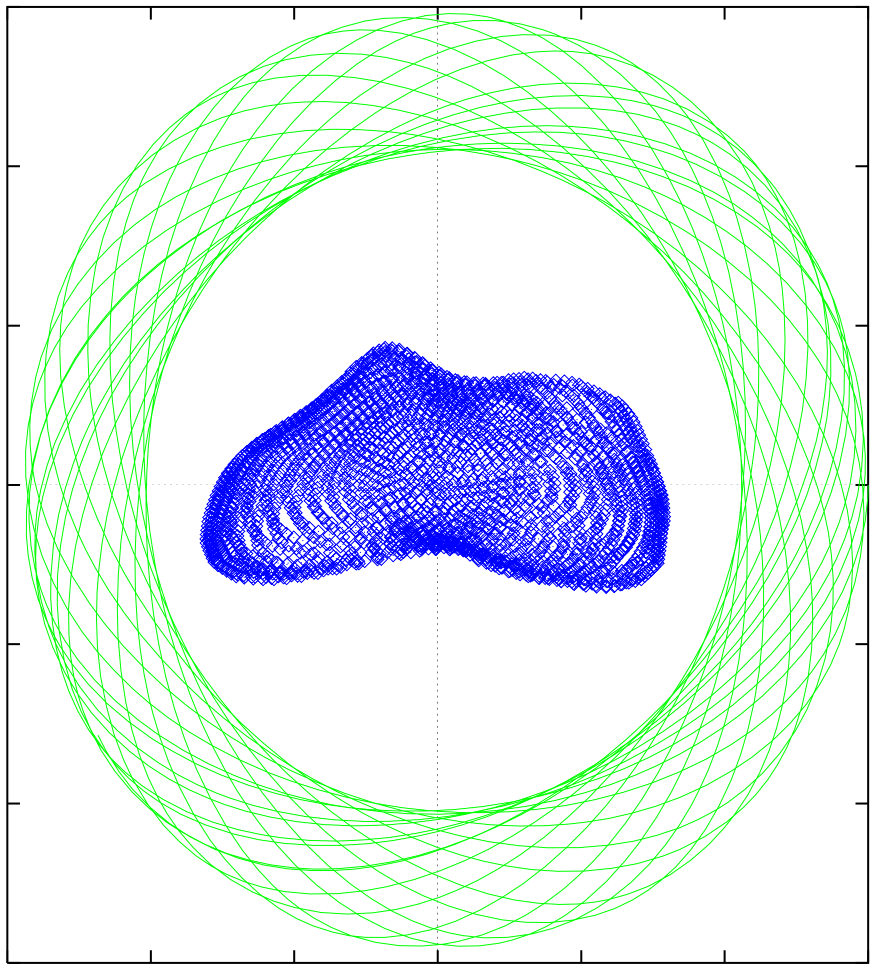
Periapsis Radius

Tuesday, January 15, 2013

What Operations are Feasible?

- ***NEAR at Eros***: Gravity dominated, controlled by avoiding resonances with the rotating mass distribution

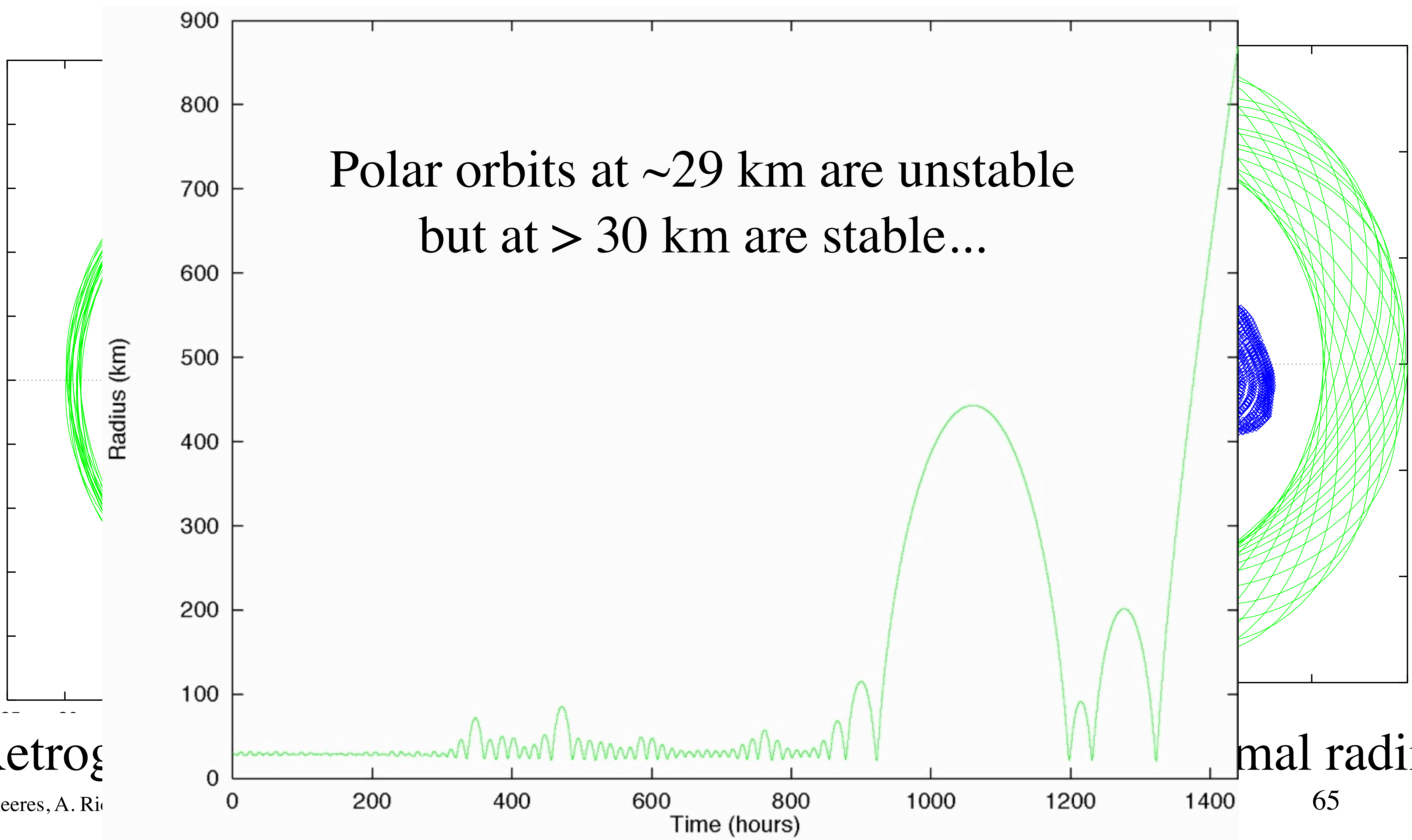




Retrograde orbits with inclination > 135° are stable to minimal radii

What Operations are Feasible?

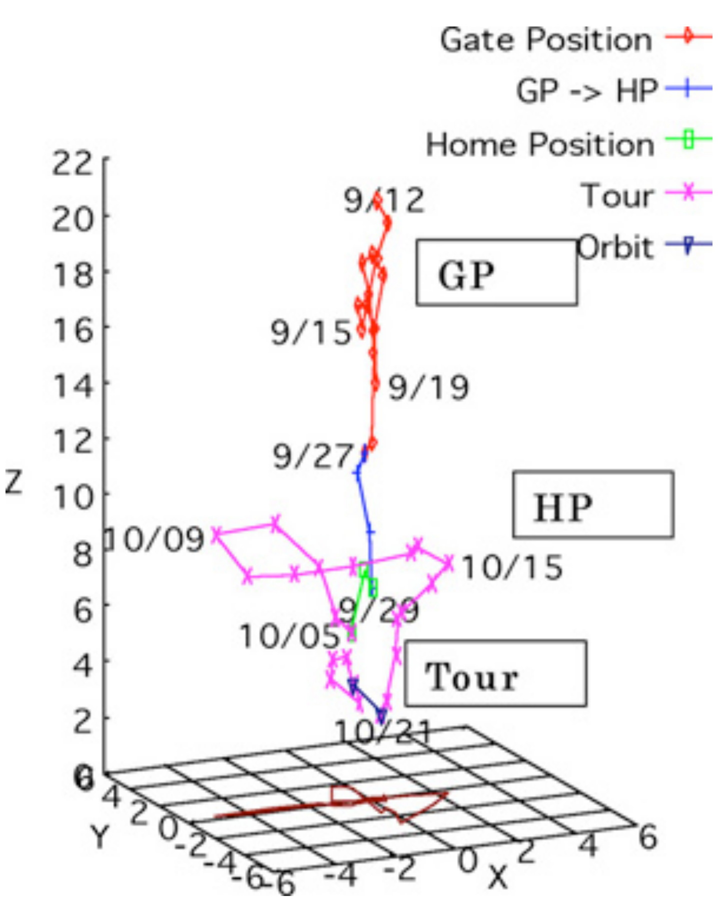
- ***NEAR at Eros***: Gravity dominated, controlled by avoiding resonances with the rotating mass distribution



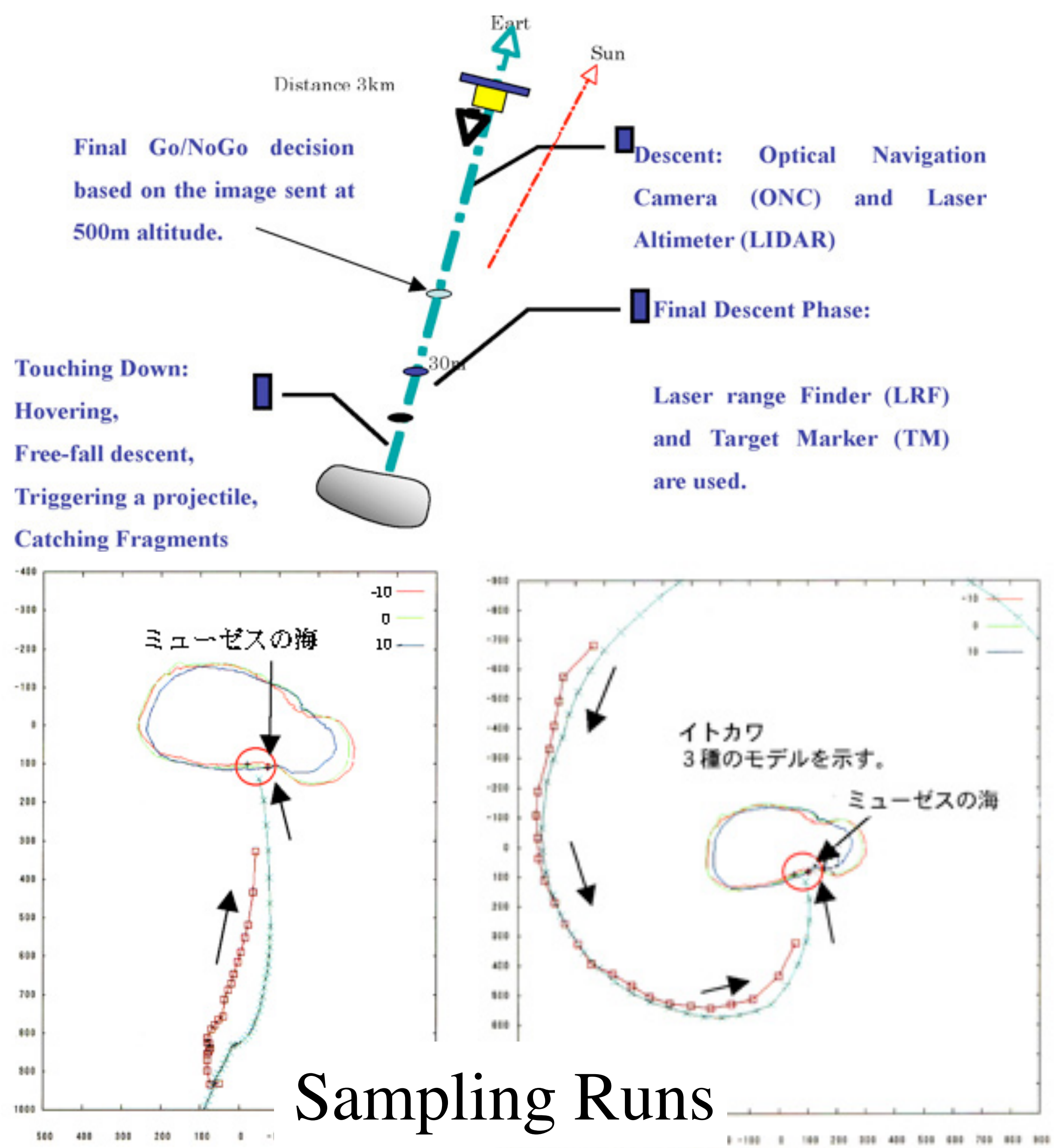
Retrog mal radii

What Operations are Feasible?

- ***Hayabusa at Itokawa***: Solar radiation pressure dominated, controlled through a hovering operations approach



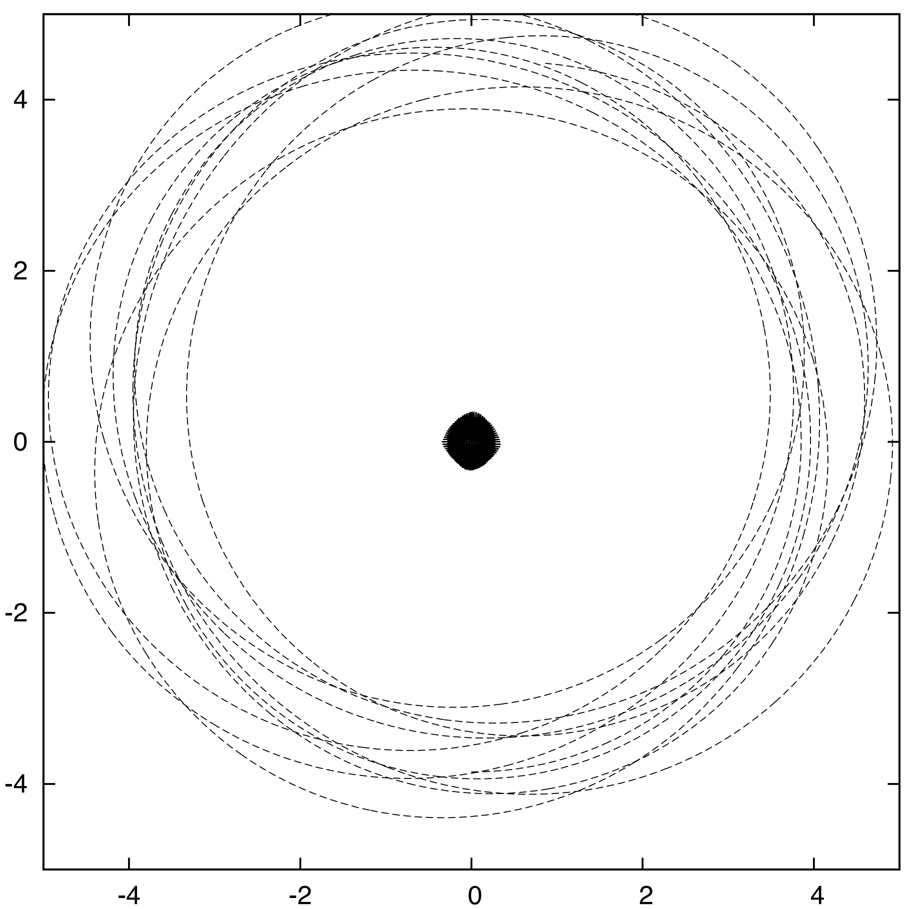
Mission Tour



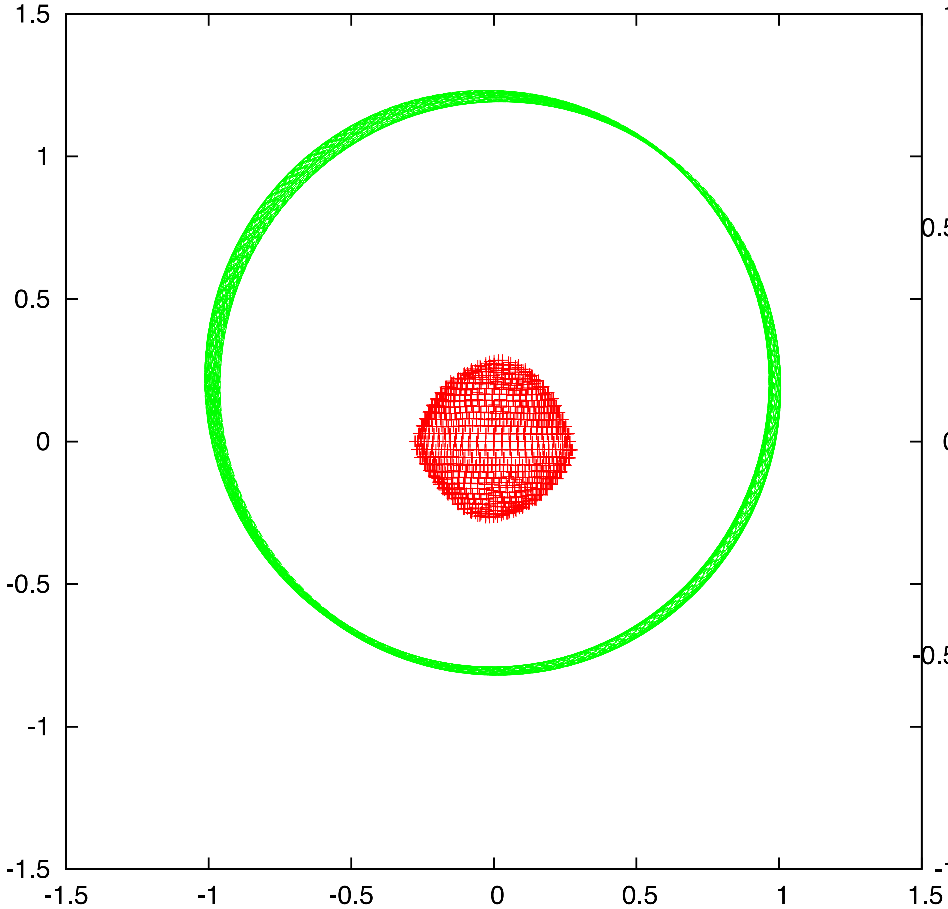
Sampling Runs

What Operations are Feasible?

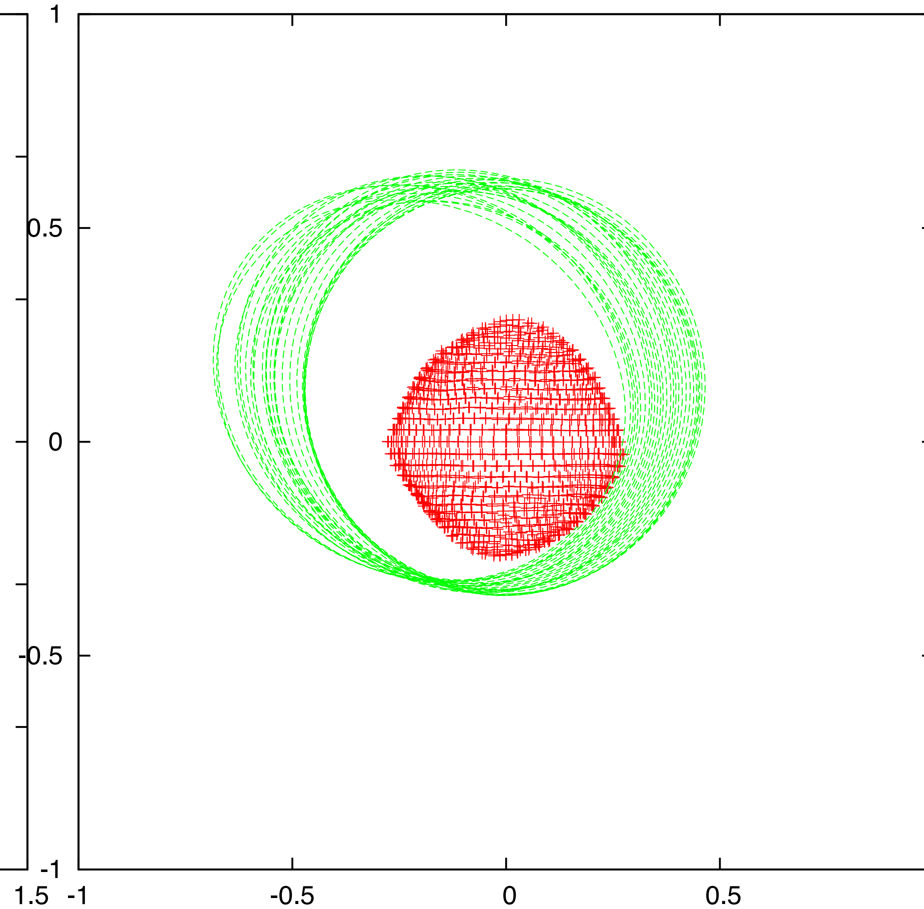
- ***OSIRIS-REx at 1998 RQ36***: Mixed perturbations dominated, will combine a mixture of slow flybys with terminator orbits



~ 4 km: Too High
Will Escape at Perihelion



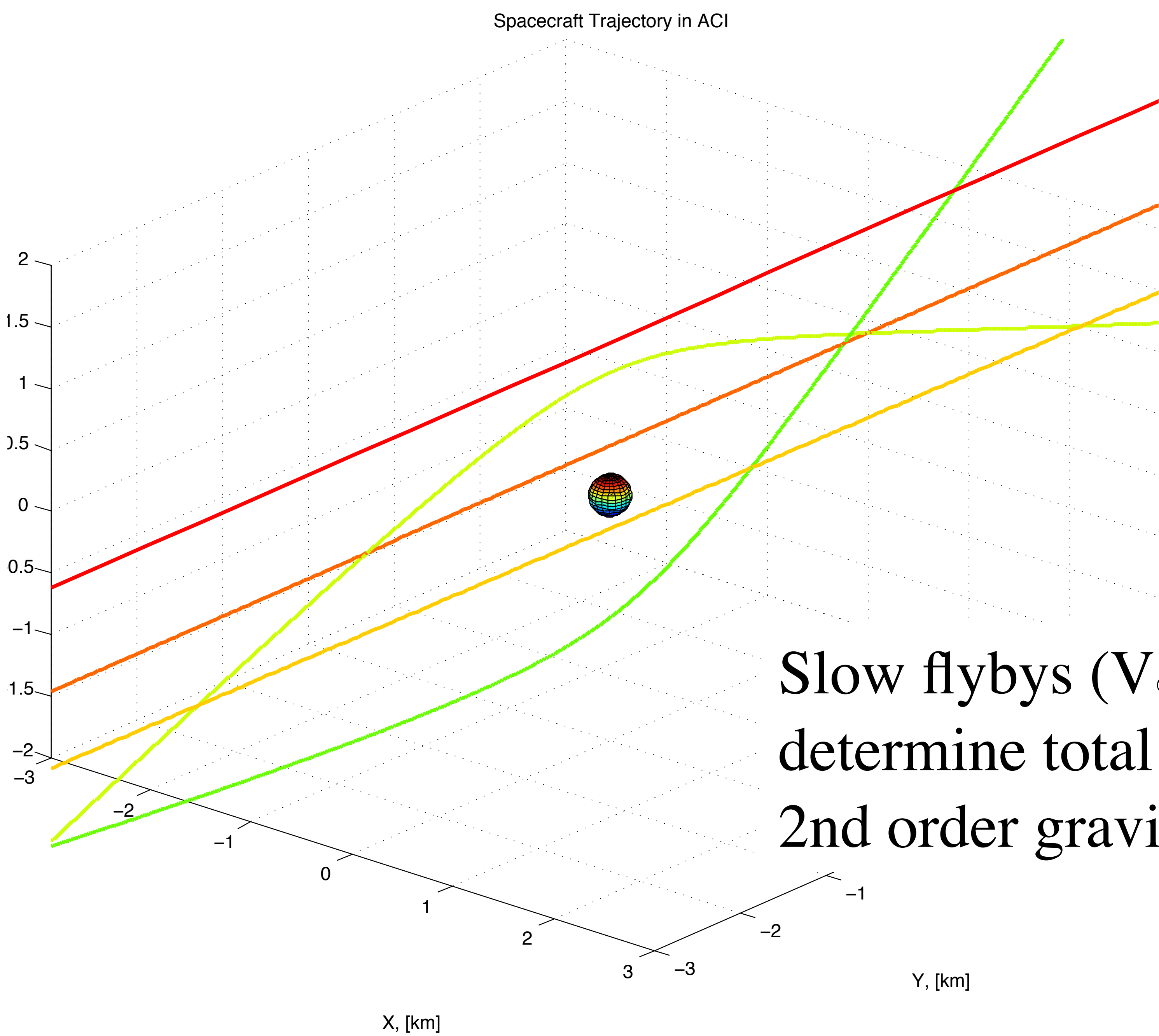
~ 1 km: Just Right
Stable over multiple years



~ 0.5 km: Too Low
Impacts after 50 days

What Operations are Feasible?

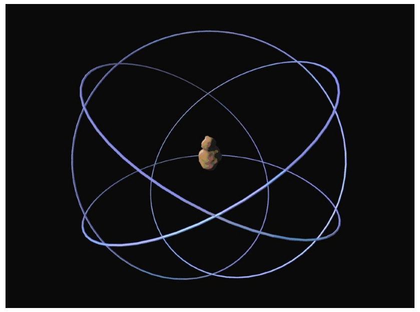
- ***OSIRIS-REx at 1998 RQ36***: Mixed perturbations dominated, will combine a mixture of slow flybys with terminator orbits



Slow flybys (V_∞ ~ 10's cm/s) can determine total mass and 1st and 2nd order gravity field terms.

D.J. Scheeres, A. Richard Seebass Chair, University of Colorado at Boulder

Conclusions



- The main issues involved with orbital mechanics of a particle in orbit about an asteroid can be identified:
  - Limits for bounded orbits
  - Criterion for destabilization of orbits
  - Characterization of coupled perturbations
- Additional study is still needed, as the dynamical systems involved are non-integrable
  - Improved theories for coupled perturbations
  - Orbit lifetimes for extended missions
  - Stripping criterion for dust on the surface
  - Landing trajectories
  - etc...

en a considerable increase in
robes to minor Solar System
d comets. This new field of
d on by the discovery of many
Kuiper Belt objects. However,
out such small Solar System
complex, but is a challenging
elds of celestial mechanics,
rodynamics.

urbed Environments

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engineering material;

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missions.

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Scheeres

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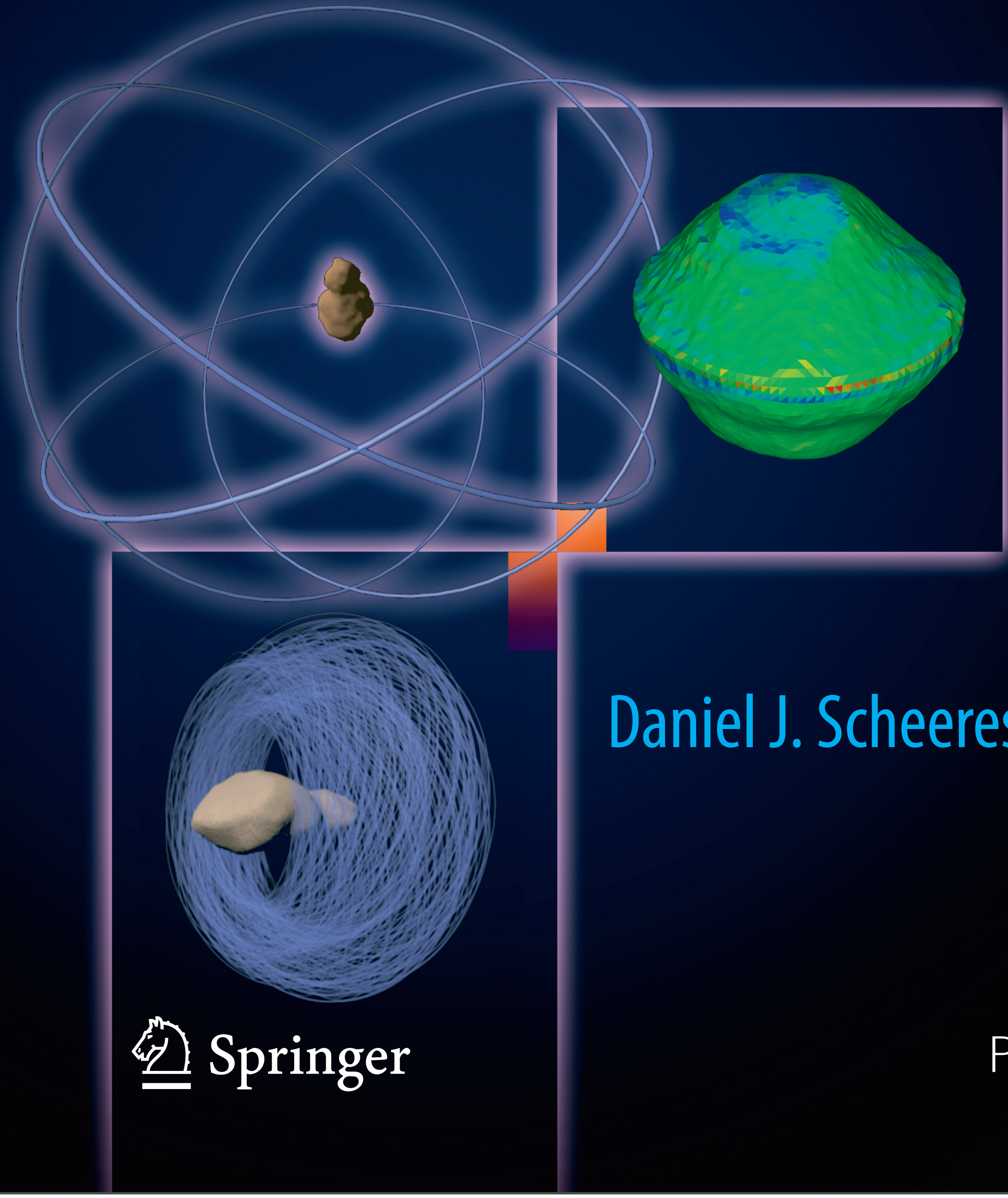
Applications to Asteroid, Comet and Planetary Satellite Orbiters



Daniel J. Scheeres

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