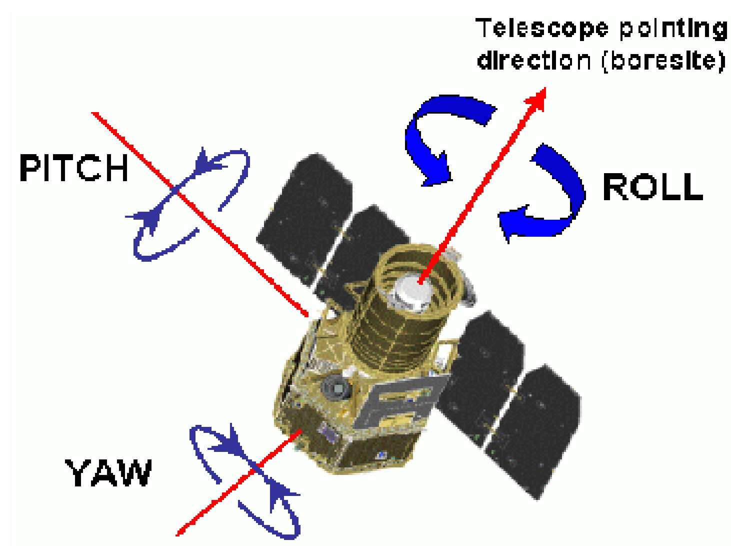
AstroNet-II

The Astrodynamics Network

Attitude dynamics and control

First AstroNet-II Training School
"Astrodynamics of natural and artificial satellites: from regular to chaotic motions"



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Outline

- Attitude determination and control overview:
  - Definitions
  - Hardware: Sensors and Actuators
  - Control overview
- Spacecraft dynamics:
  - Natural Dynamics
  - Spin stabilization
- Kinematic representations
- Perturbed dynamics and active Control
  - Perturbations
  - Detumbling
  - Re-pointing

- Attitude dynamics and control overview:
  - Definitions
  - Attitude modelling
  - Hardware: Sensors (determination) and Actuators (control)
  - Control methods

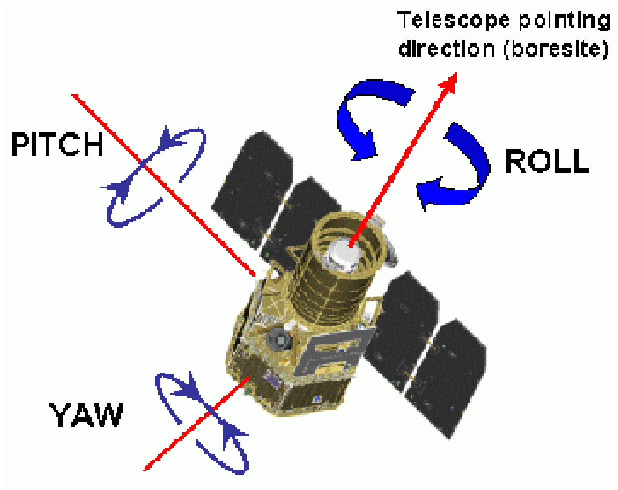
Attitude definitions

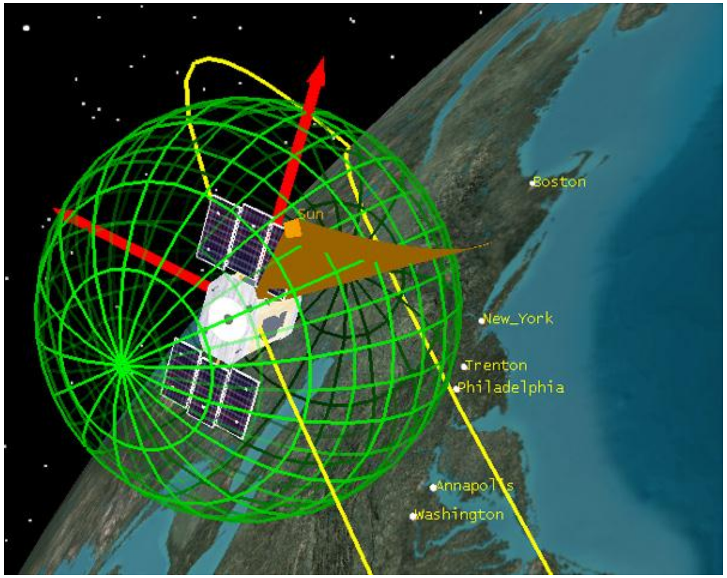
•Attitude: orientation of spacecraft body axes relative to fixed frame

• Attitude **determination**: use of **sensors** to estimate attitude (real-time)

• Attitude **control**: maintain specified attitude with given precision using **actuators**.

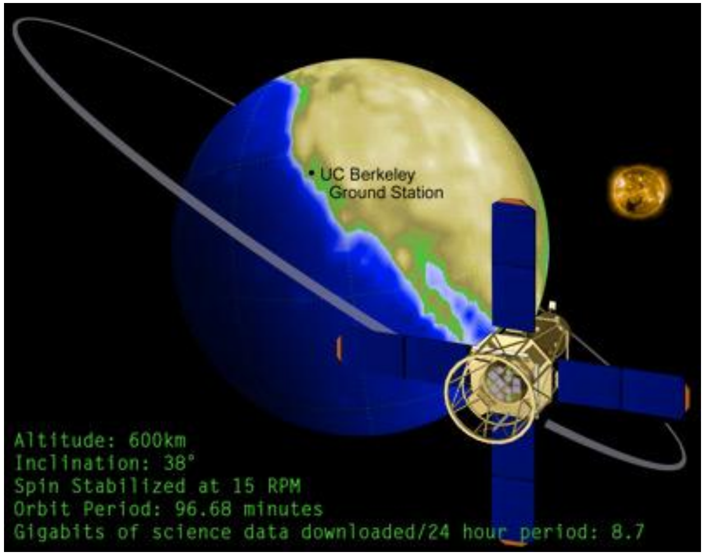
• Attitude error: difference between true and desired spacecraft attitude





Attitude control requirements

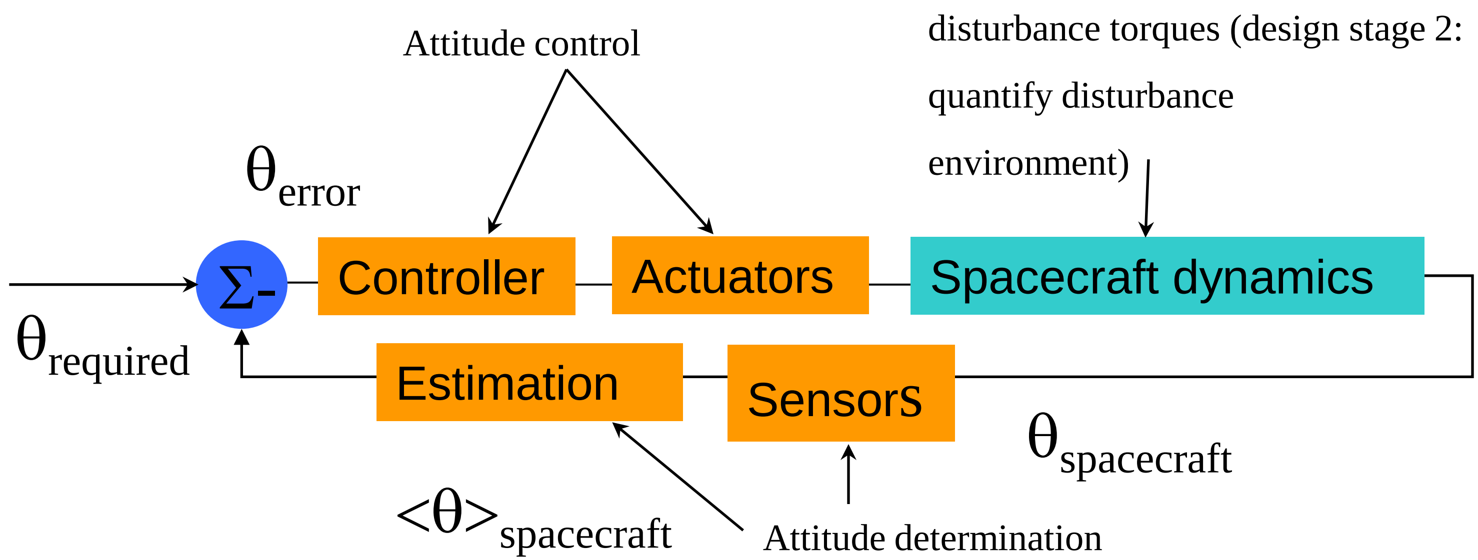
- Principal requirement is to point the spacecraft payload (instrument, antenna, solar array)
- Required accuracy depends on payload + long/short-term pointing accuracy
- Achievable accuracy depends on actuators and hardware.



Sun-pointing for RHESSI solar physics mission

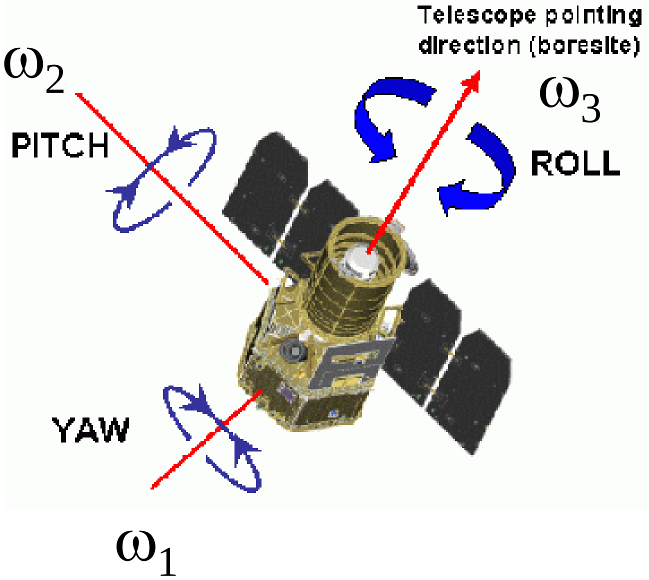
Attitude control loop (Active)

- Normally have full closed-loop for spacecraft attitude with on-board control
- Have some required attitude (point payload to target, antenna to Earth etc)
- Sensors estimate true attitude of spacecraft to generate attitude error signal
- Attitude error signal input to controller which drives the control actuators
- Control signal is then some function of the error



Attitude Dynamics of a spacecraft (Euler 1765):

$$I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 = T_1$$
$$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 = T_2$$
$$I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 = T_3$$



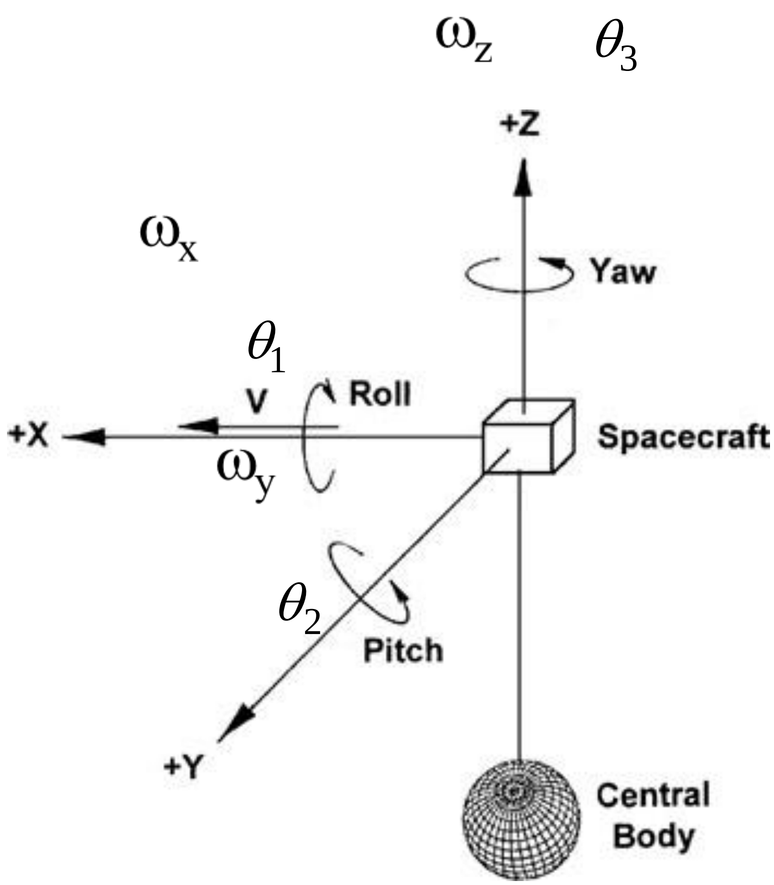
Attitude kinematics of a spacecraft:

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\\ \dot{q}_4\end{bmatrix} = \frac{1}{2}\begin{pmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{pmatrix}\begin{bmatrix} q_1\\ q_2\\ q_3\\ q_4\end{bmatrix} \quad \text{or} \quad \begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix} = \frac{1}{\cos\theta_2}\begin{pmatrix} \cos\theta_2 & \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2 \\ 0 & \cos\theta_1\cos\theta_2 & -\sin\theta_1\cos\theta_2 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{pmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix}$$

Attitude determination

•In order to determine the spacecraft's current attitude it requires the use of sensors.

•Various definitions of attitude angles (popular Earth-centred group below)



<u>Simulation Model</u>

Strong assumption in much theoretical work that state is perfectly known.

<u>Reality – require sensors to measure angles</u>

<u>Angular position</u>

$\theta_1 \; \theta_2 \; \theta_3$

<u>Angular velocities</u>

$\omega_x \; \omega_y \; \omega_z$

Attitude sensors

There are two types of measurement system:

•**Reference sensor** – measures discretely the attitude with respect to some reference frame defined by the position of objects in space e.g. the Sun, Earth or stars.

•**Inertial sensors** – measure continuously changes in attitude relative to a gyroscopic rotor.

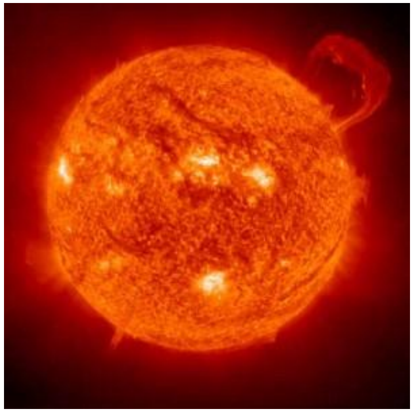
•Reference sensors fixes the reference but there can be problems during periods of eclipse for Sun sensors.

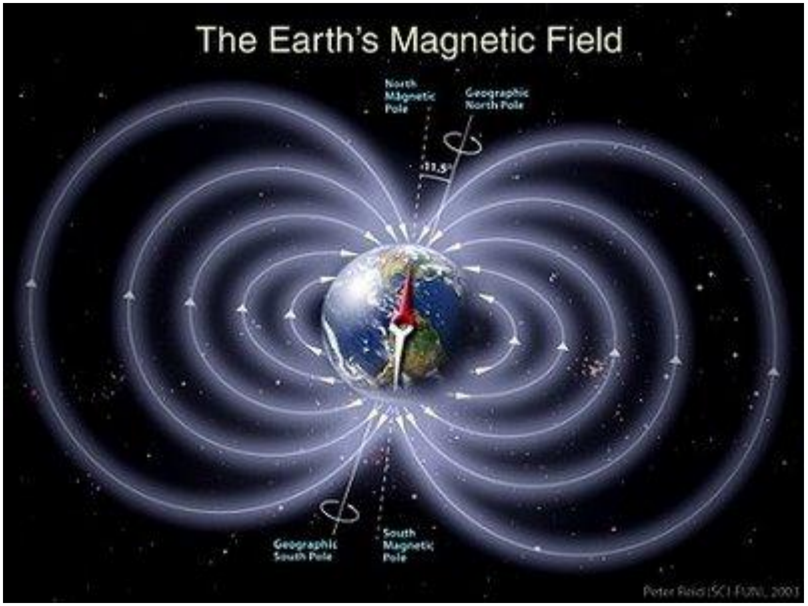
•Inertial sensors the errors progressively increase when making continuous measurements.

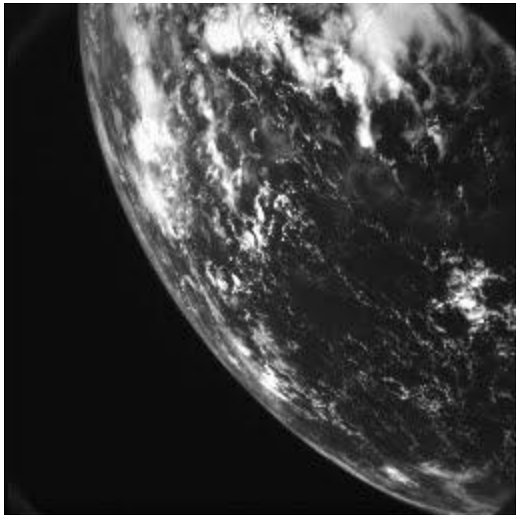
They are often used in combination where the reference sensors can be used to calibrate the inertial sensors at discrete times. The inertial sensors can then measure continuously between each calibration.

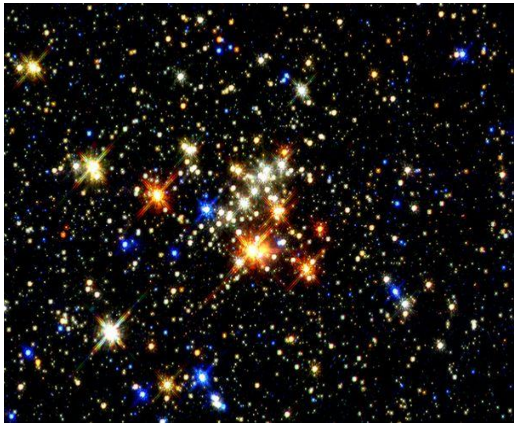
Sensor references

- External references must be used to determine the spacecrafts absolute attitude.
- the Sun
- the stars
- The Earth's IR horizon
- The Earth's magnetic field direction



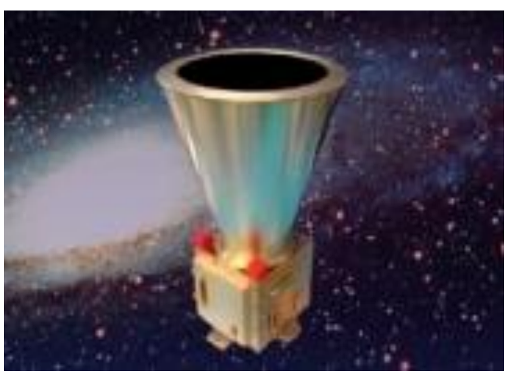




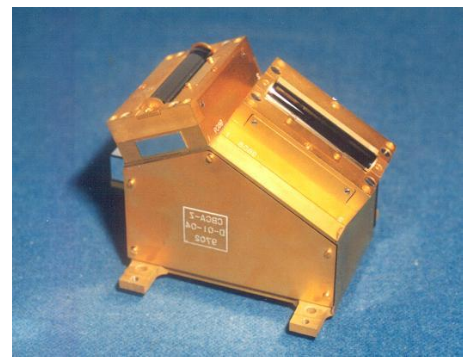


Attitude sensors

- Simple Sun sensors provides unit vector to Sun using masked photocells
- Star camera searches star catalogue database to determine view direction



star camera

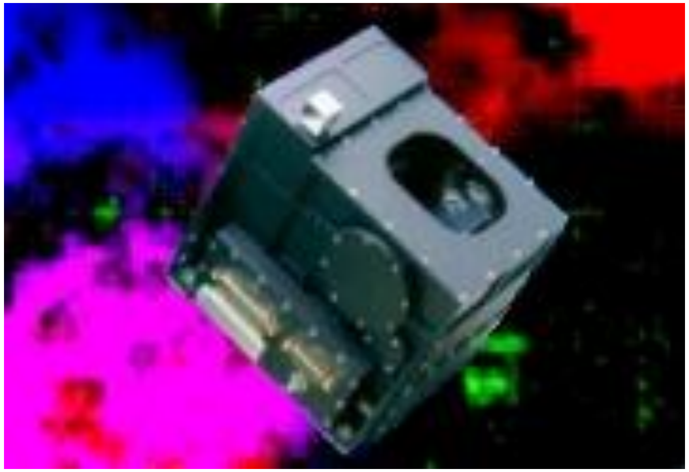


2-axis Sun sensor

Attitude sensors

•Earth sensors operating principle is based on electromechanical modulation of the radiation from the Earth's horizon.

• A magnetometer is a measuring instrument used to measure the strength or direction of the Earth's magnetic field.



IRES – Infrared earth sensor

Magnetometer used on the THEMIS

Attitude sensor data

Reference Object	Potential accuracy
Stars	1 arc second
Sun	1 arc minute
Earth (horizon)	6 arc minutes
Magnetometer	30 arc minutes
Narstar GPS	6 arc minutes

1 arcminute is 1/60th of a degree

1 arcsecond is 1/360th of a degree

Control algorithm is only as good as the hardware.

Attitude actuators

There are two classes of actuator system:

•Active controllers –to actively control the system means to use a propulsion system to re-point the spacecraft:

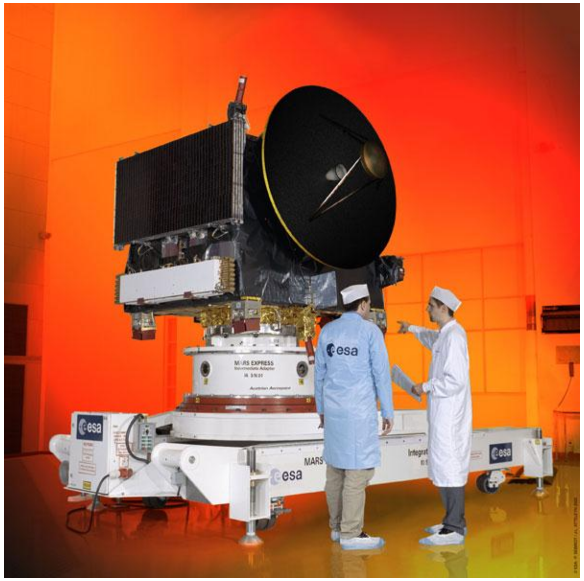
- •Thrusters
- •Magnetic Torque
- •Momentum Wheels
- •Control Moment Gyros

•Passive controllers– use mechanisms that exploit the orbit environment to naturally correct the pointing direction such as solar radiation pressure, Gravity Gradient, Earth's magnetic field, spacecraft design:

- •Permanent magnet
- •Gravity Gradient
- •Spin Stabilisation

Active Control - Three-axis stabilisation

- More complex and expensive method, but delivers high precision pointing
- Require actuators for each rotational axis of spacecraft + control laws
- Best method for missions with frequent payload slews (space telescopes)



ESA Mars Express

Hubble space telescope

Typical ADCS Actuators

Actuator	Typical Performance Range	Weight (kg)	Power (W)
Thrusters	1μN to 9N	1-10	1-100
Reaction & Momentum Wheels	max torque from to 4×10^{-6} Nm to 1Nm	0.2 to 20	0.6 to 110
CMG	25 to 500 Nm	> 40	90 to 150
Magnetic Torquers	At 800km around the Earth from 4e-7 to 0.18 Nm	0.05 to 50	0.6 to 16

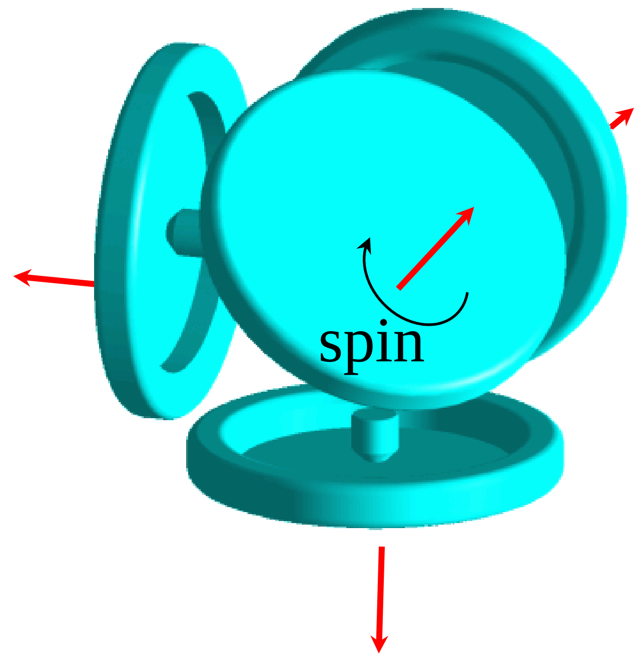
Thrusters

The conventional, low-risk solution is to use thrusters (usually monopropellant rockets), organized in a **Reaction control system**. However, they use fuel.

Momentum/reaction wheel

- Wheel with reversible DC motor to spin up/spin down metal disc assembly
- Transfer momentum to/from wheel and spacecraft axes (total conserved)
- Have 3 wheels (1 per axis) + spare canted relative to 3 main wheels

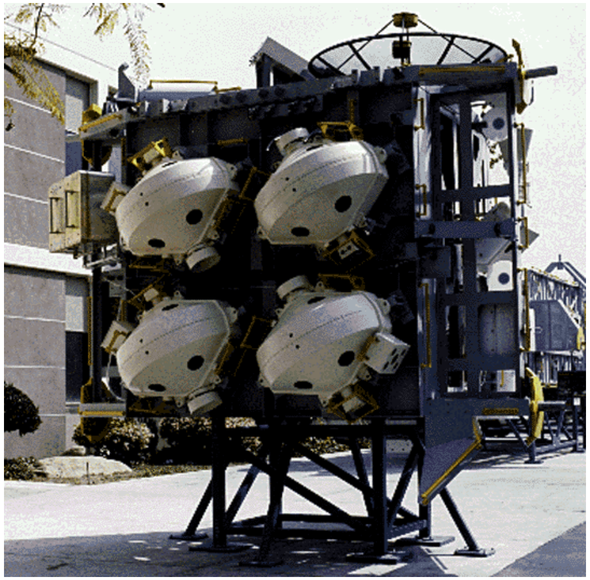
reaction wheel assembly



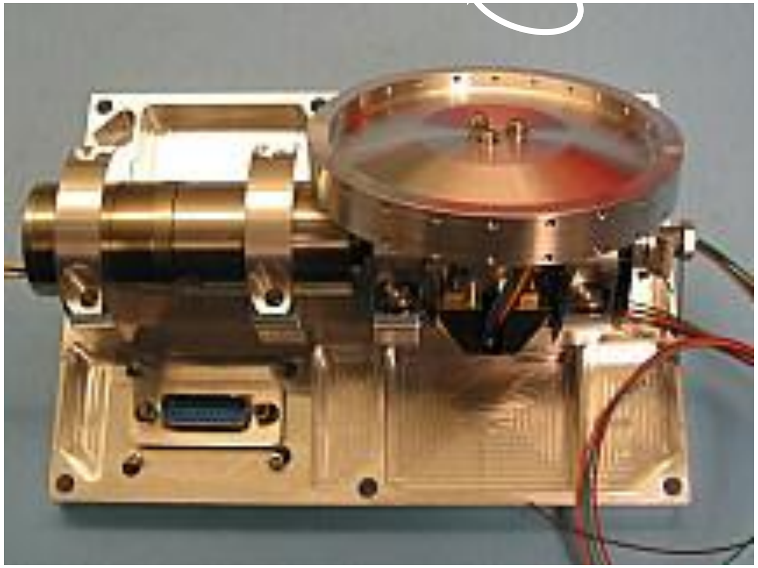
reaction wheel assembly

Control moment gyro

- Use spinning wheel which can be quickly rotated to change its spin axis
- Rapid change in direction of angular momentum vector results in large torque
- Used for fast attitude slews and for applications with high torque demands

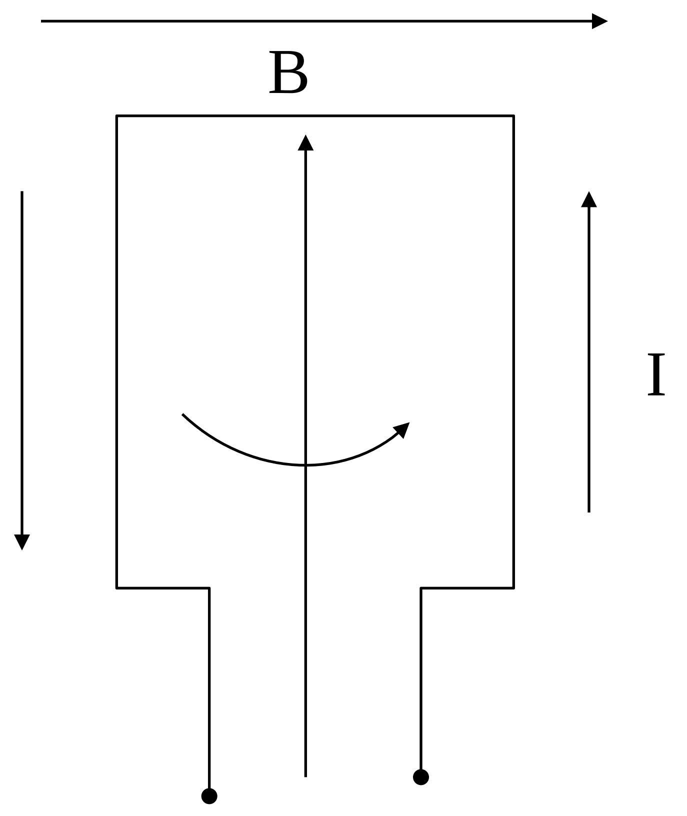


Space station CMGs



Mini-satellite CMG

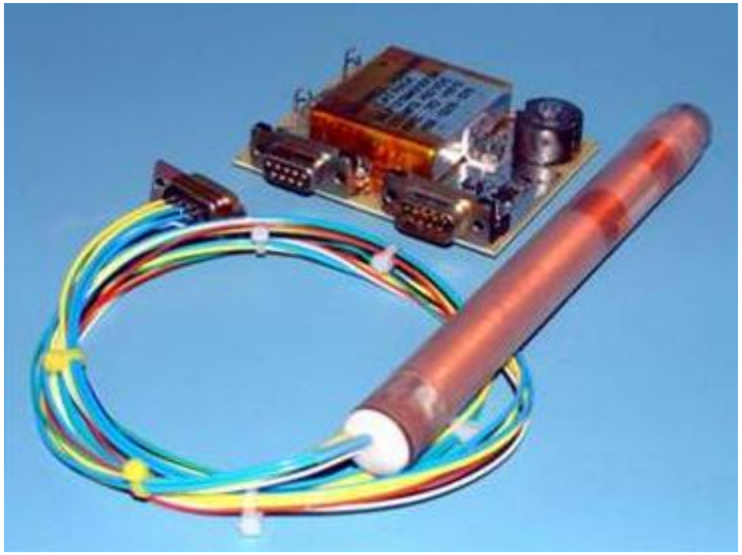
Magnetic Torques



Simple systems based on the interaction of an electric current with a magnetic field.

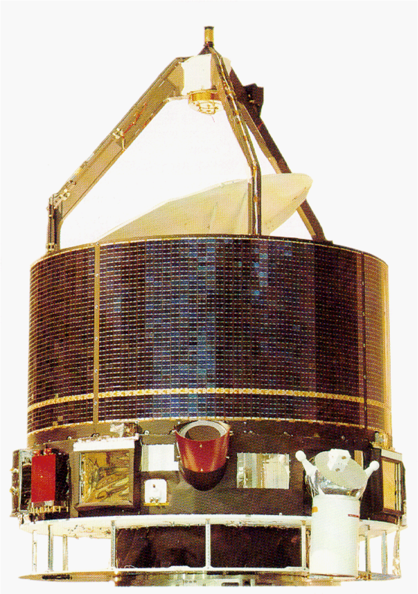
They are generally good for medium to small satellite flying at low altitude around the Earth.

They can be used as passive or active devices. The control algorithm needs a good knowledge of the magnetic field of the planet.

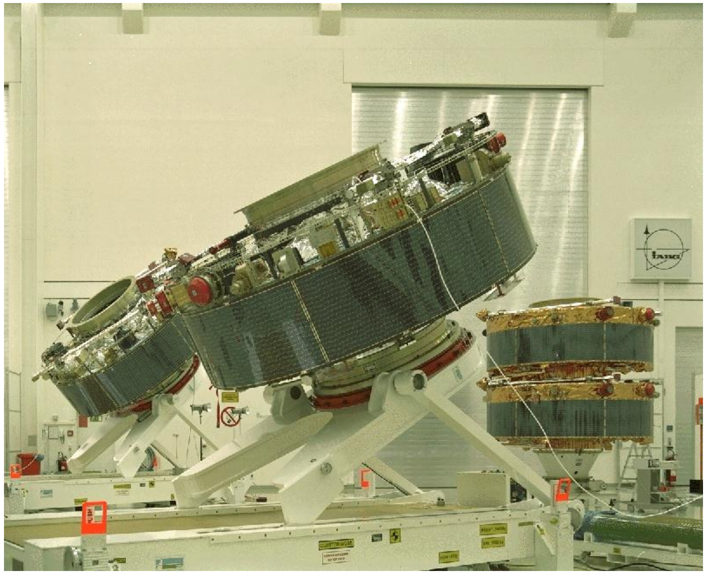


Spin stabilisation

- Simple and low cost method of attitude stabilisation (largely passive)
- Generally not suitable for imaging payloads (but can use a scan platform)
- Poor power efficiency since entire spacecraft body covered with solar cells



ESA Giotto spacecraft (de-spun antenna)



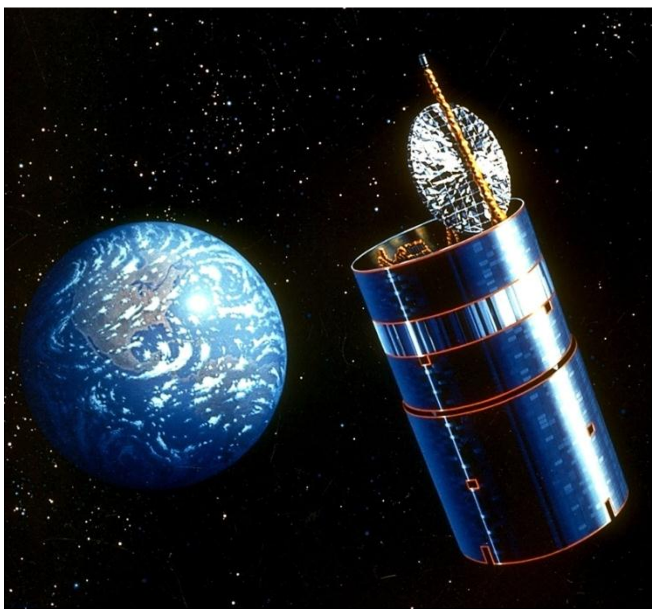
ESA Cluster spacecraft

Dual-spin stabilisation

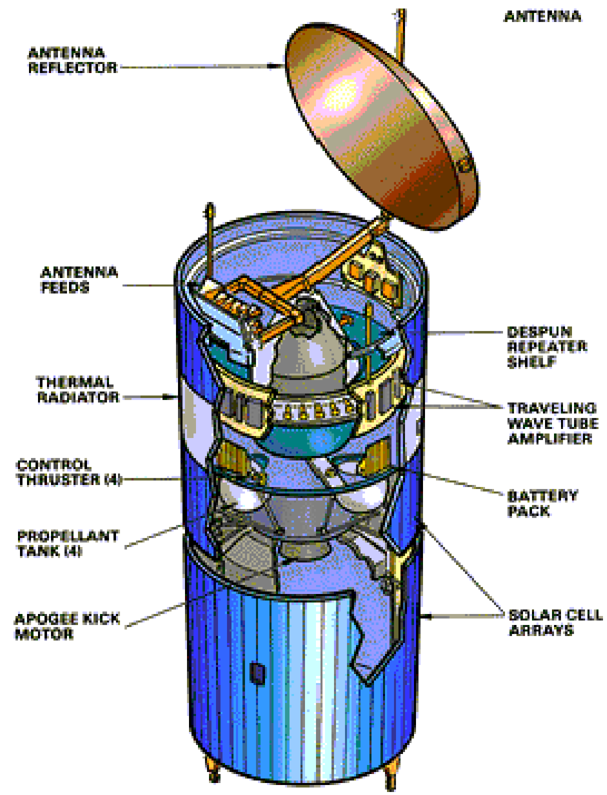
•Simplicity of spin-stabilised spacecraft, but de-spun platform at top

• Mount payload on de-spun platform for better pointing, but passive stability

• Popular for some GEO comsats



Boeing SBS 6

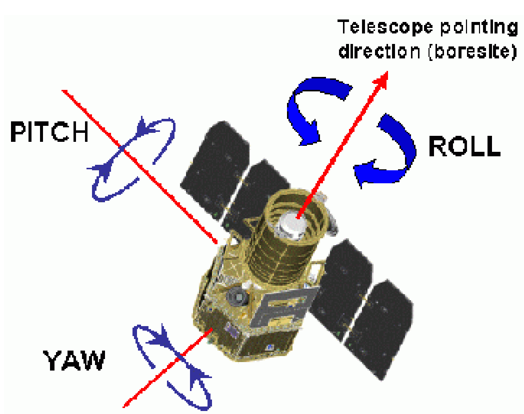


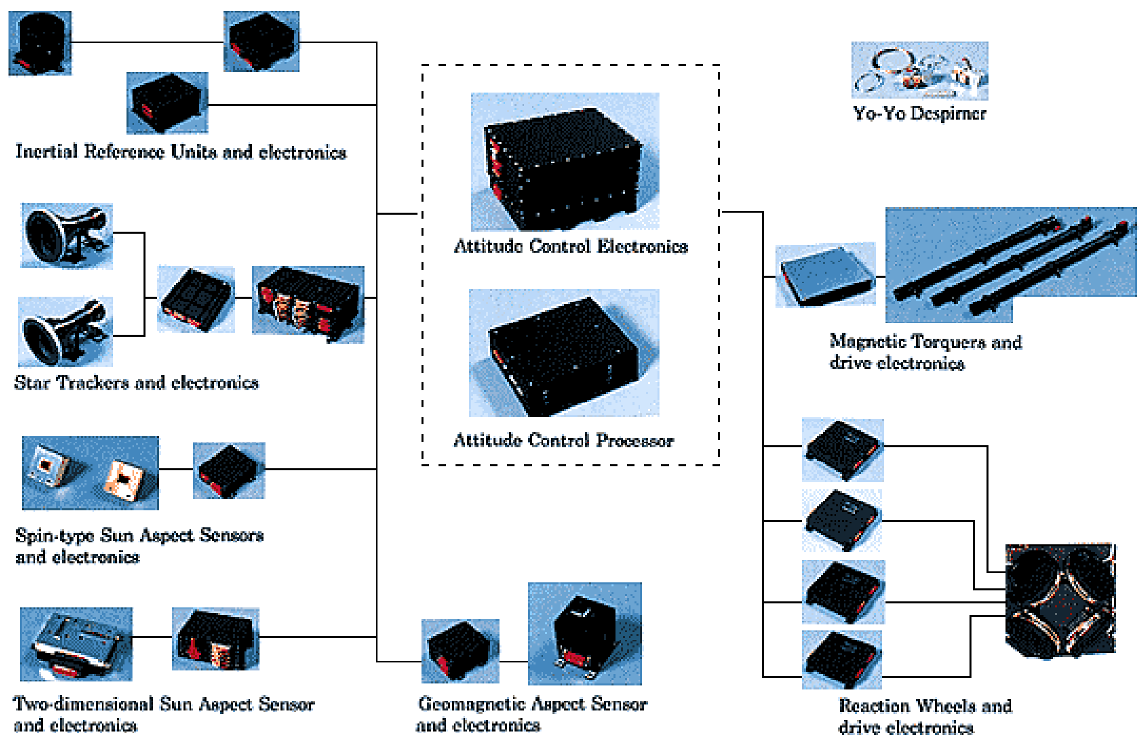
Attitude control trade-off

Type	Advantages	Disadvantages
Spin-stabilised **(~1° accuracy)**	Simple, passive, long-life, provides scan motion, gyroscopic stability for large burns	Poor manoeuvrability, low solar cell efficiency (cover entire drum), no fixed pointing
3-axis stabilised **(~0.001° accuracy)**	High pointing accuracy, rapid attitude slews possible, generate large power (Sun-facing flat solar arrays)	Expensive (~2 x spinner), complex, requires active closed-loop control, actuators for each body axis
Dual-spin stabilised **(~0.1° accuracy)**	Provides both fixed pointing (on de-spun platform) and scanning motion, gyroscopic stability for large burns	Require de-spin mechanism, low solar cell efficiency (cover entire drum), cost can approach 3-axis if high accuracy
Gravity-gradient **(~5° accuracy)**	Simple, low cost totally passive, long-life, provides simple passive Earth pointing mode	Low accuracy, almost no manoeuvrability, poor yaw stability, require deployment mechanism
Magnetic **(~1° accuracy)**	Simple, low cost, can be passive with use of permanent magnet or active with use of electromagnets	Poor accuracy (uncertainty in Earth's magnetic field), magnetic interference with science payload

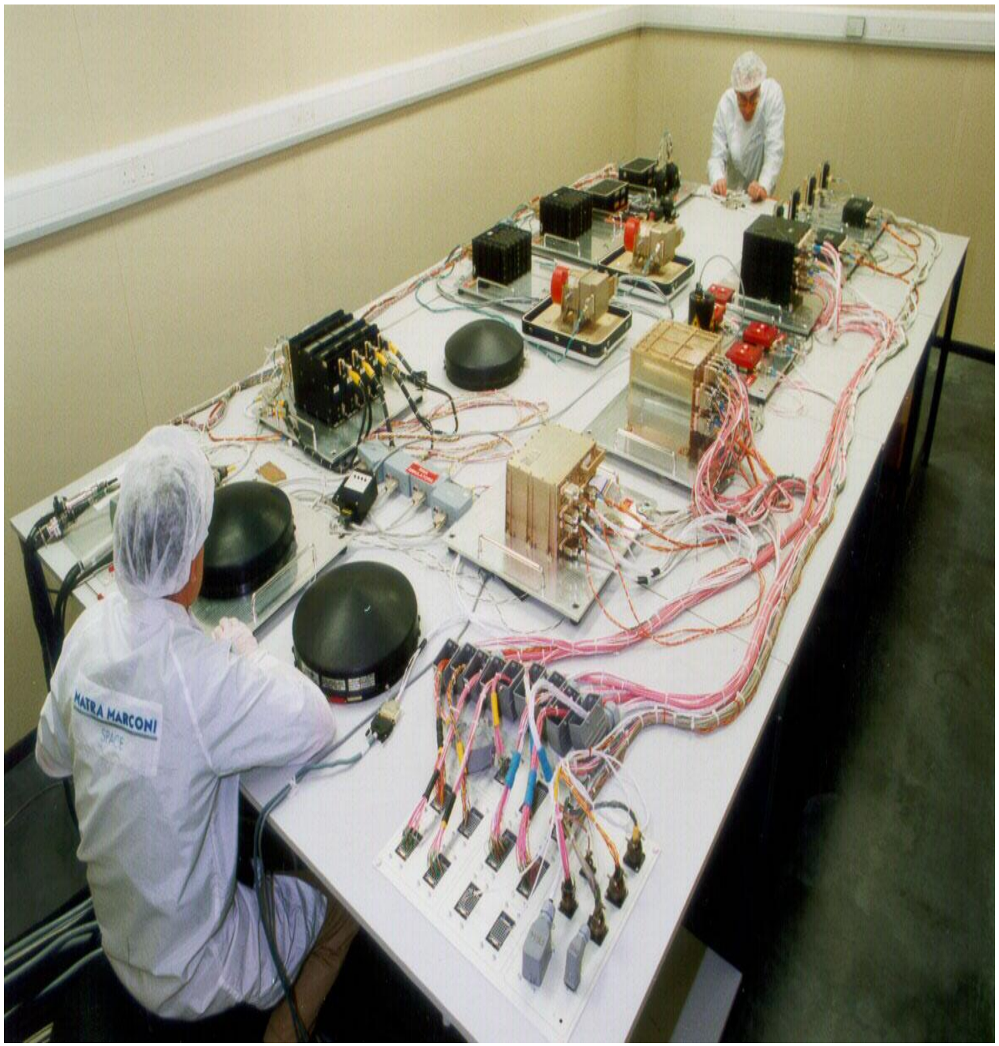
Complete attitude control system

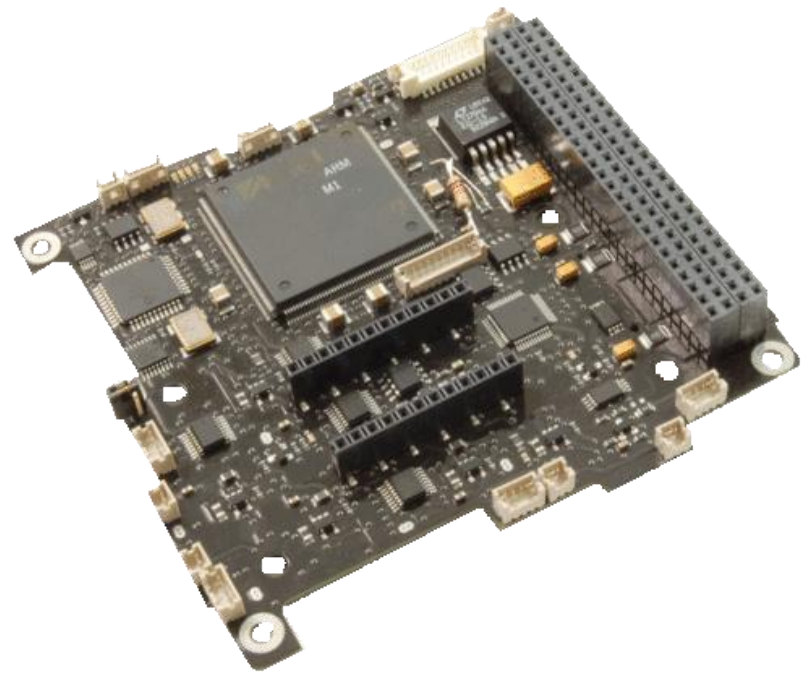
- Require complete, integrated attitude control system for spacecraft
- May have extensive software on-board for control law implementation
- Can test some hardware in the loop prior to launch (sensors/actuators)





Attitude control systems test





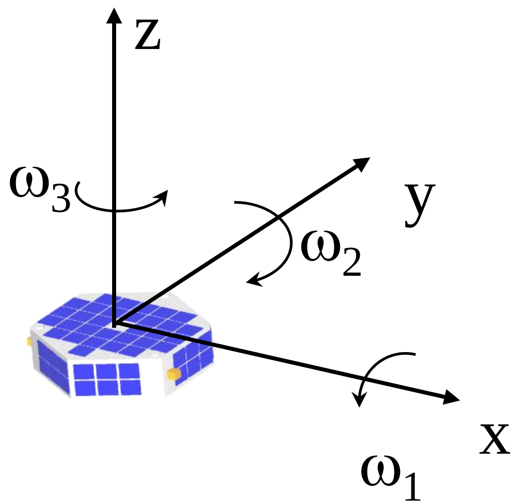
- Spacecraft natural dynamics:
  - Symmetric Spacecraft
  - Jacobi elliptic functions
  - Asymmetric spacecraft
  - Spin-stabilisation.

Euler's equations

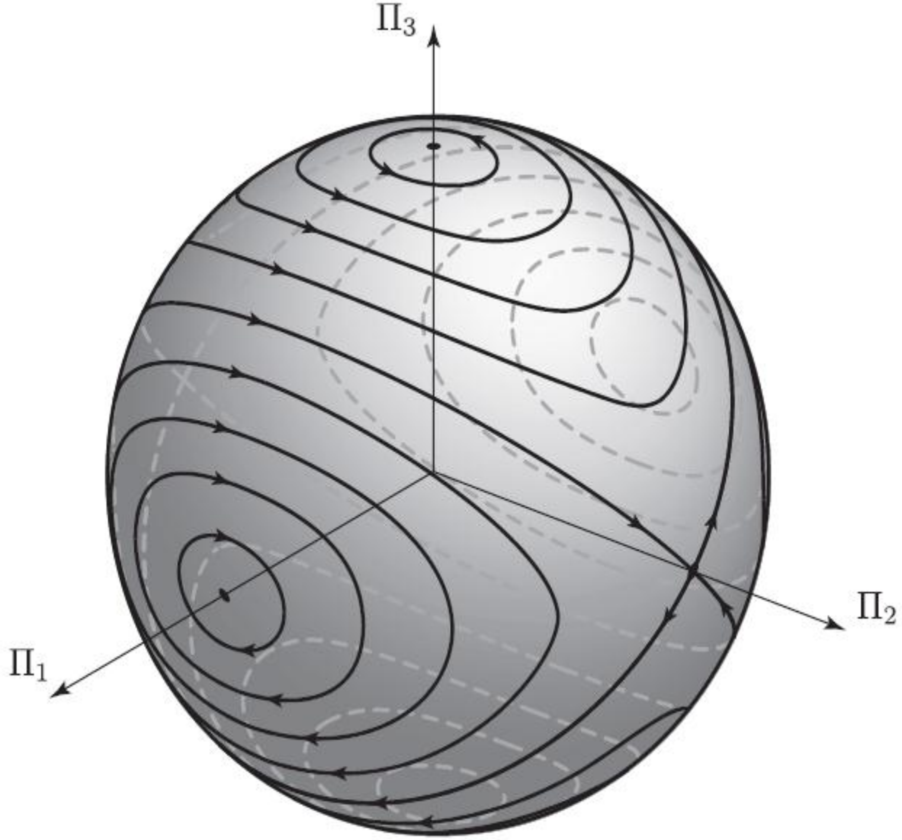
- Have Euler's equations to describe the attitude dynamics of a rigid body
- Body axes coincide with principal axes of inertia so products of inertia zero
- Real spacecraft have flexible modes (solar arrays, booms, fuel slosh . . .)

$$I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 = 0$$
$$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 = 0$$
$$I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 = 0$$

Euler's equations



spacecraft body axes



$$E = \frac{1}{2}\left(I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2\right)$$

$$M^2 = I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$$

$$E = \frac{1}{2}\left(\frac{\Pi_1^2}{I_1} + \frac{\Pi_2^2}{I_2} + \frac{\Pi_3^2}{I_3}\right)$$

$$M^2 = \Pi_1^2 + \Pi_2^2 + \Pi_3^2$$

Example **analytic functions:**

1. Any polynomial function
2. The exponential function
3. Trigonometric functions.
4. Hyperbolic functions.
5. Jacobi Elliptic functions (1829).

Exponential function

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

e.g. Exponential function

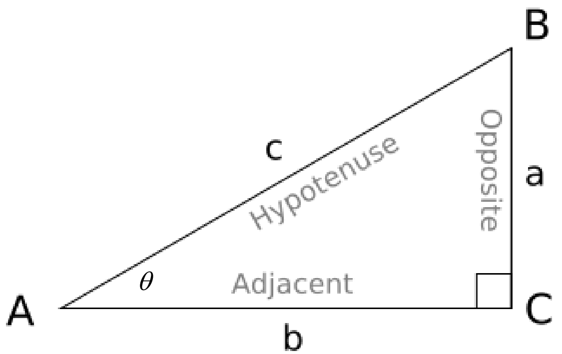
$$\frac{d}{dt} e^x = e^x \qquad y = e^x \qquad \frac{dy}{dx} = y \qquad y = e^{i\theta}$$

Geometric definition of sine and cosine:

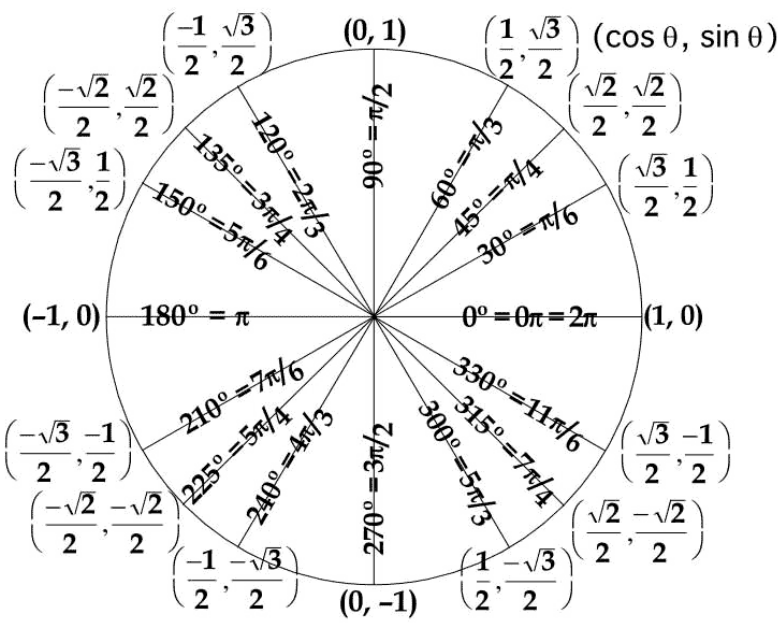
$$\frac{a}{c} = \sin\theta$$

$$\frac{b}{c} = \cos\theta$$

$$c^2 = a^2 + b^2$$



Parameterisation of the circle



Dynamical Systems definition of sine and cosine:

The trigonometric functions $x = \sin t$ and $y = \cos t$ are defined as the solution of the first order equations:

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -x$$

$$x(0) = 0, y(0) = 1$$

$$\frac{d}{dt}\sin t = \cos t$$

$$\frac{d}{dt}\cos t = -\sin t$$

Conserved quantity the **dynamical systems** definition with the **geometric** one

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -x$$

The function $1 = x^2 + y^2$ is conserved

An integral of motion.

$$\frac{d}{dt}(x^2 + y^2) = 2x\dot{x} + 2y\dot{y} = 2xy - 2yx = 0$$

Free dynamics defined by Euler equations:

$$I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 = 0$$
$$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 = 0$$
$$I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 = 0$$

Conserved quantities:

Kinetic Energy:

$$E = \frac{1}{2}\left(I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2\right)$$

Magnitude of angular momentum:

$$M^2 = I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$$

Exercise: Prove that these quantities are conserved for a free rigid body.

Natural Spin Motions

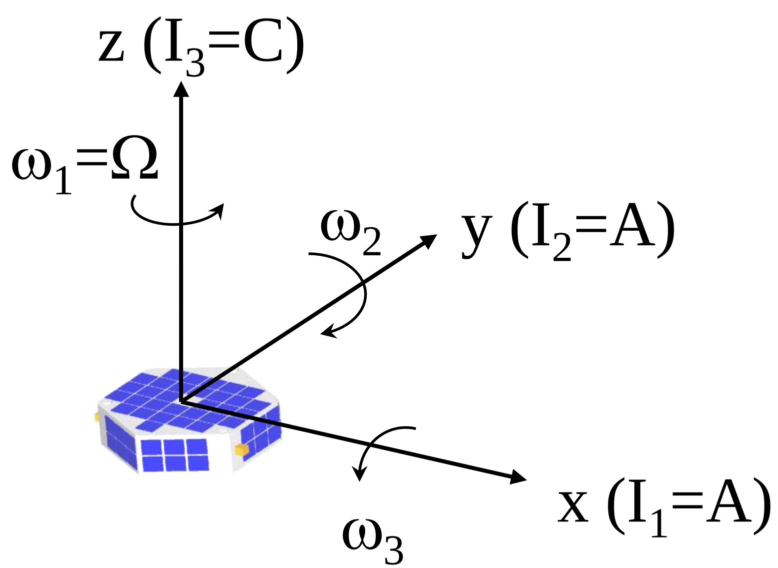
- Assume have perfectly symmetric spacecraft with I_1=I_2=A and I_3=C
- Also assume no external torques acting of spacecraft so that M=0
- See that z-axis spin rate ω_3=constant, what about ω_2 and ω_3 ?

symmetric spacecraft

$$A\dot{\omega}_1 - (A - C)\omega_2\omega_3 = 0$$

$$A\dot{\omega}_2 - (C - A)\omega_3\omega_1 = 0$$

$$C\dot{\omega}_3 = 0$$



Spacecraft nutation

- Combine Euler equations for x- and y- body axes to form a single equation
- Have oscillatory solution for x-axis angular velocity (and also y-axis)
- Solution describes nutation (precession) of the spacecraft body axes

$$\dot{\omega}_1 = \frac{(A-C)}{A}\Omega\omega_2$$

$$\dot{\omega}_2 = \frac{(C-A)}{A}\Omega\omega_1$$

$$\omega_3 = \Omega$$

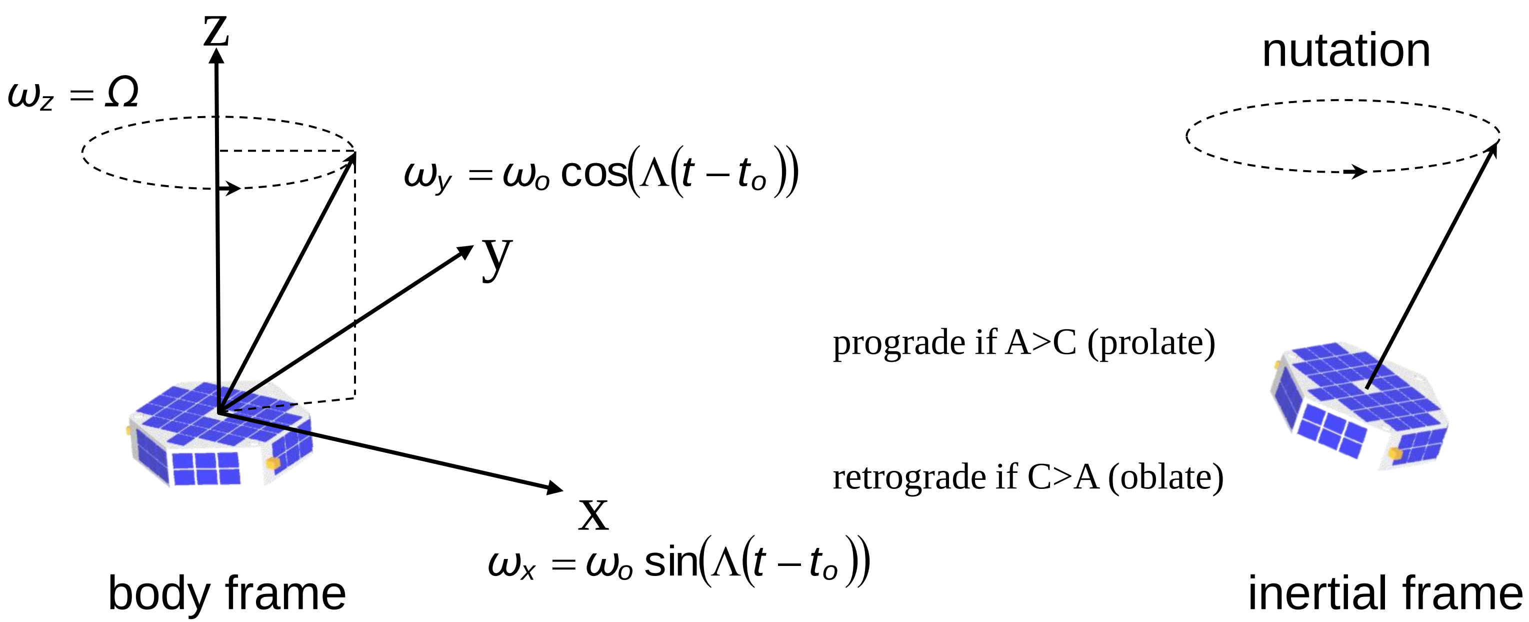
$$E = \frac{1}{2}\left(I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2\right)$$

$$\frac{2E - C\Omega^2}{A} = \omega_1^2 + \omega_2^2$$

Exercise: Try $\begin{aligned}\omega_1 &= r\sin\theta \\ \omega_2 &= r\cos\theta\end{aligned}$ to find solution in terms of initial conditions and constants of motion.

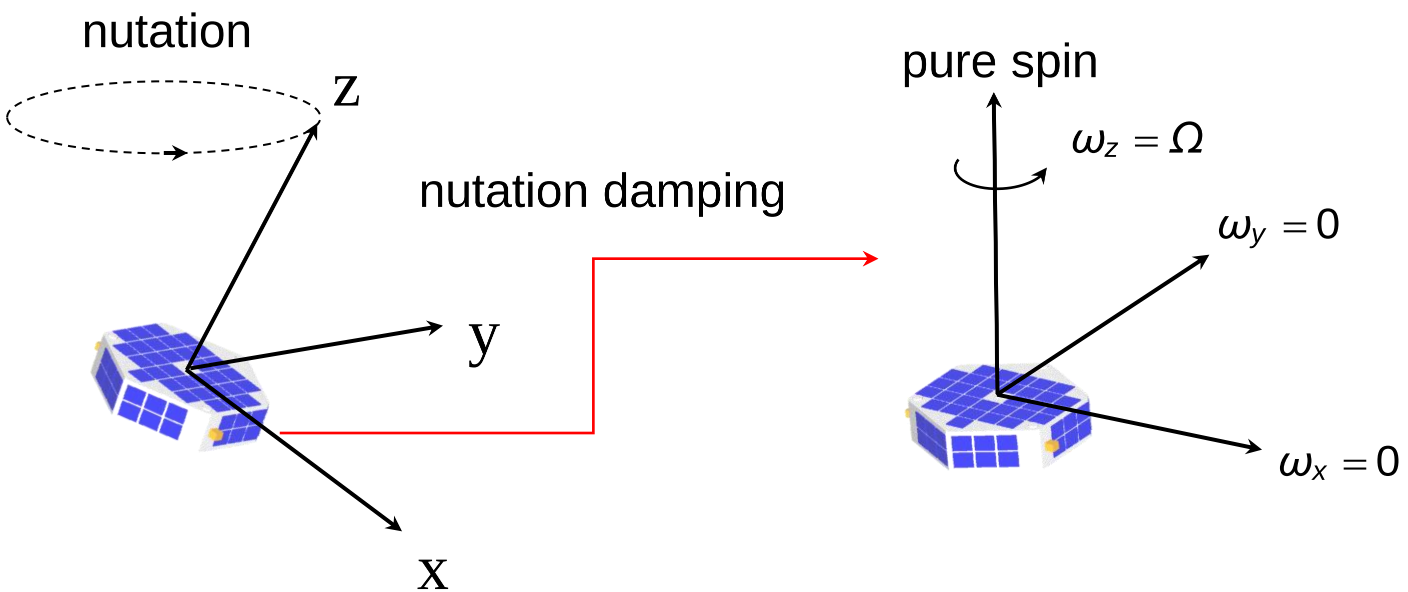
Nutation geometry

- See from solution to Euler's equations that $\underline{w}$ executes a coning motion
- The angular momentum vector $\underline{H}$ is fixed since have no external torque
- Have nutation of spacecraft due to angular momentum about x- and y-axes



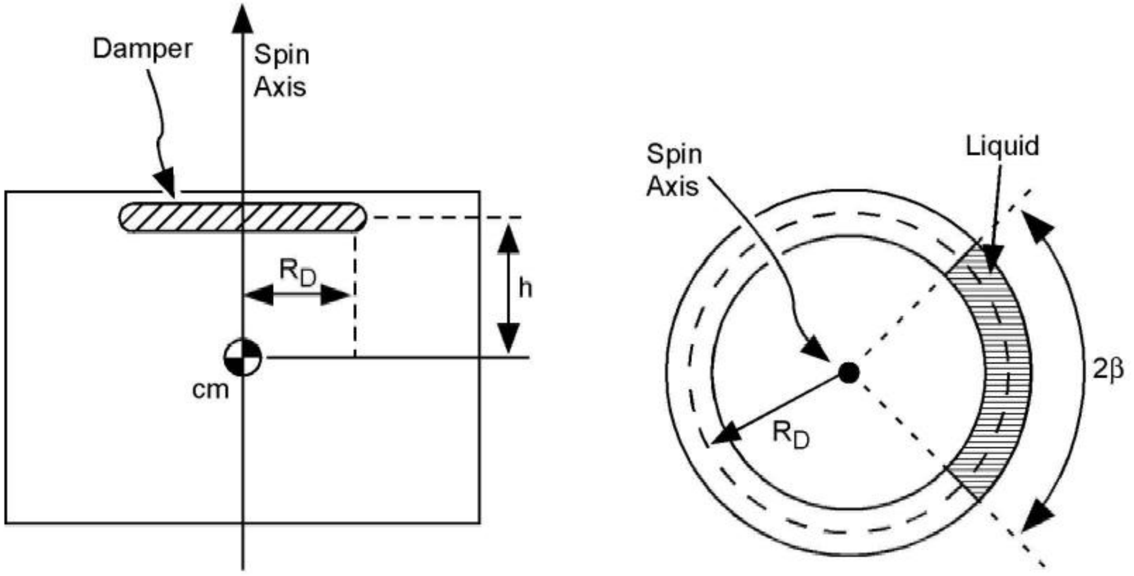
Nutation damping

- Nutation undesirable for a spin-stabilised satellite (require pure spin state)
- Can damp angular momentum about x- and y-axes using nutation dampers
- Leads to pure spin about z-axis (+ small spin-up to conserve momentum)



Liquid ring nutation damper

- Can use viscous fluid in partially filled ring to damp nutation of spacecraft
- Nutation causes fluid motion along tube - viscous effects lead to dissipation
- Simple, passive means of damping nutation and achieving pure spin state



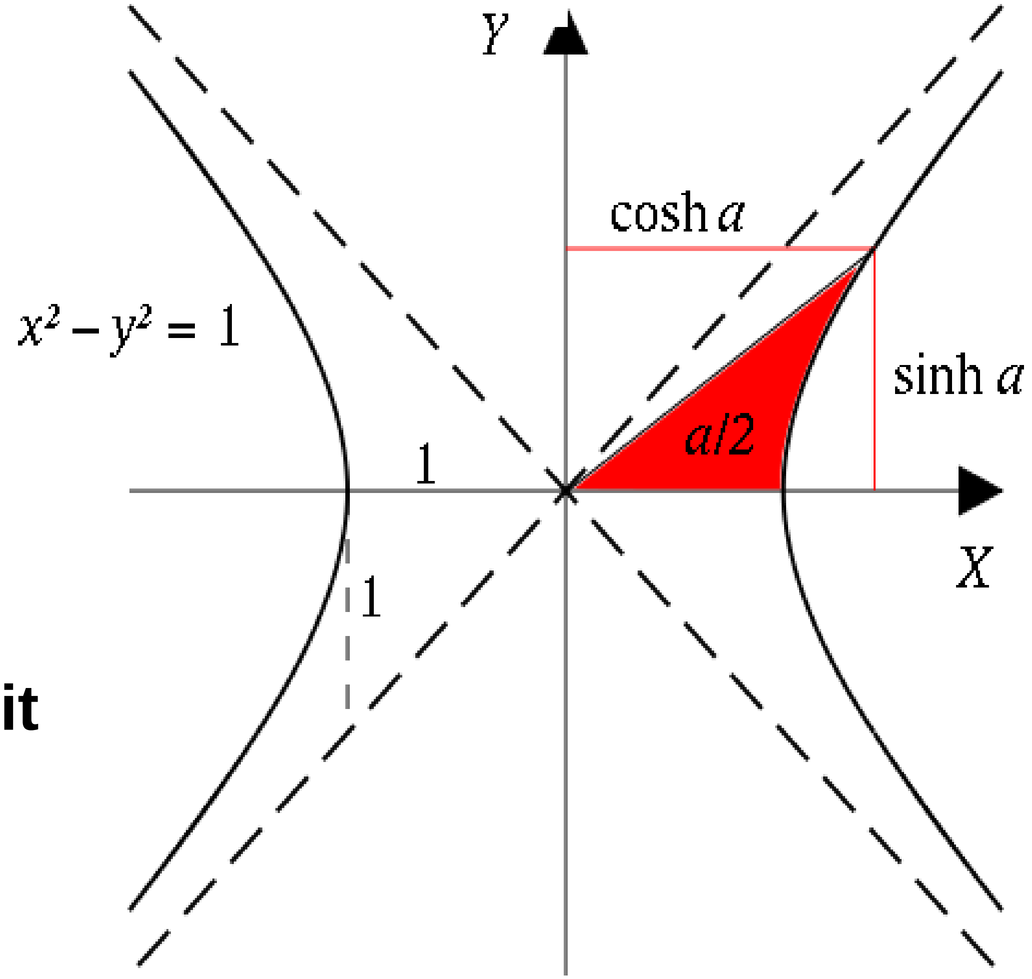
Side view of the vehicle

Top view of the damper tube

Geometric definition of sinh and cosh:

$$x = \sinh a$$

$$y = \cosh a$$



Parameterise the unit hyperbola

Dynamical Systems definition of sinh and cosh:

The hyperbolic functions $x = \sinh t$, $y = \cosh t$ are defined as the solution of the first order equations:

$$\frac{dx}{dt} = y,$$
$$\frac{dy}{dt} = x$$

$$x(0) = 0, y(0) = 1$$

$$\frac{d}{dt}\sinh t = \cosh t$$
$$\frac{d}{dt}\cosh t = \sinh t$$

Conserved quantity connects the **dynamical systems** definition with the **geometric** one

$$\frac{dx}{dt} = y, \qquad x(0) = 0, y(0) = 1$$
$$\frac{dy}{dt} = x$$

The function $1 = x^2 - y^2$ is conserved

$$\frac{d}{dt}(x^2 - y^2) = 2x\dot{x} - 2y\dot{y} = 2xy - 2yx = 0$$

Parameterise the hyperbola.

Define a Jacobi Elliptic function

Let m be a number (0,1) and let t denote a real variable that we interpret as time. The Jacobi Elliptic Functions:

$$x = sn(t,m), y = cn(t,m), z = dn(t,m)$$

Are defined as solutions of the differential equations:

$$\frac{dx}{dt} = yz$$

$$\frac{dy}{dt} = -zx$$

$$\frac{dz}{dt} = -mxy$$

Where $sn(0,m) = x(0) = 0, cn(0,m) = y(0) = 1, dn(0,m) = z(0) = 1$

$$\frac{dx}{dt} = yz$$

$$\frac{dy}{dt} = -zx$$

$$\frac{dz}{dt} = -mxy$$

It is easily shown that $1 = x^2 + y^2$ and $1 = mx^2 + z^2$

are both conserved quantities for these differential equations.

Therefore the Jacobi elliptic functions can be interpreted geometrically as parameterising the circle in the x-y plane and an ellipse in the x-z plane.

$$\frac{dx}{dt} = yz$$
$$\frac{dy}{dt} = -zx$$
$$\frac{dz}{dt} = -mxy$$
$$x(0) = 0, y(0) = 1, z(0) = 1$$

Jacobi Elliptic function

$$x = sn(t, m),$$
$$y = cn(t, m),$$
$$z = dn(t, m)$$

$$\dot{\omega}_1 = \frac{(I_2 - I_3)}{I_1}\omega_2\omega_3$$
$$\dot{\omega}_2 = \frac{(I_3 - I_1)}{I_3}\omega_1\omega_3$$
$$\dot{\omega}_3 = \frac{(I_1 - I_2)}{I_3}\omega_1\omega_2$$

Euler equations

$$\omega_1 = \lambda_1 sn(\alpha t + \beta, m),$$
$$\omega_2 = \lambda_2 cn(\alpha t + \beta, m),$$
$$\omega_3 = \lambda_3 dn(\alpha t + \beta, m)$$

$$\dot{\omega}_1 = \frac{(I_2 - I_3)}{I_1}\omega_2\omega_3$$

$$\dot{\omega}_2 = \frac{(I_3 - I_1)}{I_2}\omega_1\omega_3$$

$$\dot{\omega}_3 = \frac{(I_1 - I_2)}{I_3}\omega_1\omega_2$$

$$E = \frac{1}{2}\left(I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2\right)$$

$$M^2 = I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$$

Exercise: Solve Euler equations in terms of initial conditions and constants of motion (conserved quantities)

$$\omega_1 = \lambda_1 sn(\alpha t + \beta, m),$$

$$\omega_2 = \lambda_2 cn(\alpha t + \beta, m),$$

$$\omega_3 = \lambda_3 dn(\alpha t + \beta, m)$$

Derivatives and identities:

$$\frac{dsn(t,m)}{dt} = cn(t,m)dn(t,m)$$

$$\frac{dcn(t,m)}{dt} = -sn(t,m)dn(t,m)$$

$$\frac{ddn(t,m)}{dt} = -msn(t,m)cn(t,m)$$

$$1 = sn^2(t,m) + cn^2(t,m)$$

$$1 = msn^2(t,m) + dn^2(t,m)$$

Some interesting special solutions:

$$m \to 0$$

$$sn(t,m) \to \sin t$$

$$cn(t,m) \to \cos t$$

$$dn(t,m) \to 1$$

$$m \to 1$$

$$sn(t,m) \to \tanh t$$

$$cn(t,m) \to \mathrm{sech} t$$

$$dn(t,m) \to \mathrm{sech} t$$

Other representations

Co-ordinate form

$$\dot{\Pi}_1 = \frac{(I_2 - I_3)}{I_2 I_3}\Pi_2\Pi_3$$

$$\dot{\Pi}_2 = \frac{(I_3 - I_1)}{I_1 I_3}\Pi_2\Pi_3$$

$$\dot{\Pi}_3 = \frac{(I_1 - I_2)}{I_1 I_2}\Pi_1\Pi_2$$

$$H = \frac{1}{2}\left(\frac{\Pi_1^2}{I_1} + \frac{\Pi_2^2}{I_2} + \frac{\Pi_3^2}{I_3}\right)$$

$$M^2 = \Pi_1^2 + \Pi_2^2 + \Pi_3^2$$

Vector form

$$L = \Delta M^2$$

$$\dot{L} = L \times \Delta H$$

Matrix form

$$\dot{L} = [L, \Delta H]$$

$$[X, Y] = XY - YX$$

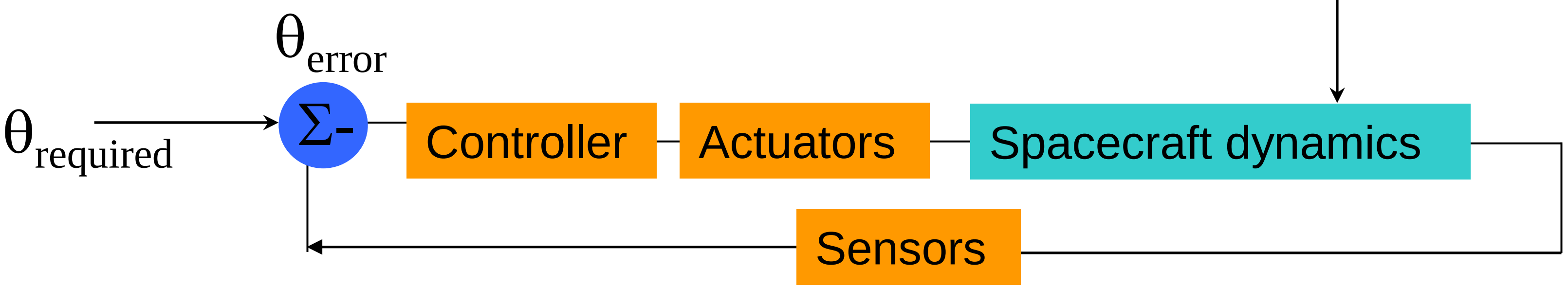
Hamiltonian systems on Lie groups

- Spacecraft Kinematic representations:
  - Kinematics
  - Euler angles
  - Quaternions
  - Lie Groups

- Active pointing control of spacecraft requires knowledge of the kinematics.
- Previous lecture focused on the natural dynamics:
- We need to know how the angular velocities relate to the orientation of the spacecraft e.g.

$$
\begin{aligned}
I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 &= T_1 \\
I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 &= T_2 \\
I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 &= T_3
\end{aligned}
\quad + \quad
\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \frac{1}{\cos\theta_2}\begin{pmatrix} \cos\theta_2 & \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2 \\ 0 & \cos\theta_1\cos\theta_2 & -\sin\theta_1\cos\theta_2 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{pmatrix}\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}
$$

- Then we can design feedback control systems:



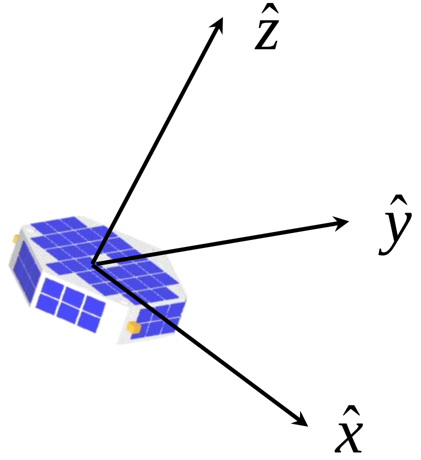
Kinematic representations

Perhaps the most intuitive (global) representation of rotation is the 3 x 3 rotation matrix comprised of three orthogonal vectors:

$R(t) = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \end{pmatrix}$ **each column is composed on a unit vector** $\hat{x}, \quad \hat{y}, \quad \hat{z} \in \mathbb{R}^3$

$\hat{x}, \quad \hat{y}, \quad \hat{z}$ **are orthogonal unit vectors**

$$R(t) \in SO(3)$$



Body-fixed orthonormal frame

It is a **<u>Lie group</u>** having the properties of a group and a differentiable manifold

Group – set with an operator (e.g. matrix multiplication) that satisfies four conditions of closure, associativity, identity, inverse.

Kinematic representations

•The structure of a Lie group means that we can define the kinematics:

$$\frac{dR(t)}{dt} = R(t)\begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} \in so(3)$$

Is a screw-symmetric matrix whose components are the angular velocities.

$$R(t) = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \end{pmatrix}$$

$$R(0) = \begin{pmatrix} \hat{x}(0) & \hat{y}(0) & \hat{z}(0) \end{pmatrix}$$

Mathematically this is the Lie algebra of SO(3). It has the structure of an algebra.

The Lie algebra of a Lie group is considered to be the tangent space to the group at the identity:

$$X = \left.\frac{dR(t)}{dt}\right|_{t=0}$$

Consider a rotation about the third axis in time:

$$R(t) = \begin{pmatrix} \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left.\frac{dR(t)}{dt}\right|_{t=0} = \left.\begin{pmatrix} -\sin t & \cos t & 0 \\ -\cos t & -\sin t & 0 \\ 0 & 0 & 0 \end{pmatrix}\right|_{t=0} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Kinematic representations

Euler's Theorem: Any two independent orthonormal coordinate frames may be related by

a minimum sequence of rotations (less than four) about coordinate axes, where no two successive rotations may be about the same axis.

It is possible to bring a rigid body into an arbitrary orientation by performing three successive rotations (12 sets are possible):

1. The first rotation is about any axis
2. The second rotation is about any axis not used for the first
3. The third rotation is either of the two axis not used for the second rotation.

Use Euler angles to represent these simple rotations about each axis.

Parameterize *R(t)* using Euler angles e.*g.*

$$R(t) = \begin{pmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{pmatrix}$$

Kinematic representations

•Matrix representation is highly computation – requires integrating 9 couple ODEs.

• Euler angles representation simplifies the procedure and design of controls.

$$R(t) = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \end{pmatrix}$$

$\hat{x}, \quad \hat{y}, \quad \hat{z}$ **are orthogonal vectors**

Parameterize *R(t)*

$$R(t) = \begin{pmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{pmatrix}$$

Kinematics in matrix form

$$\frac{dR(t)}{dt} = R(t) \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}$$

Euler angle representation of kinematics

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \frac{1}{\cos\theta_2} \begin{pmatrix} \cos\theta_2 & \sin\theta_1 \sin\theta_2 & \cos\theta_1 \sin\theta_2 \\ 0 & \cos\theta_1 \cos\theta_2 & -\sin\theta_1 \cos\theta_2 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

Kinematic representations

•Euler angles representation is useful for small manoeuvres but has singularities at $\cos\theta_2 = 0$

$$\begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \end{bmatrix} = \frac{1}{\cos\theta_2} \begin{pmatrix} \cos\theta_2 & \sin\theta_1 \sin\theta_2 & \cos\theta_1 \sin\theta_2 \\ 0 & \cos\theta_1 \cos\theta_2 & -\sin\theta_1 \cos\theta_2 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

•Quaternions often used to represent kinematics on-board spacecraft (non-intuitive)

$$\begin{bmatrix} \dot q_1 \\ \dot q_2 \\ \dot q_3 \\ \dot q_4 \end{bmatrix} = \frac{1}{2} \begin{pmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{pmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

Quaternion representations (Hamilton 1843)

•Quaternions often used to represent kinematics on-board spacecraft (non-intuitive)

$$\hat{q} = [q_1, q_2, q_3, q_4]^T$$

$$\hat{q} = q_1 + q_2 i + q_3 j + q_4 k$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2} \begin{pmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{pmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

$$ii = jj = kk = -1$$

$$ij = k,\ jk = i,\ ki = j$$

These two rotations are equivalent:

$$R(t)x = x'$$ Where $x = [x_1, x_2, x_3]^T$ is in its vector form

$$\hat{q}x\hat{q}^{-1} = x'$$ Where $x = [0, x_1, x_2, x_3]^T$ is in its quaternion form

$$\hat{q} = [q_1, q_2, q_3, q_4]^T, R(t) = \begin{pmatrix} q_1^2 + q_2^2 - q_3^2 - q_4^2 & 2q_2q_3 - 2q_1q_4 & 2q_2q_4 + 2q_1q_3 \\ 2q_2q_3 + 2q_1q_4 & q_1^2 - q_2^2 + q_3^2 - q_4^2 & 2q_3q_4 - 2q_1q_2 \\ 2q_2q_4 - 2q_1q_3 & 2q_3q_4 + 2q_1q_2 & q_1^2 - q_2^2 - q_3^2 + q_4^2 \end{pmatrix}$$

Eigen-axis rotation

Euler's (1st) Theorem

$$R(t)=\begin{pmatrix}\cos\theta_3 & \sin\theta_3 & 0\\ -\sin\theta_3 & \cos\theta_3 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\cos\theta_2 & 0 & -\sin\theta_2\\ 0 & 1 & 0\\ \sin\theta_2 & 0 & \cos\theta_2\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta_1 & \sin\theta_1\\ 0 & -\sin\theta_1 & \cos\theta_1\end{pmatrix}=R_x(\theta_3)R_y(\theta_2)R_z(\theta_1)$$

Euler's (2nd) Theorem: Any two independent orthonormal coordinate frames may be related by a single rotation about some axis.

Call the rotation angle about this eigenaxis θ then we can define the quaternions as:

$$\hat{q}=q_1+q_2 i+q_3 j+q_4 k \qquad q_1=\cos\frac{\theta}{2}, q_2=\sin\frac{\theta}{2}\cos\beta_x, q_3=\sin\frac{\theta}{2}\cos\beta_y, q_4=\sin\frac{\theta}{2}\cos\beta_z$$

$$\hat{q}=q_1+q_2 i+q_3 j+q_4 k$$

$\cos\beta_x, \cos\beta_y, \cos\beta_z$ are the direction cosines locating the single axis of rotation

e.g. if the eigenaxis is the x axis then $q_1=\cos\frac{\theta}{2}, q_2=\sin\frac{\theta}{2}, q_3=0, q_4=0$

$$\boxed{\hat{q}=\cos\frac{\theta}{2}+\sin\frac{\theta}{2}i}$$

Euler's (1st) Theorem

$$R(t)=\begin{pmatrix}\cos\theta_3 & \sin\theta_3 & 0\\ -\sin\theta_3 & \cos\theta_3 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\cos\theta_2 & 0 & -\sin\theta_2\\ 0 & 1 & 0\\ \sin\theta_2 & 0 & \cos\theta_2\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta_1 & \sin\theta_1\\ 0 & -\sin\theta_1 & \cos\theta_1\end{pmatrix}=R_x(\theta_3)R_y(\theta_2)R_z(\theta_1)$$

Euler angle representation

$$R(t)=\begin{pmatrix}q_1^2+q_2^2-q_3^2-q_4^2 & 2q_2q_3-2q_1q_4 & 2q_2q_4+2q_1q_3\\ 2q_2q_3+2q_1q_4 & q_1^2-q_2^2+q_3^2-q_4^2 & 2q_3q_4-2q_1q_2\\ 2q_2q_4-2q_1q_3 & 2q_3q_4+2q_1q_2 & q_1^2-q_2^2-q_3^2+q_4^2\end{pmatrix}$$

Quaternion representation

Equating components we can conveniently compute equations for the Euler angles in terms of the quaternions.

Using the eigenaxis definition of quaternions.

$$\hat{q}=q_1+q_2i+q_3j+q_4k$$

$$q_1=\cos\frac{\theta}{2},\ q_2=\sin\frac{\theta}{2}\cos\beta_x,\ q_3=\sin\frac{\theta}{2}\cos\beta_y,\ q_4=\sin\frac{\theta}{2}\cos\beta_z$$

We can express each rotation matrix about a fixed axes as a quaternion.

$$\hat{q}=R_x(\theta_3)R_y(\theta_2)R_z(\theta_1)=[\cos\frac{\theta_3}{2}+\sin\frac{\theta_3}{2}i][\cos\frac{\theta_2}{2}+\sin\frac{\theta_2}{2}j][\cos\frac{\theta_1}{2}+\sin\frac{\theta_1}{2}k]$$

This provides a convenient expression for the quaternions in terms of the Euler angles.

Quaternion in matrix form

- It can sometimes be convenient to use the matrix form called SU(2):

$$H \rightarrow SU(2)$$

$$\hat{q} = q_1 + q_2 i + q_3 j + q_4 k \qquad g = \begin{bmatrix} q_1 + q_2 i & q_3 + q_4 i \\ -q_3 + q_4 i & q_1 - q_2 i \end{bmatrix}$$

$$\hat{q}_1 = x_1 + x_2 i + x_3 j + x_4 k \qquad g_1 = \begin{bmatrix} x_1 + x_2 i & x_3 + x_4 i \\ -x_3 + x_4 i & x_1 - x_2 i \end{bmatrix}$$

Quaternion multiplication — Matrix Multiplication

Then the kinematics can be defined on the SU(2):

$$\frac{dg}{dt} = \frac{1}{2} g \begin{pmatrix} i\omega_1 & \omega_2 + i\omega_3 \\ -\omega_2 + i\omega_3 & -i\omega_1 \end{pmatrix}$$

Summary

- We have considered four different representations of the kinematics: SO(3), Euler angles, quaternions, SU(2).
- **SO(3)** – global coordinate system – requires the integration of 9 coupled ODEs.
- **Euler angles** – local coordinate system – requires the integration of 3 coupled ODEs.

  non-unique – suffers from gimbal lock – intuitive
- **Quaternions** – global coordinate system – requires the integration of 3 coupled ODEs

  unique – no singularities – non-intuitive.
- Quaternions in matrix form **SU(2)** – as above – useful form for certain computations.

Nonlinear model

$$I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 = T_1$$
$$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 = T_2$$
$$I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 = T_3$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2}\begin{pmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{pmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

Re-pointing problem – find $T_1, \quad T_2, \quad T_3$ that drives the spacecraft from

$$q(0) = [q_1(0), q_2(0), q_3(0).q_4(0)]^T, \omega(0) = [\omega_1(0), \omega_2(0), \omega_3(0)]^T,$$

to

$$q(T) = [q_1(T).q_2(T), q_3(T).q_4(T)]^T, q(0) = [\omega_1(T), \omega_2(T), \omega_3(T)]^T,$$

Lie Group representation of kinematics: $\frac{dg(t)}{dt} = g(t)\Omega$

where $g(t) \in G$ is a Lie Group and Ω is a Lie algebra.

When a system has this form in is often sufficient to study the Lie algebra to obtain information about the entire system.

- Controllability properties.
- The co-ordinate free maximum principle
- Lax Pair Integration

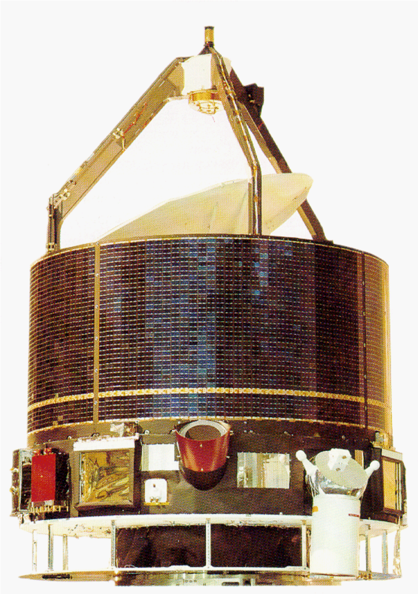
Other systems include robotics e.g. unmanned air vehicle, and autonomous underwater vehicles, quantum control, fine needle surgery.

Attitude Dynamics and Control

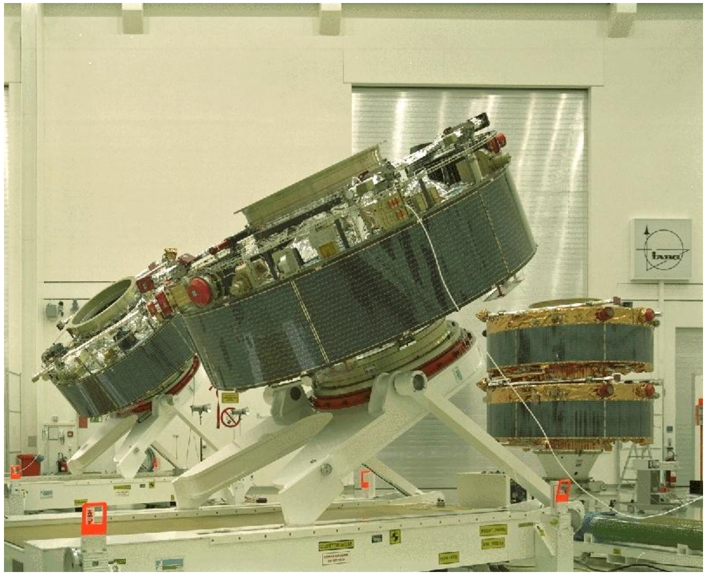
- Spin Stabilization

Spin stabilisation

- Simple and low cost method of attitude stabilisation (largely passive)
- Generally not suitable for imaging payloads (but can use a scan platform)
- Poor power efficiency since entire spacecraft body covered with solar cells



ESA Giotto spacecraft (de-spun antenna)



ESA Cluster spacecraft

Spin axis stability

- Consider rotation about z-axis with angular velocity $\omega_3 = \Omega, \omega_1 = \omega_2 = 0$
- Now perturb state of pure spin such that $\omega_3 = \Omega + \partial\omega_3, \omega_1 = \partial\omega_1, \omega_2 = \partial\omega_2,$
- Under what conditions does the spacecraft spin remain stable?

$\partial\omega_2\partial\omega_3 \sim 0 \qquad \partial\omega_1\partial\omega_3 \sim 0$

$$\partial\dot{\omega}_1 = \frac{(I_2 - I_3)}{I_1}\partial\omega_2\Omega$$

$$\partial\dot{\omega}_2 = \frac{(I_3 - I_1)}{I_2}\partial\omega_1\Omega$$

$$\partial\dot{\omega}_3 = 0$$

Conditions for stability

- Combine perturbed Euler equations x- and y-axis to form a single equation
- Obtain single 2nd order differential equation with constant coefficient
- Obtain (stable) oscillatory solution in coefficient positive (unstable if -ve)

$$\partial\ddot{\omega}_1 = \frac{(I_2 - I_3)}{I_1}\partial\dot{\omega}_2\Omega \longrightarrow \partial\ddot{\omega}_1 = \frac{(I_2 - I_3)(I_3 - I_1)}{I_1 I_2}\Omega^2\partial\omega_1$$

$$\partial\ddot{\omega}_1 + Q^2\partial\omega_1 = 0$$

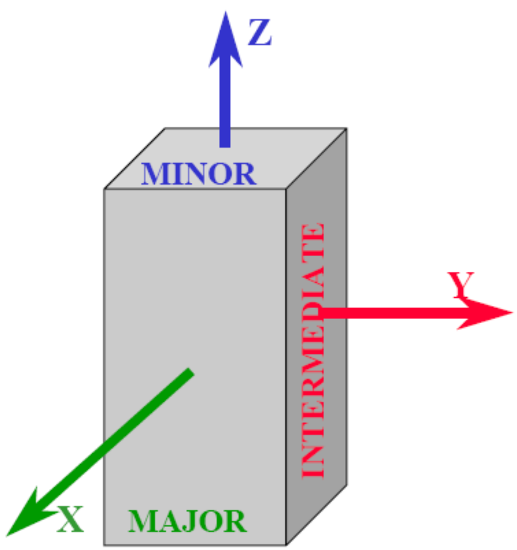
$$Q^2 = \frac{(I_3 - I_2)(I_3 - I_1)}{I_1 I_3}\Omega^2$$

$Q^2 > 0$ Stable

$Q^2 < 0$ Unstable

Major axis rule

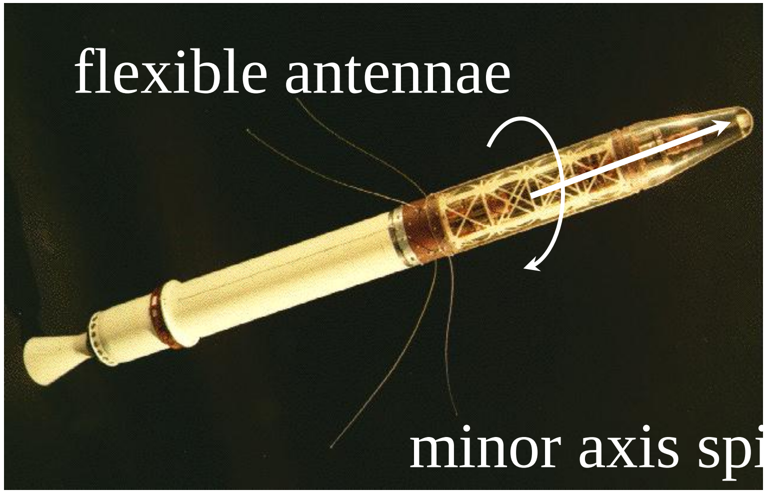
- See that minor axis spin is stable ($Q^2>0$)
- See that major axis spin is stable ($Q^2>0$)
- But, intermediate axis spin is unstable ($I_3>I_2$ but $I_3<I_1$) since then $Q^2<0$

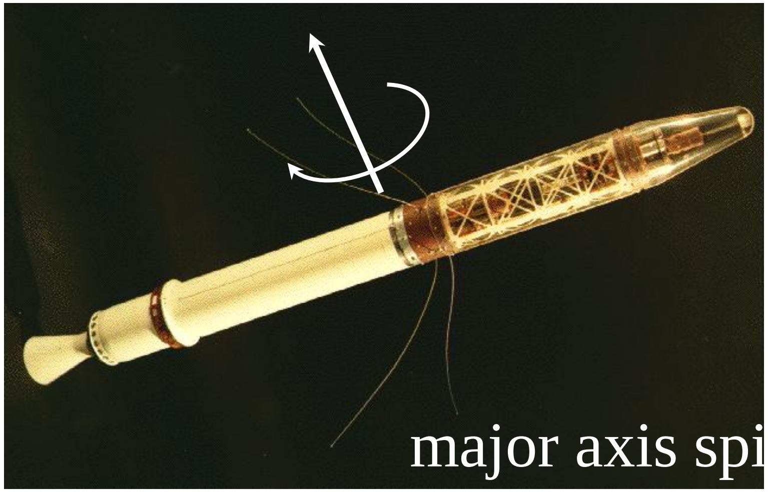


- $I_{xx} > I_{yy} > I_{zz}$
- Major axis spin is stable
- Minor axis spin is stable
- Intermediate axis spin is unstable
- Energy dissipation changes these results
  $\rightarrow$ Minor axis spin becomes unstable
- This is called the Major-Axis Rule

Explorer 1 flat spin

- Explorer 1 (first US satellite, 1958) designed as a minor axis spinner !
- Via energy dissipation spacecraft experienced transition to major axis spin
- Energy dissipation caused by flexing of wire antennae on spacecraft body





Example of a dissipative system with spherical fuel slug

$$(I_1 - I)\dot{\omega}_1 = (I_2 - I_3)\omega_2\omega_3 + \mu\sigma_1$$
$$(I_2 - I)\dot{\omega}_2 = (I_3 - I_1)\omega_3\omega_1 + \mu\sigma_2$$
$$(I_3 - I)\dot{\omega}_3 = (I_1 - I_2)\omega_2\omega_3 + \mu\sigma_3$$
$$\dot{\sigma}_1 = -\dot{\omega}_1 - (\mu / I)\sigma_1 - \omega_2\sigma_3 + \omega_3\sigma_2$$
$$\dot{\sigma}_2 = -\dot{\omega}_2 - (\mu / I)\sigma_2 - \omega_3\sigma_1 + \omega_1\sigma_3$$
$$\dot{\sigma}_3 = -\dot{\omega}_3 - (\mu / I)\sigma_3 - \omega_1\sigma_2 + \omega_2\sigma_1$$

I Spherical fuel slug moment of inertia

μ Viscous damping coefficient

$\sigma_1, \sigma_2, \sigma_3$ Relative rates between the rigid body and the fuel slug

$$H = \frac{1}{2}((I_1 - I)\omega_1^2 + (I_2 - I)\omega_2^2 + (I_3 - I)\omega_3^2 + I((\omega_1 + \sigma_1)^2 + (\omega_2 + \sigma_2)^2 + (\omega_3 + \sigma_3)^2))$$

Perturbations – disturbance torques

$$I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 = M_1$$

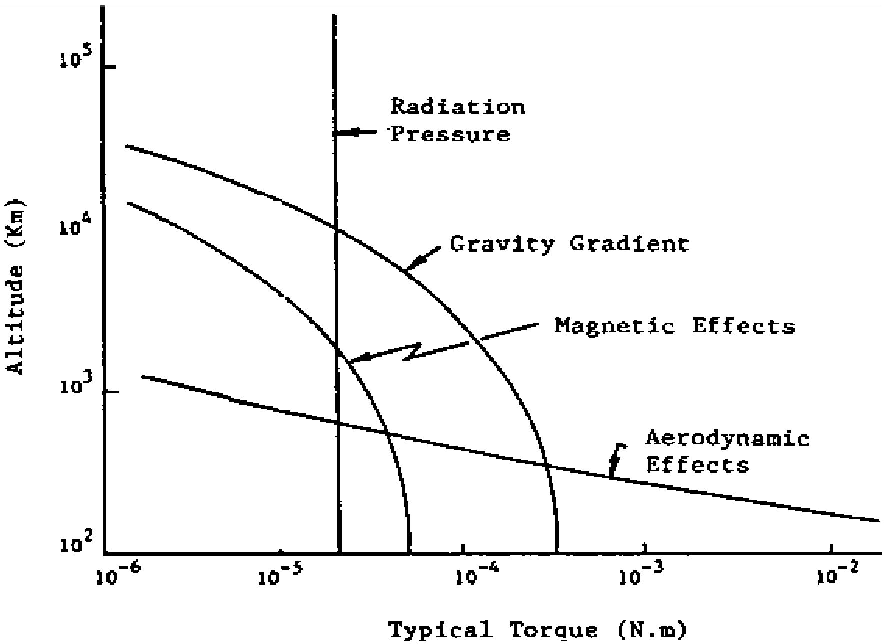
$$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 = M_2$$

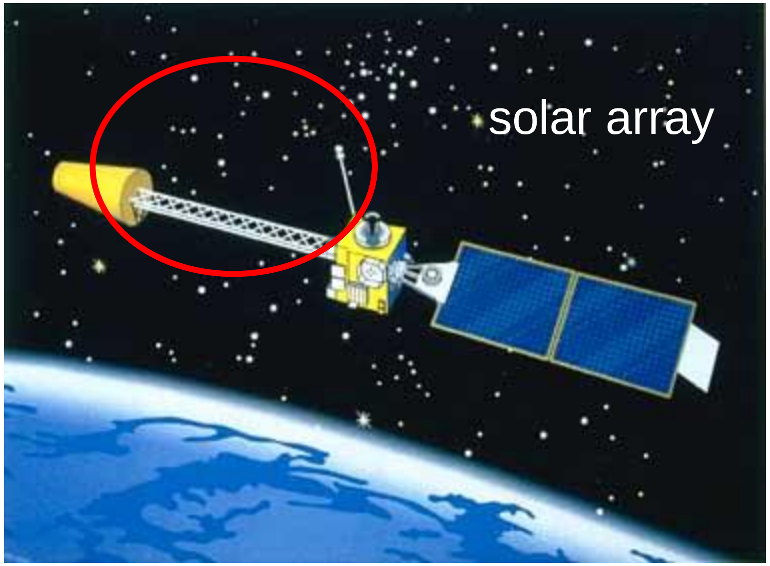
$$I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 = M_3$$

- **Air drag**
- **Solar pressure**
- **Gravity Gradient**
- **Magnetic field**
- **Spherical harmonics**

Disturbance torques

- Disturbance torques lead to reduction in pointing accuracy of spacecraft
- Torque magnitudes dependent on spacecraft orbit type and orbit altitude
- Air drag, gravity gradient torques in LEO, solar pressure torques in GEO





GOES-J solar pressure boom

ADCS Sizing: Momentum wheels, reaction wheels, magnetic torques

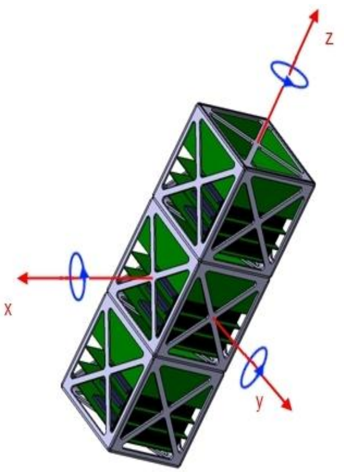
1. Calculate disturbance torques
2. Compute time integral of disturbance torques for each control axis. The resulting angular impulse on each axis L_x,L_y,L_z represent the accumulated angular momentum:

$$L_x = \int_{t_0}^{t_f} M_{dx} dt \qquad L_y = \int_{t_0}^{t_f} M_{dy} dt \qquad L_z = \int_{t_0}^{t_f} M_{dz} dt$$

3. Identify cyclic and secular components of disturbing forces
4. Size torquers: wheels and CMGs are sized for cyclic terms, thrusters/magnet torquers are sized for secular terms if used with wheels (desaturation)
5. This is often referred to a momentum dumping.

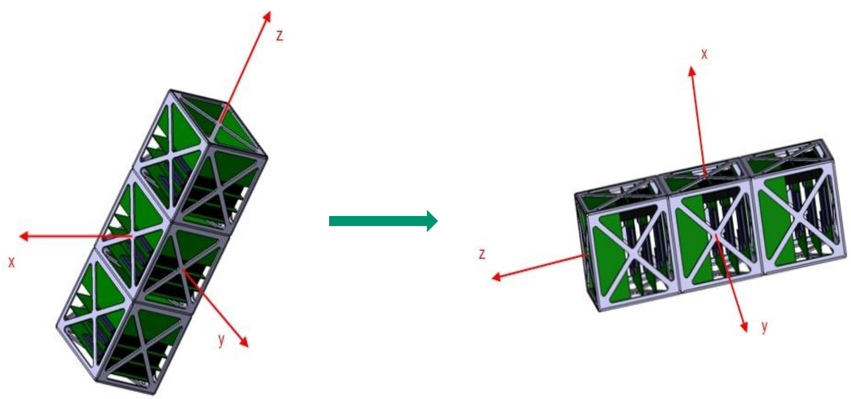
Objectives

1 – Detumbling – Dynamic equations



Tumbling motion must be stabilised or mission will fail.

2 – Repointing – Dynamics and kinematics



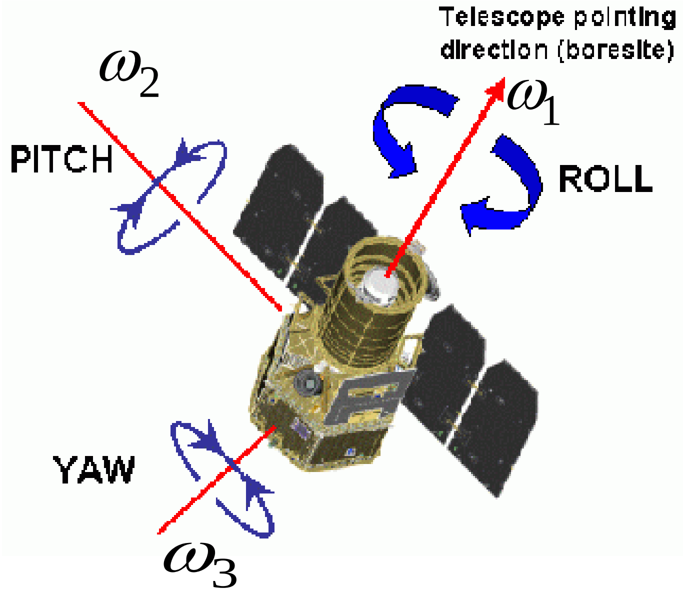
Reorient spacecraft to target specific point (e.g. point antenna to ground station, point solar cells towards sun for maximum power.)

Detumbling with thrusters

•Spacecraft have a tendency to tumble post-seperation.

• Can use continuos torque to de-tumble the spacecraft (three reaction wheels are required four for redundancy)

• Micro and nano spacecraft often use magnetic-torquers (underactuated)



$$I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 = T_1$$

$$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 = T_2$$

$$I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 = T_3$$

Use proportional control

$$T_1 = -k_1\omega_1, \quad T_2 = -k_2\omega_2, \quad T_3 = -k_3\omega_3.$$

Detumbling with magnetic torquers

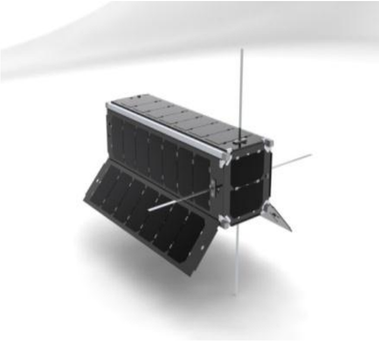
- With magnetorquers controllable parameter is magnetic dipole N:-

$$\boldsymbol{T} = \boldsymbol{N} \times \boldsymbol{B}$$

- Convention is to use BDOT, which uses rate of change of magnetic field as control (requires a magnetometer and Kalman filter):-

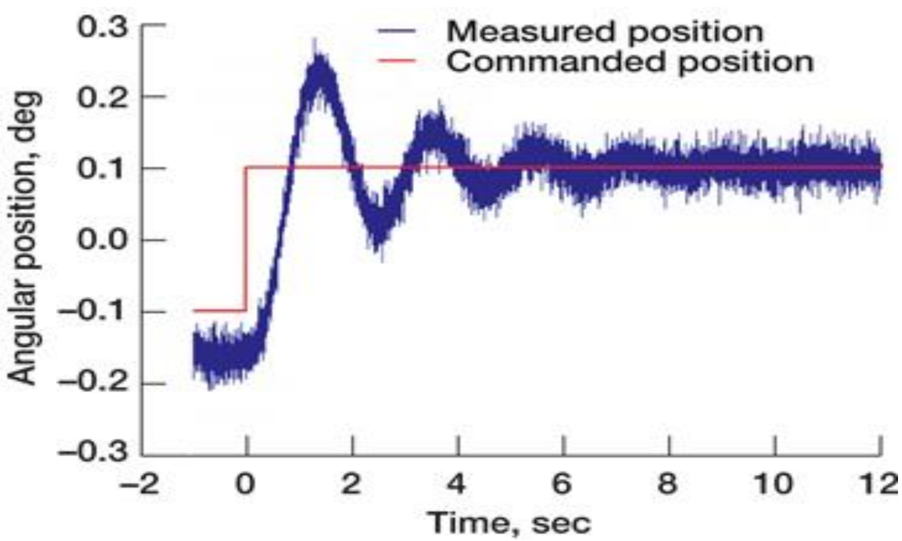
$$\boldsymbol{N} = -K(\dot{\boldsymbol{B}})$$

- BDOT used on several magnetically actuated spacecraft (e.g. Compass-1, UKube-1.).



Attitude control laws

- Can use classical linear control laws for fine pointing (PID controller)
- Also, need to ensure robustness of control laws due to disturbance torques



classical PID controller

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{d}{dt} e(t)$$

$$e(t) = [\theta_1 - \theta_{1d}, \theta_2 - \theta_{2d}, \theta_3 - \theta_{3d}]^T$$

$$I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 = T_1$$
$$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 = T_2$$
$$I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 = T_3$$

$$u(t) = [T_1, T_2, T_3]^T$$

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\,d\tau + K_d \frac{d}{dt} e(t)$$

Parameter	Overshoot	Settling time	Steady-state error
K_p	$\uparrow$	-	$\downarrow$
K_i	$\uparrow$	$\uparrow$	Eliminates
K_d	$\downarrow$	$\downarrow$	-

Estimated to be used in 95% of industrial processes.

Full nonlinear model – quaternion representation

$$I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 = M_1 + T_1$$
$$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 = M_2 + T_2$$
$$I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 = M_3 + T_3$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2}\begin{pmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{pmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

Re-pointing problem – find $T_1, \quad T_2, \quad T_3$ that drives the spacecraft from

$$q(0) = [q_1(0), q_2(0), q_3(0).q_4(0)]^T, \omega(0) = [\omega_1(0), \omega_2(0), \omega_3(0)]^T,$$

to

$$q(T) = [q_1(T).q_2(T), q_3(T).q_4(T)]^T, q(0) = [\omega_1(T), \omega_2(T), \omega_3(T)]^T,$$

A simple quaternion feedback controller (rest-to-rest)

$$T_1 = -k_1 e_1 - c_1 \omega_1$$

$$T_2 = -k_2 e_2 - c_2 \omega_2$$

$$T_3 = -k_3 e_3 - c_3 \omega_3$$

where

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{pmatrix} q_4(T) & q_3(T) & -q_2(T) & -q_1(T) \\ -q_3(T) & q_4(T) & q_1(T) & -q_2(T) \\ q_2(T) & -q_1(T) & q_4(T) & -q_3(T) \\ q_1(T) & q_2(T) & q_3(T) & q_4(T) \end{pmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

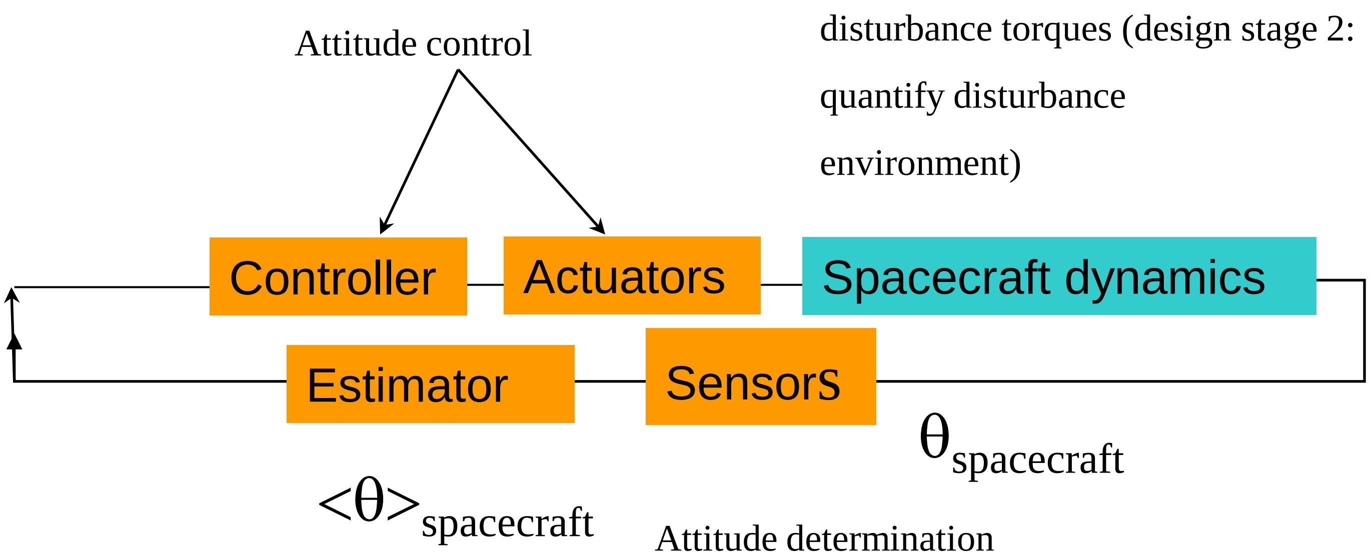
Algorithm is flight tested and stability can be verified. Requires tuning of the parameters:

$$k_1, c_1, k_2, c_2, k_3, c_3$$

Can use adaptive tuning methods e.g fuzzy logic

Attitude control loop (precision spacecraft)

- Actuators: Reaction wheels and jet torques.
- Sensors: Star and Sun sensors.
- Estimator – Kalman filter, low pass filter, nonlinear filter.
- Appropriate feedback law e.g. quaternion feedback law



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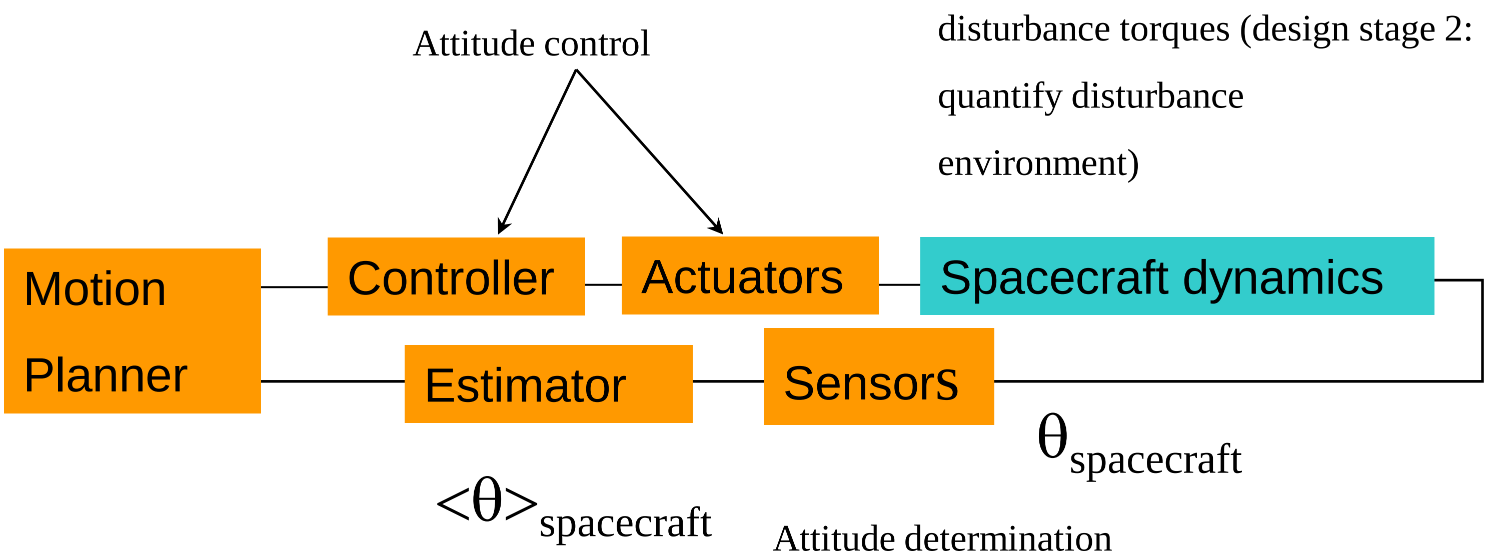
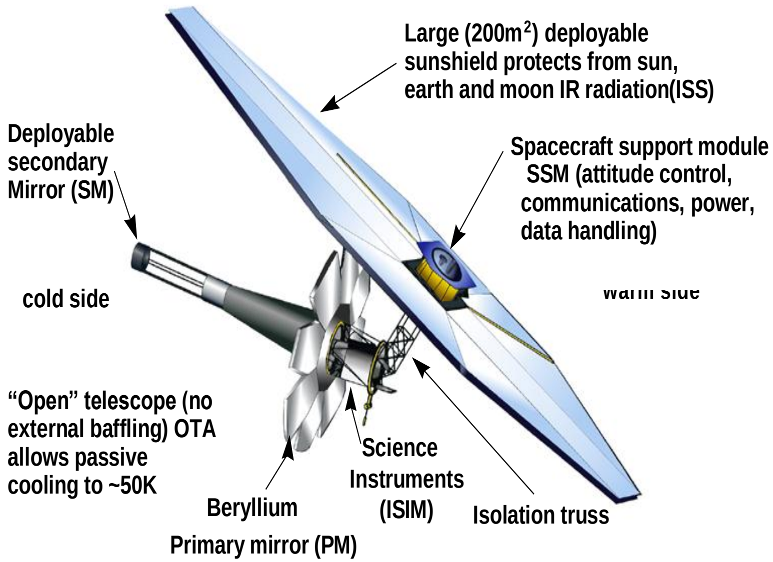


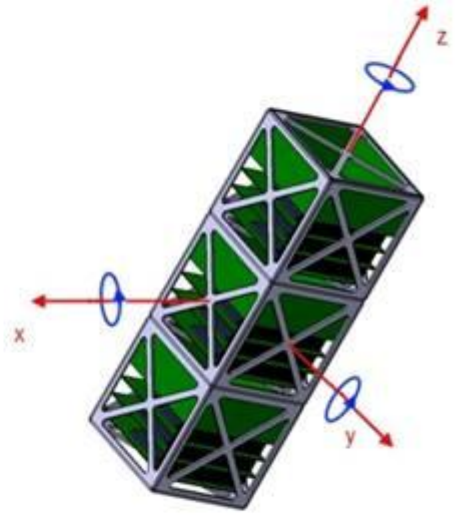
Future developments

- Future spacecraft will pose exciting challenges for attitude control design
- Large deployable, highly flexible membrane structures (telescope Sun-shade)
- Fast slews for agile science/Earth observation (monitor transient phenomena)
- Attitude control of nano and micro spacecraft – low-onboard computational power and small torque available.



next generation space telescope (NGST)

Rapideye



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