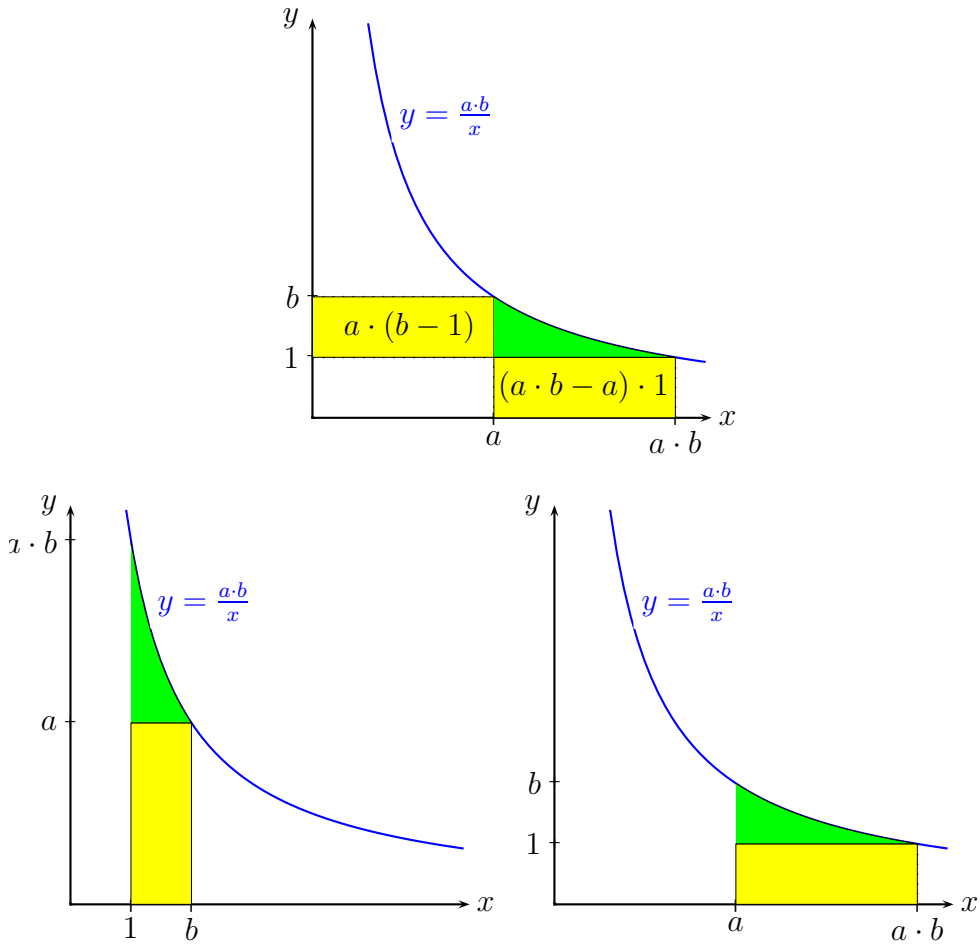
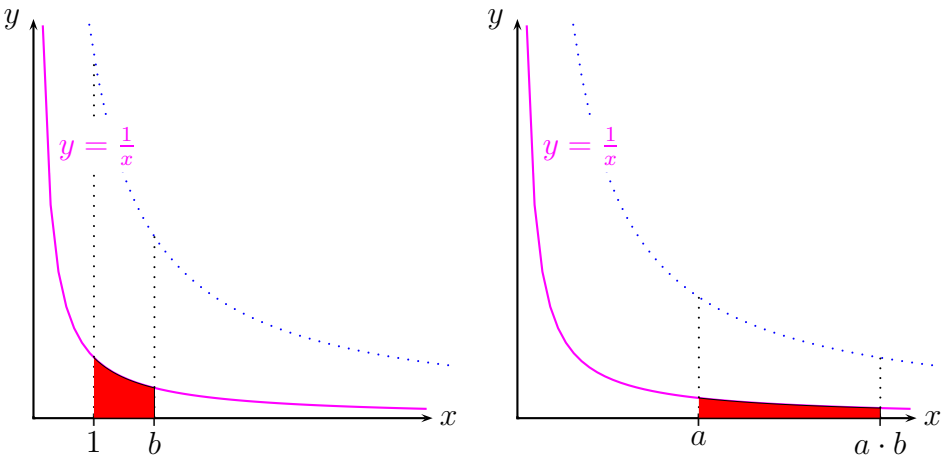
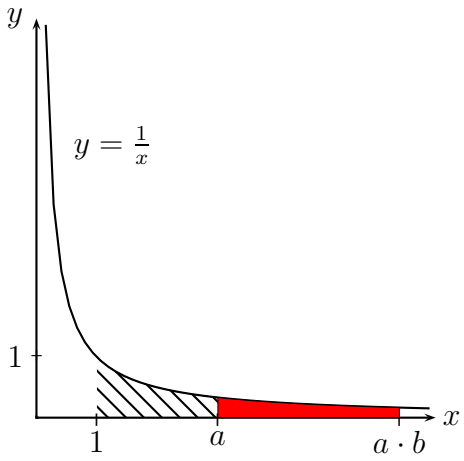
Proof without words: $\ln(a \cdot b) = \ln a + \ln b$



$$\text{Area}\left(\tfrac{a\cdot b}{x}\ ,\ [1\ ,\ b]\right) \ = \ \text{Area}\left(\tfrac{a\cdot b}{x}\ ,\ [a\ ,\ a\cdot b]\right)$$



$$\text{Area}\left(\tfrac{1}{x}\ ,\ [1\ ,\ b]\right)\ =\ \text{Area}\left(\tfrac{1}{x}\ ,\ [a\ ,\ a\cdot b]\right)$$



$$\text{Area}\left(\tfrac{1}{x}\ ,\ [1\ ,\ a]\right) + \text{Area}\left(\tfrac{1}{x}\ ,\ [1\ ,\ b]\right)\ =\ \text{Area}\left(\tfrac{1}{x}\ ,\ [1\ ,\ a\cdot b]\right).$$

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