

Analisi Matematica I
Integrali e integrali impropri

Esercizio 1. Calcolare i seguenti integrali indefiniti

(1) $\int e^x \left(\frac{1}{2}x^2 + x\right) dx$

(2) $\int (x - 4)^2 \sin x dx$

(3) $\int x^4 \cos x dx$

(4) $\int \frac{dx}{\sqrt[3]{2x+1}}$

(5) $\int x\sqrt{1+4x^2} dx$

(6) $\int \sin^2 x \cos^7 x dx$

(7) $\int \frac{\sin^3 x}{\cos^2 x} dx$

(8) $\int \cos^4 x dx.$

(9) $\int \cos^2 x (\cos x + \sin x)^2 dx.$

(10) $\int x(\sin 2x - \sin x) dx$

(11) $\int e^{2\sqrt{t}} dt$

(12) $\int e^{1-\sqrt[3]{x}} dx$

(13) $\int \frac{\log \log x}{x} dx$

(14) $\int \arcsin(\sqrt{x}) dx$

(15) $\int \frac{1}{\cos^2 \sqrt{x}} dx$

Esercizio 2. Calcolare i seguenti integrali indefiniti

(1) $\int \frac{2x+1}{x(x^2+1)} dx$

(2) $\int \frac{2x+1}{x(x^2-1)} dx$

(3) $\int \frac{x-3}{x(x-1)(x-2)} dx$

(4) $\int \frac{x^3+1}{x(x-1)^2} dx$

(5) $\int \frac{2x^4+3}{x(x+1)^2} dx$

- (6) $\int \frac{x^2 + 3x + 1}{x(x^2 - 1)^2} dx$
- (7) $\int \frac{1}{x^2 + 2x + 2} dx$
- (8) $\int \frac{x^4 + 1}{x^2 + 2x + 2} dx$
- (9) $\int \frac{3x - 2}{(x - 1)(x^2 - 2x + 2)} dx$
- (10) $\int \frac{1}{x^2 + x + 1} dx$
- (11) $\int \frac{3x + 1}{x^2 + x + 1} dx$
- (12) $\int \frac{1}{x(x^2 + x + 1)} dx$
- (13) $\int \frac{x^2 + 5x + 2}{(x^2 + 1)(x^2 - x)} dx$
- (14) $\int \frac{1}{(x^2 + 1)(x^2 + 4)} dx$
- (15) $\int \frac{1}{(x^2 + 1)^2} dx$
- (16) $\int \frac{1}{x^2(x^2 + 1)^2} dx$

Esercizio 3. Calcolare i seguenti integrali indefiniti

- (1) $\int \frac{\sqrt{x}}{1 + \sqrt{x}} dx$
- (2) $\int \frac{x + 1}{2x(x - 3\sqrt{x} + 2)} dx$
- (3) $\int \frac{\sqrt{x + 1} + 3}{x + 2} dx$
- (4) $\int \frac{\sqrt{x - 1} + 1}{x + 2\sqrt{x - 1} + 2} dx$
- (5) $\int \frac{1}{e^{2x} + 9} dx$
- (6) $\int \frac{1}{e^x \sinh x} dx$
- (7) $\int (x^2 + 1) \log(x + 1) dx$
- (8) $\int \frac{\sin^4 x}{\cos^2 x} dx$

$$(9) \int \frac{\cos^2 x}{4 \cos^2 x + \sin^2 x} dx$$

$$(10) \int x \sqrt{1 + \frac{9}{4}x} dx$$

$$(11) \int \sqrt{x^2 + 1} dx$$

$$(12) \int \sqrt{\frac{1}{x^2} + \frac{1}{x^4}} dx$$

$$(13) \int \frac{\sqrt{x^2 - 1}}{x + 1} dx$$

$$(14) \int \frac{\sqrt{1 - x^2}}{x^2 + 1} dx$$

$$(15) \int x^2 \sqrt{9 - x^2} dx$$

Esercizio 4. Calcolare i seguenti integrali definiti

$$(1) \int_{-\pi/4}^{\pi/4} \frac{|\sin x|}{\cos x} dx$$

$$(2) \int_{\pi/2}^{3\pi/2} \cos^3 x dx$$

$$(3) \int_0^{2\pi} |\sin x|^3 dx$$

$$(4) \int_0^{\pi/3} \frac{1}{1 - \sin x} dx$$

$$(5) \int_0^{\pi/4} \frac{\operatorname{tg} x}{1 + \sin^2 x} dx$$

$$(6) \int_{\pi/4}^{\pi/2} \frac{\sin x \cos x}{\sqrt{1 - \cos x}} dx$$

$$(7) \int_0^1 e^x \log(4 + e^{2x}) dx$$

$$(8) \int_{-1}^0 \frac{1}{x - \sqrt{3x + 4}} dx$$

$$(9) \int_0^1 \frac{\log(x^2 - x + 1)}{(x + 1)^2} dx$$

$$(10) \int_1^e \frac{(2 \log x + 1) \operatorname{arctg}(\log x)}{x} dx$$

$$(11) \int_0^3 \operatorname{arctg} \left(\frac{\sqrt{x} + 3}{\sqrt{x} + 1} \right) dx$$

$$(12) \int_0^{\pi/4} \frac{\sin x}{\cos^2 x} \sqrt{1 + 4 \cos^2 x} dx$$

Esercizio 5. Calcolare i seguenti integrali impropri

$$(1) \int_0^1 \frac{1}{\sqrt{x(1+x)}} dx$$

$$(2) \int_0^\infty \frac{\log(1 + \sqrt{x})}{\sqrt{x}(2 + \sqrt{x})^2} dx$$

$$(3) \int_1^\infty \frac{\operatorname{arctg}(\sqrt{x-1})}{\sqrt{x-1}(x + 2\sqrt{x-1})} dx$$

$$(4) \int_0^\infty e^{-x}(x + \sqrt{e^x - 1}) dx$$

$$(5) \int_0^\infty x(1 - \cos x)e^{-x} dx$$

$$(6) \int_0^1 \frac{1}{\sqrt{x(1-x)}} dx$$

$$(7) \int_0^{\pi/2} \frac{\sin x \log(\sin x)}{\cos^2 x} dx$$

$$(8) \int_1^\infty \frac{\log x}{(x-1)^{3/2}} dx$$

$$(9) \int_1^\infty \frac{\log x}{(x-1)^{4/3}} dx$$

Esercizio 6. Discutere l'integrabilità in senso improprio dei seguenti integrali

$$(1) \int_1^{+\infty} \frac{\log(x+1)}{x^3 + 2x + 1} dx$$

$$(2) \int_1^{+\infty} \frac{\log(2+x^2)}{\sqrt{x} \operatorname{arctg}(x^2)} dx$$

$$(3) \int_0^1 \frac{\log x}{|x-1|^{5/4} \sin(x^{1/2})} dx$$

$$(4) \int_0^1 \frac{\log(x^2)}{x^{1/2} \arcsin(|x-1|^{9/4})} dx$$

$$(5) \int_0^{+\infty} \frac{1}{\sqrt{x}(x^2+1) \log(1+\sqrt{x})} dx$$

$$(6) \int_0^{+\infty} \frac{e^{-x^2/2}}{\sqrt{2x} + \operatorname{arctg}(x^{1/4})} dx$$

$$(7) \int_1^{+\infty} \frac{\sin \frac{1}{\sqrt{x}}}{(x-1)^{1/2}} dx$$

- (8) $\int_{-1}^{+\infty} \frac{e^{-x}}{(x-4)^2(x+\frac{1}{2})^{1/3}} dx$
- (9) $\int_{-1}^{+\infty} \frac{e^{-x}}{(x-3)^{1/3}|x-\frac{1}{2}|^{1/2}} dx$
- (10) $\int_{-1}^{+\infty} \frac{1}{(x-3)^{1/3}|x-\frac{1}{2}|^{1/2}} dx$
- (11) $\int_{-1}^{+\infty} \frac{1}{|x-3|^{3/4}|x-\frac{1}{2}|^{1/2}} dx$
- (12) $\int_{-1}^{+\infty} \frac{\log(3+|x|^{-1/4})}{|x-3|^{3/4}|x-\frac{1}{2}|^{1/2}} dx$
- (13) $\int_{-\infty}^{+\infty} e^{-x^2/2} dx$

Esercizio 7. Trovare i valori di $\beta \in \mathbb{R}$ per cui risultano convergenti i seguenti integrali impropri

- (1) $\int_1^{+\infty} \frac{1}{(1+x^2)(x+2)^\beta} dx$
- (2) $\int_2^{+\infty} \frac{(\log(1+\frac{1}{x}))^\beta}{\sqrt{x}+1} dx$
- (3) $\int_2^{+\infty} \frac{\operatorname{arctg}(x+7)}{x(\log(x+2))^\beta} dx$
- (4) $\int_1^{+\infty} \left(1 - \cos \frac{1}{x^3}\right)^\beta x^{\beta/2} dx$
- (5) $\int_1^{+\infty} \frac{|\sin \frac{1}{x} - \frac{1}{x}|^\beta}{\sqrt[3]{x}} dx$
- (6) $\int_0^1 \frac{(e^x - 1)^\beta}{\sqrt{x}(1-x)} dx$
- (7) $\int_1^\infty \frac{\log x}{(x-1)^\beta} dx$
- (8) $\int_{-1}^{+\infty} \frac{\operatorname{arctg}(x^2+3)}{(x+1)^\beta(x+2)} dx$
- (9) $\int_0^{+\infty} \left(\operatorname{arctg} \frac{1}{x}\right)^\beta dx$
- (10) $\int_3^{+\infty} \frac{e^{-x}}{(x-3)^\beta \sqrt{x}} dx$
- (11) $\int_0^{+\infty} (\operatorname{arctg} x)^\beta (\sqrt{x}+3)^{2\beta} dx$
- (12) $\int_0^1 \frac{\cos^2 x + 3}{x^\beta + \sqrt{x}} dx$

$$(13) \int_0^{+\infty} \left(e^{-x} + \frac{x^{2\beta} + 1}{\sqrt{x}} \right) dx$$

$$(14) \int_0^{+\infty} \frac{\operatorname{arctg}\left(\frac{1}{x^\beta}\right)}{2 + \sqrt{x}} dx$$

$$(15) \int_0^{+\infty} \frac{|\sin \frac{1}{\sqrt{x}}|^\beta}{\sqrt{x} \log(1 + \sqrt[3]{x})} dx, \quad \beta \geq 0$$

$$(16) \int_0^\infty \frac{x(1 - \cos x)e^{-x}}{\operatorname{arctg}(x^\beta)} dx$$

$$(17) \int_0^{+\infty} \frac{2x + \sin(x^\beta)}{e^x - \cos(x^\beta)} dx, \quad \beta \geq 0$$

Esercizio 8. Trovare i valori di $\beta \in \mathbb{R}$ per cui risultano convergenti i seguenti integrali impropri. Calcolare, poi, i medesimi integrali per il valore di β indicato.

$$(1) \int_0^1 \frac{\sqrt{x}}{x^\beta(x-2)} dx, \quad \beta = 1,$$

$$(2) \int_1^\infty \left(\frac{\operatorname{arctg} \frac{1}{x}}{(x-1)^2} \right)^\beta \frac{1}{x\sqrt[3]{x-1}} dx, \quad \beta = 0,$$

$$(3) \int_1^{+\infty} \frac{x+2}{x(x^2+2)} e^{\beta x^2} dx, \quad \beta = 0,$$

$$(4) \int_0^\infty \log(1+x^\beta) \left(\frac{1}{x^2} + \frac{1}{(x+3)^2} \right) dx, \quad \beta = 2,$$

$$(5) \int_0^\infty \frac{1}{\sqrt[3]{x-1}(x+7x^\beta)} dx, \quad \beta = 0,$$