

**Exercise 1.7**

... compute the following quotients and remainders.

- (a) 34787 divided by 353.
- (b) 238792 divided by 7843.
- (c) 9829387493 divided by 873485.
- (d) 1498387487 divided by 76348.

**Exercise 1.8**

... compute the following remainders, without bothering to compute the associated quotients.

- (a) The remainder of 78745 divided by 127.
- (b) The remainder of 2837647 divided by 4387.
- (c) The remainder of 8739287463 divided by 18754.
- (d) The remainder of 4536782793 divided by 9784537.

**Exercise 1.9**

Use the Euclidean algorithm to compute the following greatest common divisors.

- (a)  $\gcd(291, 252)$ .
- (b)  $\gcd(16261, 85652)$ .
- (c)  $\gcd(139024789, 93278890)$ .
- (d)  $\gcd(16534528044, 8332745927)$ .

**Exercise 1.12(c)**

Use your program to compute  $g = \gcd(a, b)$  and integer solutions to the equation  $au + bv = g$  for the following pairs  $(a, b)$ .

- (i) (527, 1258)
- (ii) (228, 1056)
- (iii) (163961, 167181)
- (iv) (3892394, 239847)

**Exercise 1.28**

Compute the following  $\text{ord}_p$  values. (qui  $\text{ord}_p(x) = \text{ordine di } x \text{ in } \mathbb{Z}_p$ )

- (a) Compute  $\text{ord}_2(2816)$ .
- (b) Compute  $\text{ord}_7(2222574487)$ .
- (c) Compute  $\text{ord}_p(46375)$  for each of  $p=3, 5, 7,$  and  $11$ .

**Exercise 2.28**

Use the Baby-Step-Giant-Step algorithm (or the Pohlig–Hellman Algorithm) to solve the discrete logarithm problem...

- (a)  $p = 433, g = 7, a = 166$ .
- (b)  $p = 746497, g = 10, a = 243278$ .
- (c)  $p = 41022299, g = 2, a = 39183497$ . (Hint:  $p=2 \cdot 295+1$ .)
- (d)  $p = 1291799, g = 17, a = 192988$ . (Hint:  $p-1$  has a factor of 709.)

**Exercises 3.5(b)**

Solve the following congruences. ...

- (i)  $x577 \equiv 60 \pmod{1463}$ .
- (ii)  $x959 \equiv 1583 \pmod{1625}$ .
- (iii)  $x133957 \equiv 224689 \pmod{2134440}$ .

### Exercises 3.8

For each of the given values of  $N=pq$  and  $(p-1)(q-1)$ , use the method described in Remark 3.10 to determine  $p$  and  $q$ . (a)  $N = pq = 352717$  and  $(p-1)(q-1) = 351520$ .

(b)  $N = pq = 77083921$  and  $(p-1)(q-1) = 77066212$ .

(c)  $N = pq = 109404161$  and  $(p-1)(q-1) = 109380612$ .

(d)  $N = pq = 172205490419$  and  $(p-1)(q-1) = 172204660344$ .

### Exercises 3.9

A decryption exponent for an RSA public key  $(N, e)$  is an integer  $d$  with the property that...

(b) Let  $N = 38749709$ . Eve's magic box tells her that the encryption exponent  $e = 10988423$  has decryption exponent  $d = 16784693$  and that the encryption exponent  $e = 25910155$  has decryption exponent  $d = 11514115$ . Use this information to factor  $N$ .

(c) Let  $N = 225022969$ . Eve's magic box tells her the following three encryption/decryption pairs for  $N$ :  $(70583995, 4911157)$ ,  $(173111957, 7346999)$ ,  $(180311381, 29597249)$ . Use this information to factor  $N$ .

(d) Let  $N = 1291233941$ . Eve's magic box tells her the following three encryption/decryption pairs for  $N$ :  $(1103927639, 76923209)$ ,  $(1022313977, 106791263)$ ,  $(387632407, 7764043)$ . Use this information to factor  $N$ .

### Exercises 3.12

Alice decides to use RSA with the public key  $N = 1889570071$ . In order to guard against transmission errors, Alice has Bob encrypt his message twice, once using the encryption exponent  $e_1 = 1021763679$  and once using the encryption exponent  $e_2 = 519424709$ . Eve intercepts the two encrypted messages  $c_1=1244183534$  and  $c_2=732959706$ . Assuming that Eve also knows  $N$  and the two encryption exponents  $e_1$  and  $e_2$ , ... help Eve recover Bob's plaintext without finding a factorization of  $N$ .

### Exercises 3.14

Use the Miller–Rabin test on each of the following numbers. ...

(a)  $n = 1105$ .

(b)  $n = 294409$ .

(c)  $n = 294439$ .

(d)  $n = 118901509$ .

(e)  $n = 118901521$ .

(f)  $n = 118901527$ .

(g)  $n = 118915387$ .

### Exercises 3.21

Use Pollard's  $p-1$  method to factor each of the following numbers.

(a) 1739 (b) 220459 (c) 48356747

### Exercises 3.27(c)

The following is a list of 20 randomly chosen numbers between 1 and 1000, sorted from smallest to largest. Which of these numbers are 10-power-smooth? Which of them are 10-smooth?

{84, 141, 171, 208, 224, 318, 325, 366, 378, 390, 420, 440, 504, 530, 707, 726, 758, 765, 792, 817}

### Exercises 5.18

Use the Elliptic Curve Factorization Algorithm to factor each of the numbers  $N$  using the given

elliptic curve E and point P.

(a)  $N=589$ ,  $E: Y^2 = X^3 + 4X + 9$ ,  $P=(2,5)$ .

(b)  $N=26167$ ,  $E: Y^2 = X^3 + 4X + 128$ ,  $P=(2,12)$ .

(c)  $N=1386493$ ,  $E: Y^2 = X^3 + 3X - 3$ ,  $P=(1,1)$ .

(d)  $N=28102844557$ ,  $E: Y^2 = X^3 + 18X - 453$ ,  $P=(7,4)$ .