Many of the following exercises require a computer and PARI/GP. Here are the links to some programs which you can paste into PARI/GP and use.

Trial division: http://www.mat.uniroma2.it/ĝeatti/ERBIL2024/naif.txt Pollard ρ: http://www.mat.uniroma2.it/ĝeatti/ERBIL2024/pollard.txt Pollard p - 1: http://www.mat.uniroma2.it/ĝeatti/ERBIL2024/pminus.txt Miller-Rabin test: http://www.mat.uniroma2.it/ĝeatti/ERBIL2024/mrtest.txt Naif versus Pollard: http://www.mat.uniroma2.it/ĝeatti/ERBIL2024/PollardVersusNaif.txt

## Trial division factoring algorithm

1. Use the "trial division" algorithm to factor

n = 1006988900088310890327954977.

2. On the base of the estimates of the previous exercise, exhibit integers whose factorisation with trial division requires one month, one year, 10 years.

## Pollard $\rho$

- 3. Let n be a composite integer of 200 digits. After 10000 iterations the Pollard  $\rho$  algorithm did not find any factors. What can we conclude about the size of the prime factors?
- 4. Let n be a composite integer of 200 digits, and let B = 10000. After how many unsuccessful iterations of the Pollard  $\rho$  algorithm we can conclude that probably n has no factor  $\leq B$ ?
- 5. Factor completely the integers (you may have to apply pollard several times):
  - $n_1 = 1077471755063687$
  - $n_2 = 1068764300000395442791$
  - $n_3 = 100058768830000000000000000000000008574912503519308271870011$