## 1. Group theory: review exercises

1. Compute explicitly the maps

$$F: \mathbf{Z}_{15} \to \mathbf{Z}_3, \ \bar{x} \mapsto \bar{x} \mod 3, \qquad G: \mathbf{Z}_{15} \to \mathbf{Z}_5, \ \bar{x} \mapsto \bar{x} \mod 5$$

- 2. Let  $\varphi$  denote the Euler  $\varphi$ -function. Compute  $\varphi(2024)$ .
- 3. Compute  $\varphi(15)$  and determine all elements of  $\mathbf{Z}_{15}^*$ .
- 4. State Lagrange's Theorem for the following groups

$$\mathbf{Z}_{11}, \quad \mathbf{Z}_{11}^*, \quad \mathbf{Z}_{12}, \quad \mathbf{Z}_{12}^*, \quad \mathbf{Z}_{100}^*, \quad \mathbf{Z}_{11} \times \mathbf{Z}_{17}, \quad \mathbf{Z}_{7^3}^*$$

5. Find the orders of all elements of  ${\bf Z}_7^*.$  \*

<sup>\*</sup> Let  $(G, \cdot)$  be a multiplicative group. The order of an element x in G is the smallest integer k such that  $x^k = e$ .