1. Calcolare il rango delle seguenti matrici

$$\begin{pmatrix} 1 & -2 & -3 \\ 3 & 0 & 3 \\ 7 & -3 & 1 \\ 2 & 1 & 4 \\ 1 & -2 & -3 \end{pmatrix} \quad \begin{pmatrix} 1 & 3 & 7 & 2 & 1 \\ -2 & 0 & -3 & 1 & -2 \\ -3 & 3 & 1 & 4 & -3 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 2 \\ -1 & 2 & -3 \\ 1 & 3 & 1 \\ 1 & -1 & -1 \end{pmatrix}.$$

2. Calcolare il determinante delle seguenti matrici

$$\begin{pmatrix} 3 & 5 \\ 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 8 & 15 \\ 7 & 13 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & -1 & 1 \\ 2 & -1 & 3 & 1 \end{pmatrix}.$$

3. Siano date le matrici

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 5 & 0 \\ 3 & 0 & -3 \\ 0 & -5 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

(i) Calcolare $\det A$, $\det B$, $\det(AB)$, $\det(BA)$, $\det A^{-1}$, $\det(A+B)$;
(ii) Calcolare $\det C$, C^{-1}, C^{-2}.

4. Siano date le matrici $A = \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}$ e $B = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$. Calcolare A^{-1}, B^{-1}, $(AB)^{-1}$, $(2BA)^{-1}$, ${}^tA^{-1}$.

5. Sia $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ a sia P il parallelogramma costruito sui vettori $\begin{pmatrix} a \\ c \end{pmatrix}$ e $\begin{pmatrix} b \\ d \end{pmatrix}$. Allora

$$Area(P) = |\det A|.$$

Disegnare i parallelogrammi costruiti sulle seguenti coppie di vettori e calcolarne l'area

$$\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}\} \quad \{\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}\}, \quad \{\begin{pmatrix} 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}\}, \quad \{\begin{pmatrix} 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \end{pmatrix}\}.$$

6. Sia $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ a sia P il parallelepipedo costruito sui vettori $\begin{pmatrix} a \\ d \\ g \end{pmatrix}$, $\begin{pmatrix} b \\ e \\ h \end{pmatrix}$ e $\begin{pmatrix} c \\ f \\ i \end{pmatrix}$. Allora

$$Vol(P) = |\det A|.$$

Calcolare i volumi dei parallelepipedi costruiti sui vettori

$$\{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}\} \quad \{\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 9 \end{pmatrix}\}, \quad \{\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\}, \quad \{\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\}.$$