Let $R$ be a 1-dimensional integral domain, let $h \in R \setminus \{0\}$ be prime, and let $\mathcal{HA}$ be the category of torsionless Hopf algebras over $R$. We call $H \in \mathcal{HA}$ a “quantized function algebra” (=QFA), resp. “quantized universal enveloping algebras” (=QrUEA), at $h$ if it is commutative, resp. cocommutative, modulo $h$ (plus technical conditions), i.e. if $H/hH = F[G]$, resp. $H/hH = U(g)$, for some Poisson group $G$, resp. Lie bialgebra $g$.

We establish an “inner” Galois’ correspondence on $\mathcal{HA}$, via the definition of two endo-functors, $(\ )^\vee$ and $(\ )'$, of $\mathcal{HA}$ such that: (a) the image of $(\ )^\vee$, resp. of $(\ )'$, is the full subcategory of all QrUEAs, resp. QFAs, at $h$; (b) the restrictions $(\ )^\vee|_{QFAs}$ and $(\ )'|_{QrUEAs}$ yield equivalences inverse to each other; (c) starting from a QFA over a Poisson group $G$, resp. from a QrUEA over a Lie bialgebra $g$, the functor $(\ )^\vee$, resp. $(\ )'$, gives a QrUEA, resp. a QFA, over the dual Lie bialgebra, resp. the dual Poisson group. In particular, (a) provides a machine to produce quantum groups of both types (either QFAs or QrUEAs), (b) gives a characterization of them among objects of $\mathcal{HA}$, and (c) gives a “global” version of the so-called “quantum duality principle” (after Drinfeld’s formulation, cf. [Dr]). A main application is to Hopf algebras of the form $k[h] \otimes_k H$ where $H$ is a Hopf algebra over the field $k$: this yields quantum groups, hence “classical” geometrical symmetries of Poisson type (Poisson groups or Lie bialgebras, via specialization) associated to the “generalized” symmetry encoded by $H$.

These notes draw a sketch of the theoretical construction leading to the “global quantum duality principle”. Besides, the principle itself, and in particular the above mentioned application, is illustrated by means of several examples: group algebras, the standard quantization of the Kostant-Kirillov structure on any Lie algebra, the quantum semisimple groups, the quantum Euclidean group and the quantum Heisenberg group.

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REFERENCES


