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"The global quantum duality principle: a survey through examples"

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ABSTRACT

Let R be a 1-dimensional integral domain, let $\hbar \in R \setminus \{0\}$ be prime, and let $\mathcal{HA}$ be the category of torsionless Hopf algebras over R. We call $H \in \mathcal{HA}$ a *"quantized function algebra" (=QFA)*, resp. *"quantized universal enveloping algebras" (=QrUEA)*, at $\hbar$ if it is commutative, resp. cocommutative, modulo $\hbar$ (plus technical conditions), i.e. if $H \big/ \hbar H = F[G]$, resp. $H \big/ \hbar H = U(\mathfrak{g})$, for some Poisson group G, resp. Lie bialgebra $\mathfrak{g}$.

We establish an "inner" Galois' correspondence on $\mathcal{HA}$, via the definition of two endofunctors, $(\ )^\vee$ and $(\ )'$, of $\mathcal{HA}$ such that: *(a)* the image of $(\ )^\vee$, resp. of $(\ )'$, is the full subcategory of all QrUEAs, resp. QFAs, at $\hbar$; *(b)* the restrictions $(\ )^\vee\big|_{\mathrm{QFAs}}$ and $(\ )'\big|_{\mathrm{QrUEAs}}$ yield equivalences inverse to each other; *(c)* starting from a QFA over a Poisson group G, resp. from a QrUEA over a Lie bialgebra $\mathfrak{g}$, the functor $(\ )^\vee$, resp. $(\ )'$, gives a QrUEA, resp. a QFA, over the dual Lie bialgebra, resp. the dual Poisson group. In particular, *(a)* provides a machine to produce quantum groups of both types (either QFAs or QrUEAs), *(b)* gives a characterization of them among objects of $\mathcal{HA}$, and *(c)* gives a "global" version of the so-called "quantum duality principle" (after Drinfeld's formulation, cf. [Dr]). A main application is to Hopf algebras of the form $\Bbbk[\hbar] \otimes_{\Bbbk} H$ where H is a Hopf algebra over the field $\Bbbk$: this yields quantum groups, hence "classical" geometrical symmetries of Poisson type (Poisson groups or Lie bialgebras, via specialization) associated to the "generalized" symmetry encoded by H.

These notes draw a sketch of the theoretical construction leading to the "global quantum duality principle". Besides, the principle itself, and in particular the above mentioned application, is illustrated by means of several examples: group algebras, the standard quantization of the Kostant-Kirillov structure on any Lie algebra, the quantum semisimple groups, the quantum Euclidean group and the quantum Heisenberg group.

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