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"Real forms of complex Lie superalgebras and supergroups"

ABSTRACT

We investigate the notion of *real form* of complex Lie superalgebras and supergroups, both in the *standard* and *graded* version. Our functorial approach allows most naturally to go from the superalgebra to the supergroup and retrieve the real forms as fixed points, as in the ordinary setting. We also introduce a more general notion of compact real form for Lie superalgebras and supergroups, and we prove some existence results for Lie superalgebras that are simple contragredient and their associated connected simply connected supergroups.

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